Mass and Energy Analysis of Control Volumes

- 1. Conservation of Mass
- 2. Flow Work and the Energy of a Flowing Fluid
 - Total Energy of a Flowing Fluid
 - Energy Transport by Mass
- 3. Energy Analysis of Steady-Flow Systems
- 4. Some Steady-Flow Engineering Devices
 - Nozzles and Diffusers / Turbines and Compressor
 - Throttling Valves and Mixing Chambers
- 5. Energy Analysis of Unsteady-Flow Processes

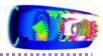




²⁰²² Spring semester



Objectives



- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.
- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy.
- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers.
- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes.







Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

Closed systems: The mass of the system remain constant during a process.

Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

Mass *m* and energy *E* can be converted to each other according to

 $E = mc^2$

c the speed of light in a vacuum $c = 2.9979 \times 10^8$ m/s.

The mass change due to energy change is negligible.

$$\begin{array}{c} 2 \text{ kg} \\ H_2 \end{array} + \begin{array}{c} 16 \text{ kg} \\ O_2 \end{array} \longrightarrow \begin{array}{c} 18 \text{ kg} \\ H_2 O \end{array}$$

FIGURE 5–1

Mass is conserved even during chemical reactions.

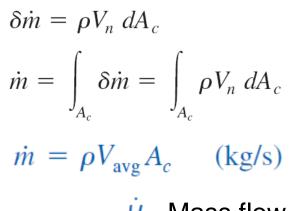


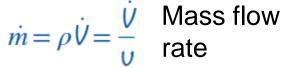


5-1. CONSERVATION OF MASS



Mass and Volume Flow Rates





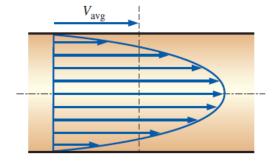


FIGURE 5–3

The average velocity V_{avg} is defined as the average speed through a cross section.

$$V_{\rm avg} = \frac{1}{A_c} \int_{A_c} V_n dA_c$$

Definition of average velocity

Volume flow rate

$$\dot{V} = \int_{A_c} V_n dA_c = V_{\text{avg}} A_c = V A_c \qquad (\text{m}^3/\text{s})$$

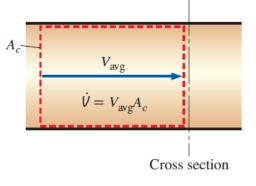


FIGURE 5-4

The volume flow rate is the volume of fluid flowing through a cross section per unit time.







Conservation of Mass Principle

 $\begin{pmatrix} \text{Total mass entering} \\ \text{the CV during } \Delta t \end{pmatrix} - \begin{pmatrix} \text{Total mass leaving} \\ \text{the CV during } \Delta t \end{pmatrix} = \begin{pmatrix} \text{Net change of mass} \\ \text{within the CV during } \Delta t \end{pmatrix}$



FIGURE 5–5

Conservation of mass principle for an ordinary bathtub.

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt .

 $m_{\rm in} - m_{\rm out} = \Delta m_{\rm CV}$ (kg)

$$\Delta m_{\rm CV} = m_{\rm final} - m_{\rm initial}$$

 $\dot{m}_{\rm in} - \dot{m}_{\rm out} = dm_{\rm CV}/dt$ (kg/s)

These equations are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.





5-1. CONSERVATION OF MASS

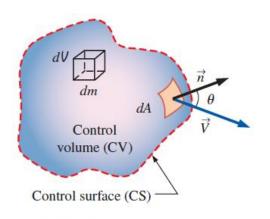


FIGURE 5-6

The differential control volume dV and the differential control surface dA used in the derivation of the conservation of mass relation. Total mass within the CV:

$$m_{\rm cv} = \int_{\rm CV} \rho \ dV$$

Rate of change of mass within the CV: $\frac{dm_{CV}}{dt} = \frac{d}{dt} \int_{CV} \rho \, dV$

Normal component of velocity: $V_n = V \cos \theta = \vec{V} \cdot \vec{n}$

Differential mass flow rate: $\delta \dot{m} = \rho V_n dA = \rho (V \cos \theta) dA = \rho (\vec{V} \cdot \vec{n}) dA$

Net mass flow rate:
$$\dot{m}_{net} = \int_{CS} \delta \dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA$$

$$\frac{d}{dt} \int_{\rm CV} \rho \ dV + \int_{\rm CS} \rho(\vec{V} \cdot \vec{n}) dA = 0$$

The time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

$$\frac{d}{dt} \int_{CV} \rho \, dV + \sum_{\text{out}} \rho |V_n| A - \sum_{\text{in}} \rho |V_n| A = 0$$

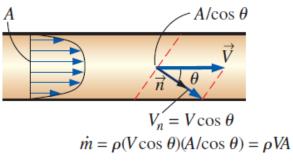
$$\frac{d}{dt} \int_{CV} \rho \, dV = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m} \quad \text{or} \quad \frac{dm_{CV}}{dt} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m} \quad \text{General conservation of mass}$$
in rate form



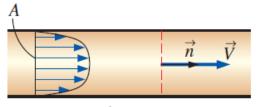


5-1. CONSERVATION OF MASS





(a) Control surface at an angle to the flow



 $\dot{m} = \rho VA$

(b) Control surface normal to the flow

FIGURE 5–7

A control surface should always be selected *normal to the flow* at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.







Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

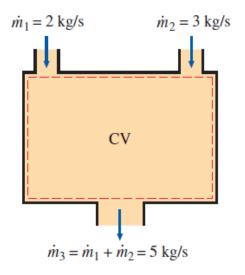


FIGURE 5–8

Conservation of mass principle for a two-inlet-one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, the mass flow rate.

 $\sum \dot{m} = \sum \dot{m}$ (kg/s) Multiple inlets and exits

 $\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$ Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).







Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.

 $\sum \dot{V} = \sum V$

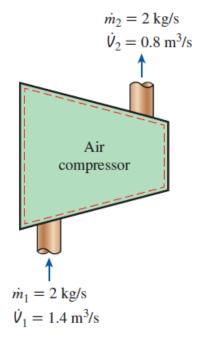


FIGURE 5-9

During a steady-flow process, volume flow rates are not necessarily conserved, although mass flow rates are. (m³/s) Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

There is no such thing as a "conservation of volume" principle.

For steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.







Flow work, or flow energy: The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

$$F = PA$$
 $W_{\text{flow}} = FL = PAL = PV$ (kJ)

 $w_{\rm flow} = P U$

(kJ/kg)

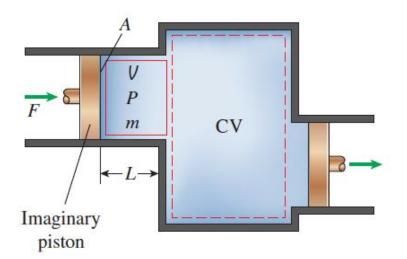


FIGURE 5-12

Schematic for flow work.

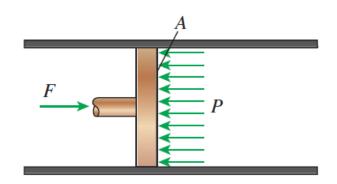


FIGURE 5-13

In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.







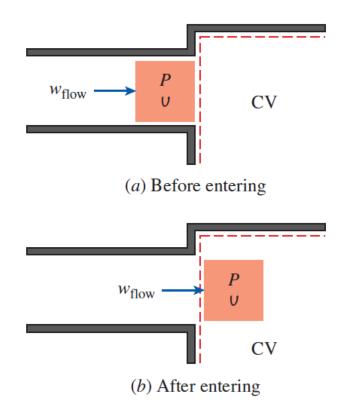


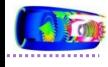
FIGURE 5-14

Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to P v.





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Total Energy of a Flowing Fluid

$$e = u + ke + pe = u + \frac{V^2}{2} + gz \quad (kJ/kg) \quad (5-25)$$

$$\theta = Pv + e = Pv + (u + ke + pe) \quad (5-26)$$

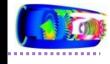
$$h = u + Pv \quad (kJ/kg)$$

$$\theta = h + ke + pe = h + \frac{V^2}{2} + gz \quad (kJ/kg) \quad (5-27)$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.







$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz$$
 (kJ/kg) (5-27)

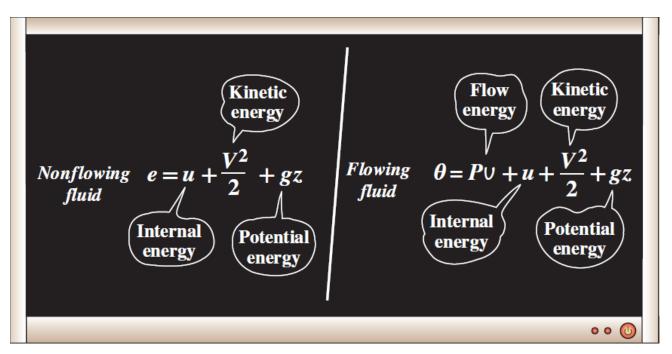
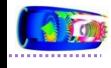


FIGURE 5-15

The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.







Energy Transport by Mass

Amount of energy transport:
$$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right)$$
 (kJ)

Rate of energy transport:
$$\dot{E}_{mass} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$
 (kW)

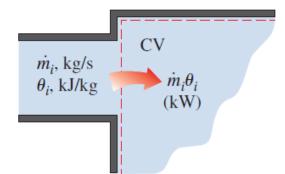


FIGURE 5–16

The product $\dot{m}_i \theta_i$ is the energy transported into control volume by mass per unit time.

When the kinetic and potential energies of a fluid stream are negligible

$$E_{\rm mass} = mh \frac{\dot{E}_{\rm mass}}{(5-27)} = \dot{m}h$$

When the properties of the mass at each inlet or exit change with time as well as over the cross section

$$E_{\rm in, mass} = \int_{m_i} \theta_i \, \delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + g z_i \right) \delta m_i$$







Steady-flow process: A process during which a fluid flows through a control volume steadily.



FIGURE 5–18

Many engineering systems such as power plants operate under steady conditions.

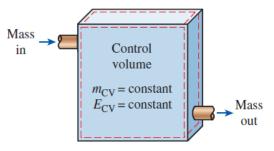


FIGURE 5–19

Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

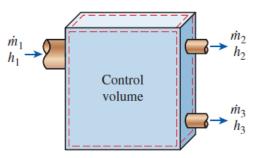


FIGURE 5–20

Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).







ennelson Not.

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Mass and Energy balances for a steady-flow process

 $\sum \dot{m} = \sum \dot{m}$ (kg/s) Mass balance $\dot{m}_1 = \dot{m}_2 \longrightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$ $\dot{E}_{\rm in} - \dot{E}_{\rm out} = dE_{\rm system}/dt$ = 0Rate of net energy transfer in Rate of change in internal, kinetic, by heat, work, and mass potential, etc., energies $E_{\rm in}$ $E_{\rm out}$ (kW)Energy balance Rate of net energy transfer in Rate of net energy transfer out by heat, work, and mass by heat, work, and mass $\dot{Q}_{\rm in} + \dot{W}_{\rm in} + \sum \dot{m}\theta = \dot{Q}_{\rm out} + \dot{W}_{\rm out} + \sum \dot{m}\theta$ $\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m} \left(h + \frac{V^2}{2} + gz \right) = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m} \left(h + \frac{V^2}{2} + gz \right)$ for each inlet for each exit





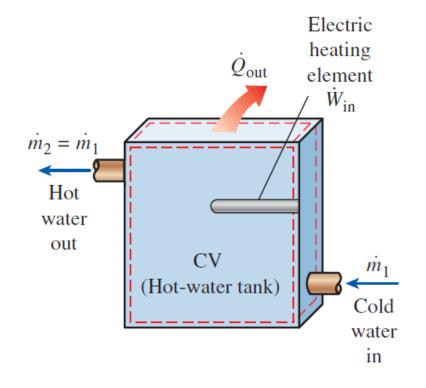


FIGURE 5–21

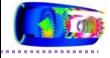
A water heater in steady operation.

SCNL Combridge

Conservation of energy principle:

The water stream experiences an increase in its total energy as it flows through the water heater that is equal to the electric energy supplied to the water minus the heat losses.





Energy balance relations with sign conventions (i.e., heat input and work output are positive)

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \dot{m} \left(h + \frac{V^2}{2} + gz \right)_{\text{for each exit}} - \sum_{\text{in}} \dot{m} \left(h + \frac{V^2}{2} + gz \right)_{\text{for each inlet}}$$
$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$
$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad q = \dot{Q}/\dot{m} \qquad w = \dot{W}/\dot{m}$$

When kinetic and potential energy changes are negligible

$$q - w = h_2 - h_1$$







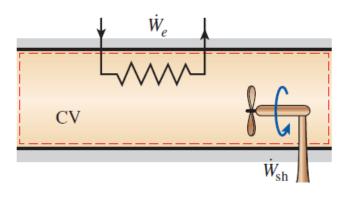


FIGURE 5–22

Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

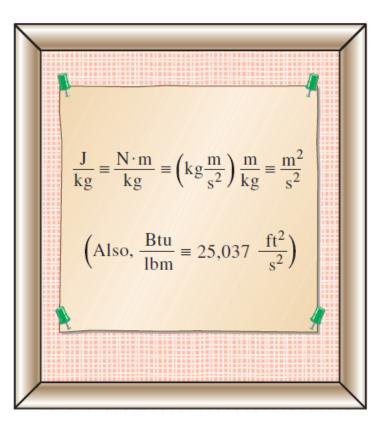


FIGURE 5–23 The units m²/s² and J/kg are equivalent.







0				
0		V_2		
	m/s	m/s	kJ/kg	
	0	45	1	
	50	67	1	
	100	110	1	
	200	205	1	
	500	502	1	
0				

FIGURE 5–24

At very high velocities, even small changes in velocities can cause significant changes in the kinetic energy of the fluid.





Many engineering devices operate essentially under the same conditions for long periods of time.

The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), operate nonstop for months before the system is shut down for maintenance.

Therefore, these devices can be conveniently analyzed as steady-flow devices.





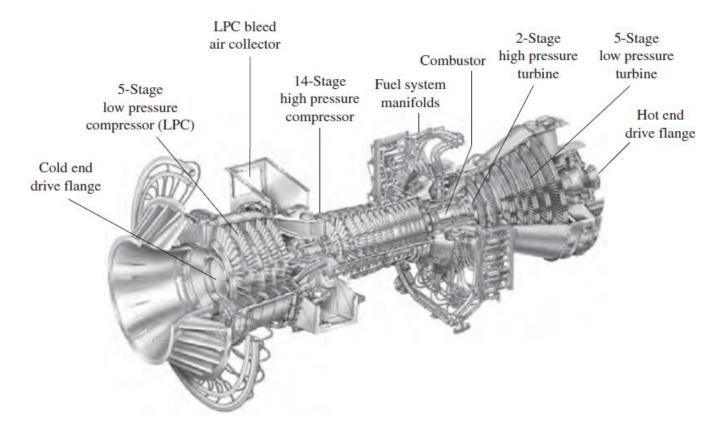


FIGURE 5-25

A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.





1) Nozzles and Diffusers

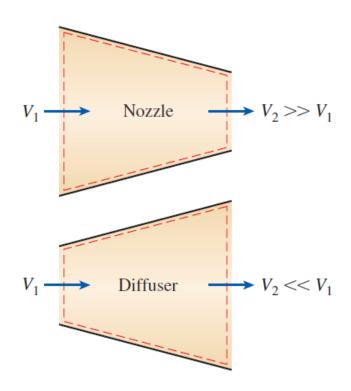


FIGURE 5–26

Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies. Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

Energy balance for a nozzle or diffuser

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$
(since $\dot{O} \cong 0$, $\dot{W} = 0$, and $\Delta pe \cong 0$





Deceleration of Air in a Diffuser



0 (steady)

0

$$\dot{E}_{\rm in} - \dot{E}_{\rm out} =$$

Rate of net energy transfer by heat, work, and mass

$$E_{\text{system}}/dt =$$

Rate of change in internal, kinetic, potential, etc., energies

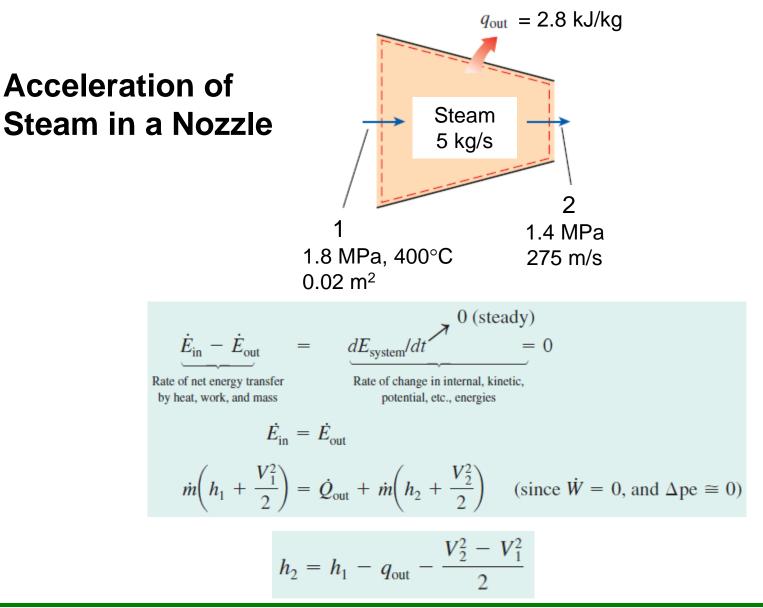
$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$
 (since $\dot{Q} \approx 0, \dot{W} =$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$



0, and $\Delta pe \cong 0$)







2) Turbines and Compressors



FIGURE 5–29 Turbine blades attached to the turbine shaft.

Turbine drives the electric generator In steam, gas, or hydroelectric power plants.

As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

Compressors, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

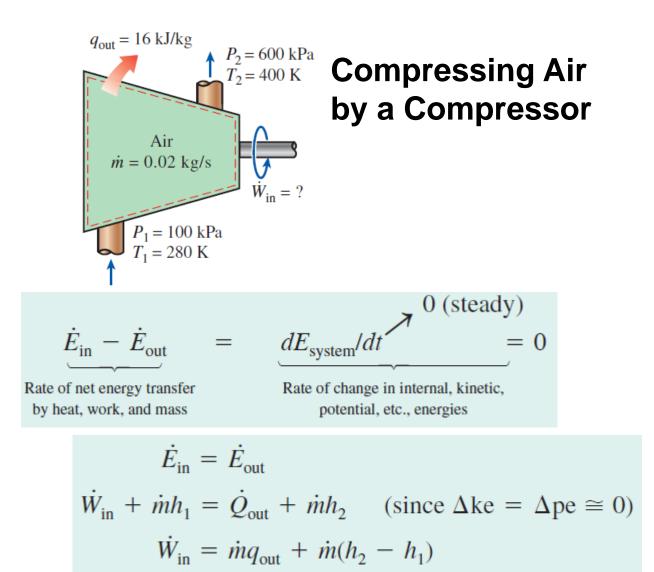
A *fan* increases the pressure of a gas slightly and is mainly used to mobilize a gas.

A *compressor* is capable of compressing the gas to very high pressures.

Pumps work very much like compressors except that they handle liquids instead of gases.

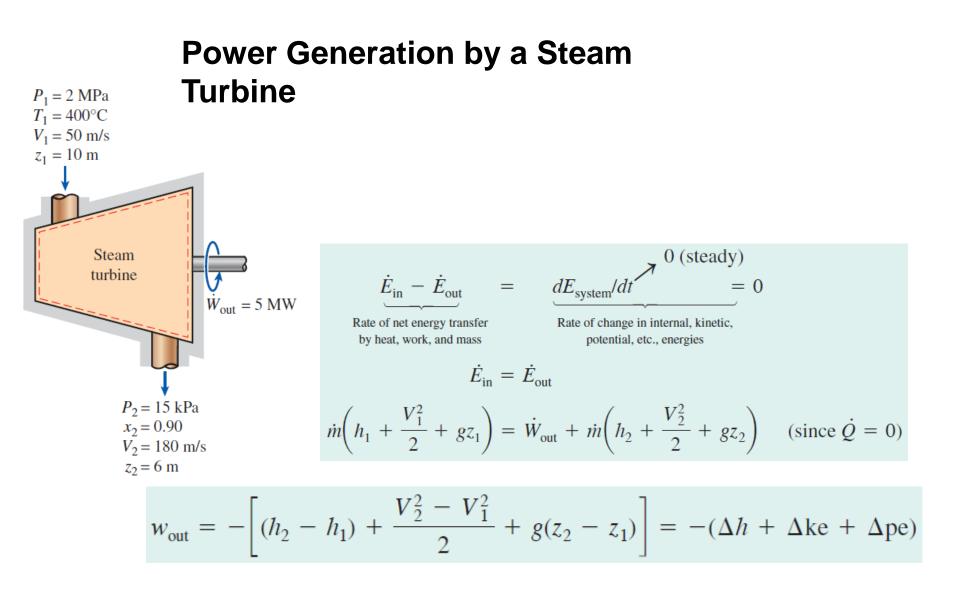
















3) Throttling valves

Throttling valves are *any kind of flowrestricting devices* that cause a significant pressure drop in the fluid.

What is the difference between a turbine and a throttling valve?

The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and airconditioning applications.

Energy balance

$$h_2 \cong h_1$$
 (kJ/kg)

$$u_1 + P_1 U_1 = u_2 + P_2 U_2$$

Internal energy + Flow energy = Constant



(a) An adjustable valve



(b) A porous plug

(c) A capillary tube

FIGURE 5–32 Throttling valves are devices that cause large pressure drops in the fluid.





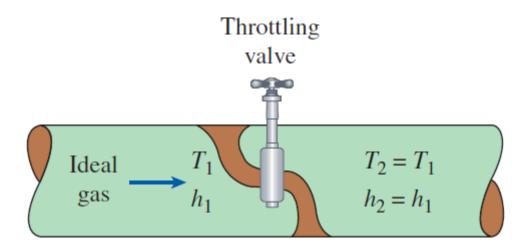
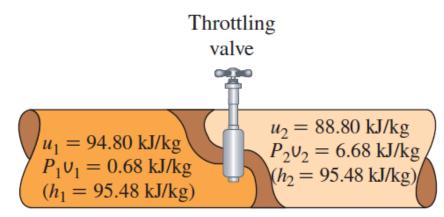


FIGURE 5–33

The temperature of an ideal gas does not change during a throttling (h = constant) process since h = h(T).







Expansion of Refrigerant-134a in a Refrigerator

FIGURE 5-34

During a throttling process, the enthalpy (flow energy + internal energy) of a fluid remains constant. But internal and flow energies may be converted to each other.

 $h_2 \cong h_1$ (kJ/kg)

$$u_1 + P_1 U_1 = u_2 + P_2 U_2$$

Internal energy + Flow energy = Constant





4) Mixing chambers

In engineering applications, the section where the mixing process takes place is commonly referred to as a **mixing chamber**.

The mixing chamber does not have to be a distinct "chamber." An ordinary T-elbow or a Y-elbow in a shower, for example, serves as the mixing chamber for the cold- and hot-water streams.

The conservation of mass principle for a mixing chamber requires that the sum of the incoming mass flow rates equal the mass flow rate of the outgoing mixture.

The conservation of energy equation is analogous to the conservation of mass equation.

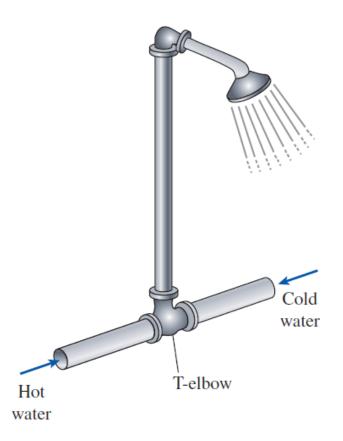


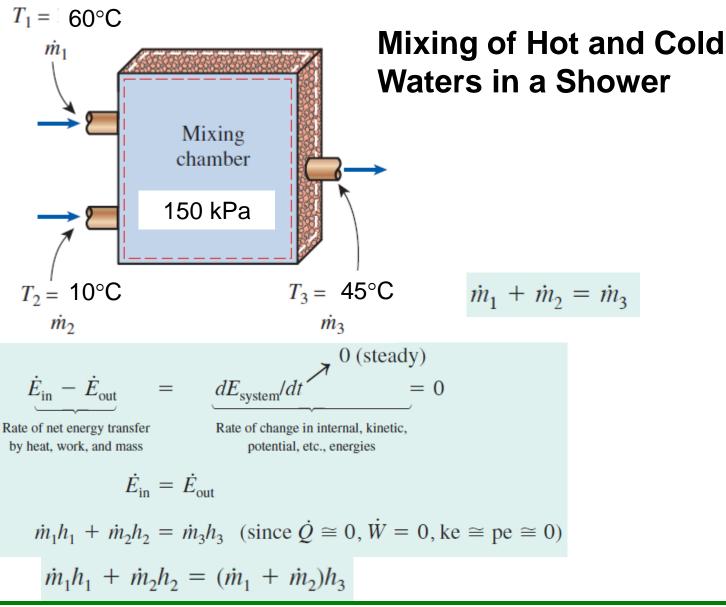
FIGURE 5–35

The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.



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5) Heat exchangers

Heat exchangers are devices where two moving fluid streams exchange heat without mixing.

Heat exchangers are widely used in various industries, and they come in various designs.

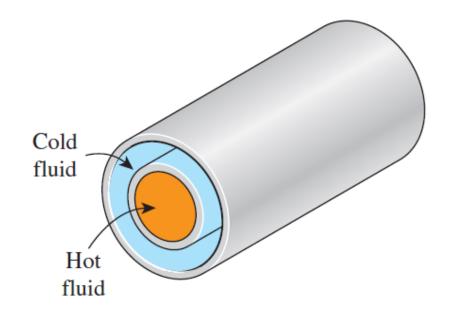


FIGURE 5-38

A heat exchanger can be as simple as two concentric pipes.





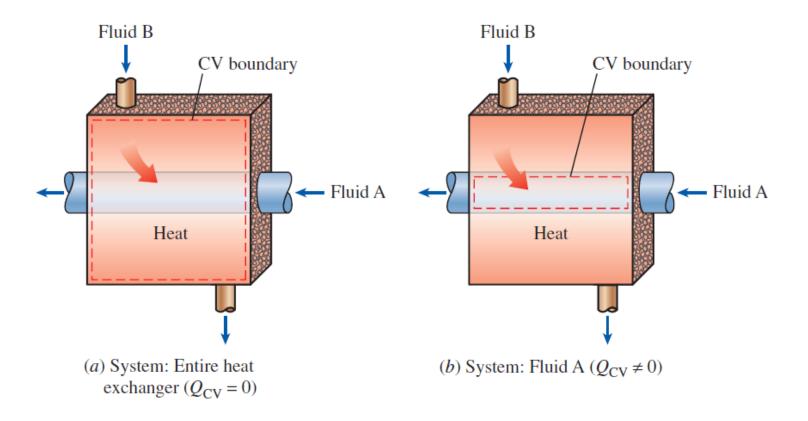
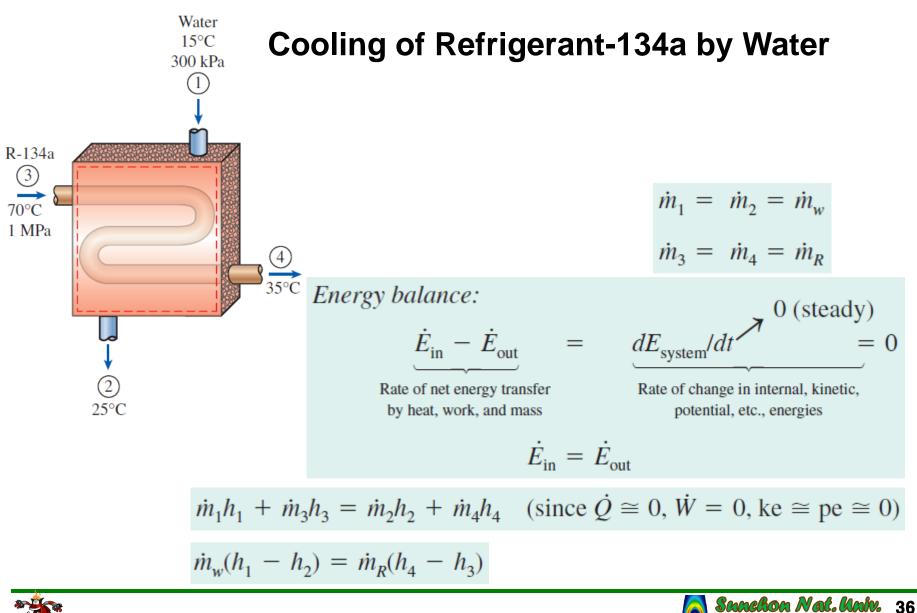


FIGURE 5–39

The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.

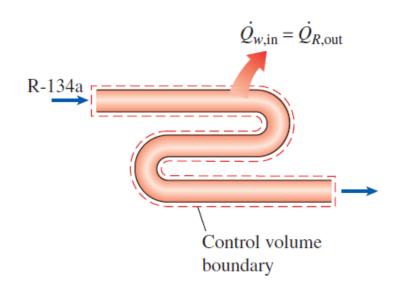








5-4. SOME STEADY-FLOW ENGINEERING DEVICES



$$\dot{E}_{\rm in} - \dot{E}_{\rm out}$$

Rate of net energy transfer by heat, work, and mass

$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$

_

$$\frac{dE_{\text{system}}/dt}{dE_{\text{system}}/dt} = 0$$

Rate of change in internal, kinetic, potential, etc., energies

$$\dot{Q}_{w,\,\mathrm{in}} + \dot{m}_w h_1 = \dot{m}_w h_2$$

$$\dot{Q}_{w,\mathrm{in}} = \dot{m}_w (h_2 - h_1)$$

FIGURE 5-41

In a heat exchanger, the heat transfer depends on the choice of the control volume.





6) Pipe and duct flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications.

Flow through a pipe or a duct usually satisfies the steady-flow conditions.

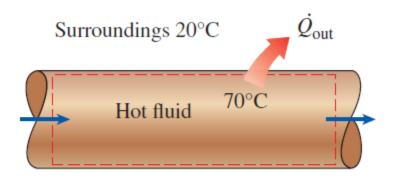


FIGURE 5-42

Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

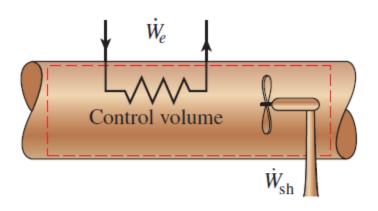


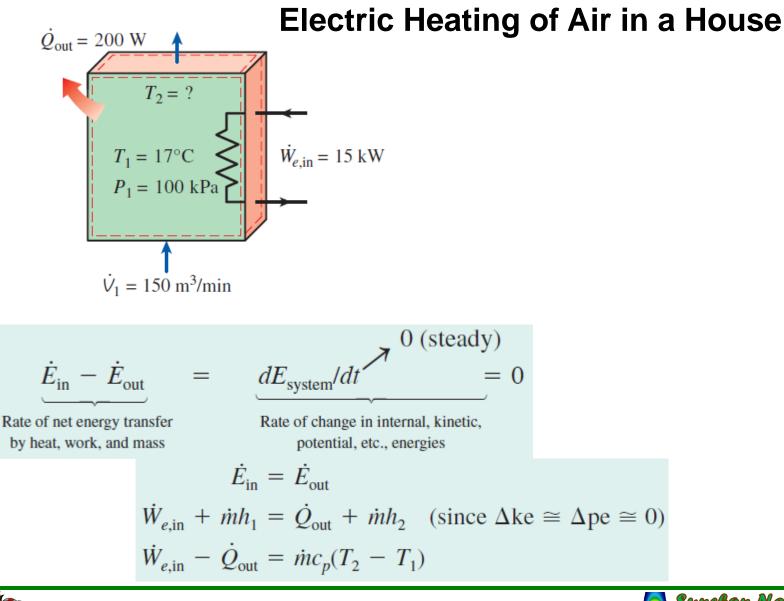
FIGURE 5-43

Pipe or duct flow may involve more than one form of work at the same time.





5-4. SOME STEADY-FLOW ENGINEERING DEVICES







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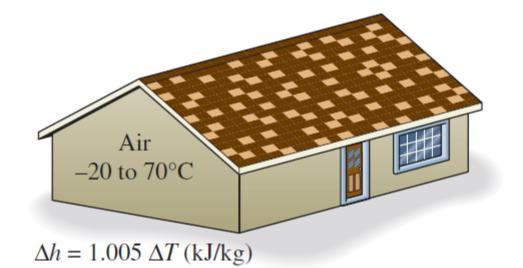


FIGURE 5-45

The error involved in $\Delta h = c_p \Delta T$, where $c_p = 1.005 \text{ kJ/kg} \cdot ^{\circ}\text{C}$, is less than 0.5 percent for air in the temperature range -20 to 70°C .







Many processes of interest, involve *changes* within the control volume with time.

Such processes are called *unsteady-flow,* or *transient-flow,* processes.

Most unsteady-flow processes can be represented reasonably well by the *uniform-flow process*.

Uniform-flow process: The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.







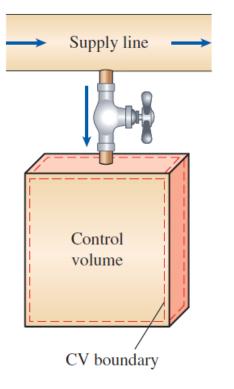


FIGURE 5-46

Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.

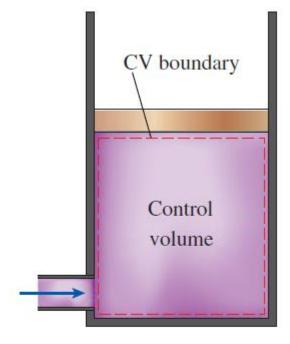


FIGURE 5-47

The shape and size of a control volume may change during an unsteady-flow process.







$$m_{\rm in} - m_{\rm out} = \Delta m_{\rm system}$$
 (kg)

$$\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$$
$$m_i - m_e = (m_2 - m_1)_{\text{CV}}$$

Energy balance

$$E_{\rm in} - E_{\rm out} = \Delta E_{\rm system}$$
 (kJ)

Net energy transfer by heat, work, and mass Change in internal, kinetic, potential, etc., energies

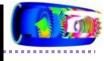
$$\left(Q_{\rm in} + W_{\rm in} + \sum_{\rm in} m\theta\right) - \left(Q_{\rm out} + W_{\rm out} + \sum_{\rm out} m\theta\right) = (m_2 e_2 - m_1 e_1)_{\rm system}$$

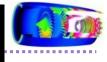
$$\theta = h + \text{ke} + \text{pe}$$
 $e = u + \text{ke} + \text{pe}$

$$Q - W = \sum_{\text{out}} mh - \sum_{\text{in}} mh + (m_2 u_2 - m_1 u_1)_{\text{system}}$$
$$Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}} \quad W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$$









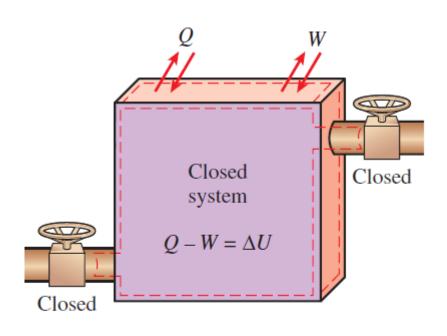


FIGURE 5-48

The energy equation of a uniformflow system reduces to that of a closed system when all the inlets and exits are closed.

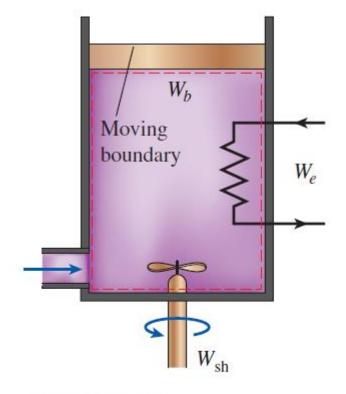


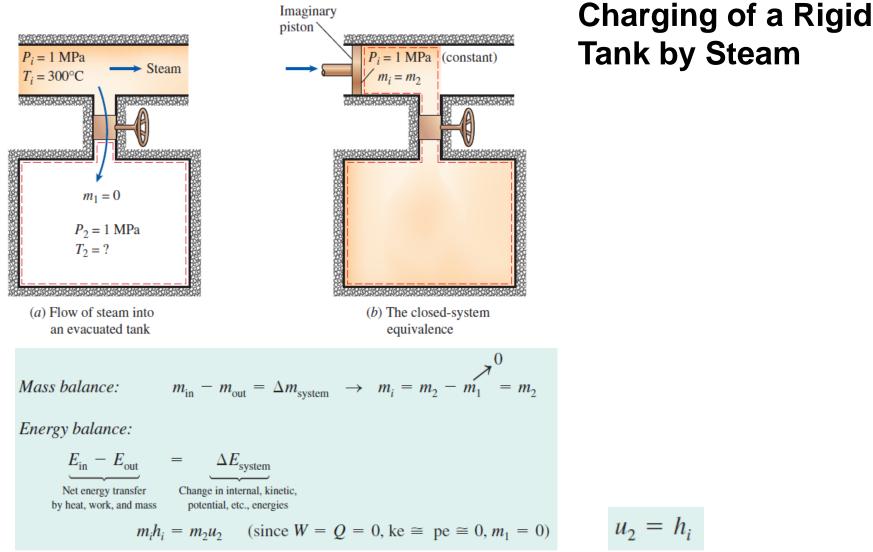
FIGURE 5-49

A uniform-flow system may involve electrical, shaft, and boundary work all at once.

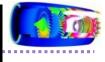












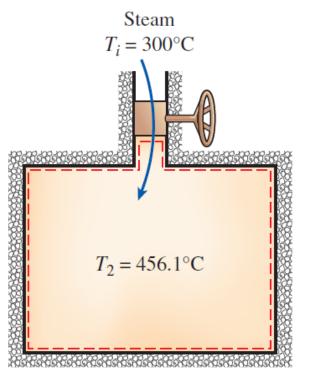


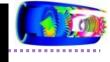
FIGURE 5–51

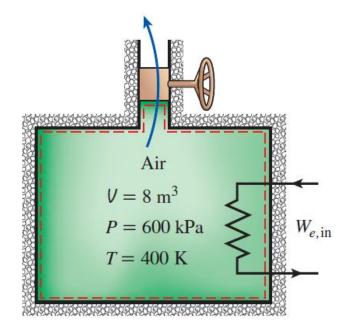
The temperature of steam rises from 300 to 456.1°C as it enters a tank as a result of flow energy being converted to internal energy.





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Discharge of Heated Air at Constant Temperature

Mass balance: $m_{\rm in} - m_{\rm out}$

by

$$-m_{\rm out} = \Delta m_{\rm system} \rightarrow m_e = m_1 - m_2$$

Energy balance: E_{in}

$$E_{out} - E_{out}$$

 $\Delta E_{\mathrm{system}}$

Change in internal, kinetic, potential, etc., energies

$$W_{e,\text{in}} - m_e h_e = m_2 u_2 - m_1 u_1 \text{ (since } Q \cong \text{ ke} \cong \text{ pe} \cong 0)$$







Conservation of mass

- Flow work and the energy of a flowing fluid
- Energy analysis of steady-flow systems
- Some steady-flow engineering devices
- Energy analysis of unsteady-flow processes



