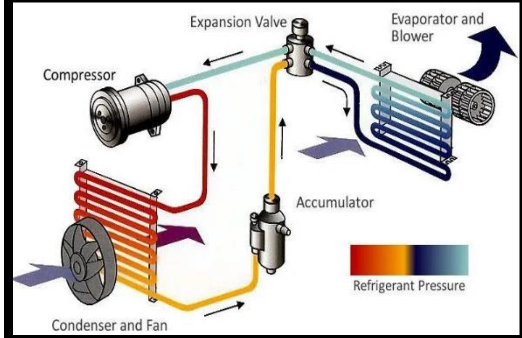
CHAPTER

11

Refrigeration Cycles

1. Refrigerators and Heat Pumps
2. The Reversed Carnot Cycle
3. The Ideal Vapor-Compression Refrigeration Cycle
4. Actual Vapor-Compression Refrigeration Cycle
5. Selecting The Right Refrigerant
6. Heat Pump Systems
7. Innovative Vapor-Compression Refrigeration Systems
8. Gas Refrigeration Cycles
9. Absorption Refrigeration Systems



2022 Fall semester

Objectives

- Introduce the concepts of refrigerators and heat pumps and the measure of their performance.
- Analyze the ideal vapor-compression refrigeration cycle.
- Analyze the actual vapor-compression refrigeration cycle.
- Perform second-law analysis of vapor-compression refrigeration cycle.
- Review the factors involved in selecting the right refrigerant for an application.
- Discuss the operation of refrigeration and heat pump systems.
- Evaluate the performance of innovative vapor-compression refrigeration systems.
- Analyze gas refrigeration systems.
- Introduce the concepts of absorption-refrigeration systems.

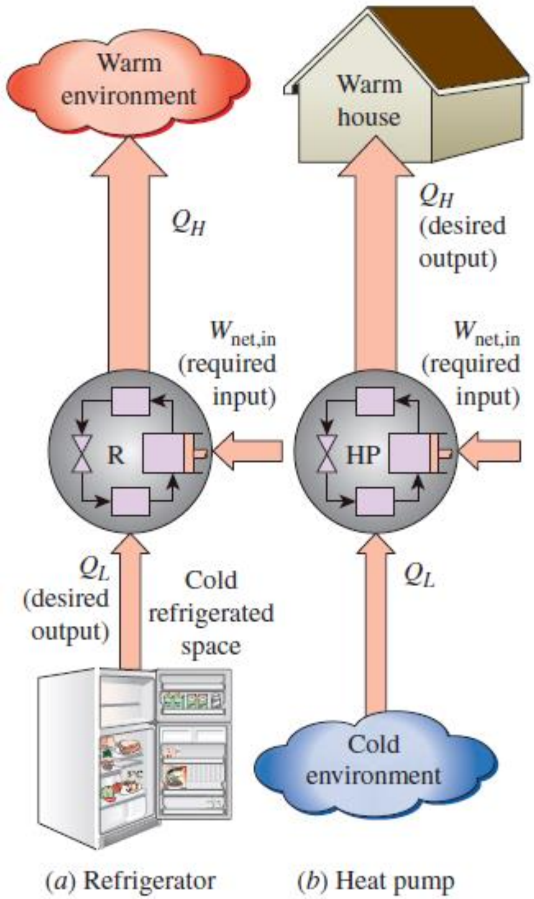


FIGURE 11–1
The objective of a refrigerator is to remove heat (Q_L) from the cold medium; the objective of a heat pump is to supply heat (Q_H) to a warm medium.

The transfer of heat from a low-temperature region to a high-temperature one requires special devices called **refrigerators**.

Another device that transfers heat from a low-temperature medium to a high-temperature one is the **heat pump**.

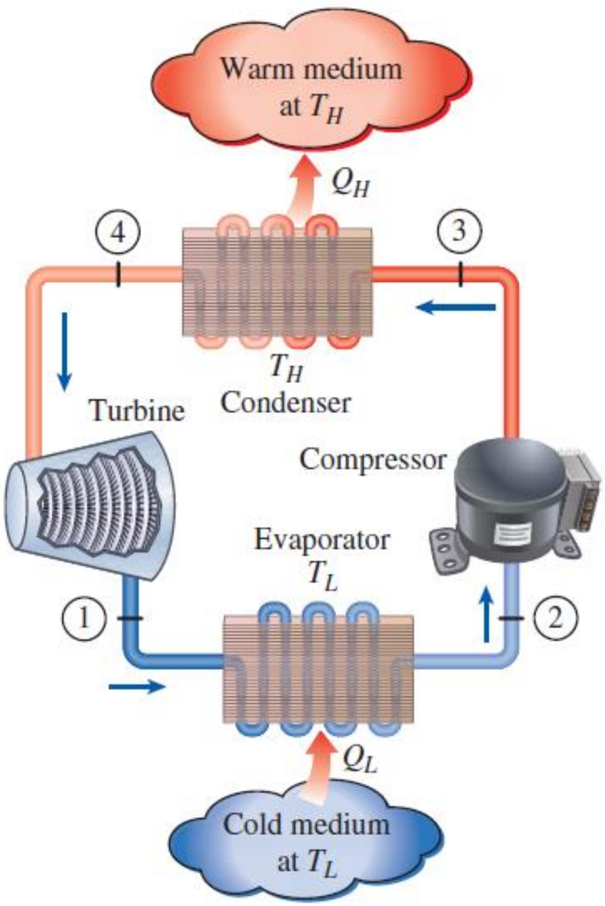
Refrigerators and heat pumps are essentially the same devices; they differ in their objectives only.

$$COP_R = \frac{\text{Desired output}}{\text{Required input}} = \frac{\text{Cooling effect}}{\text{Work input}} = \frac{Q_L}{W_{net,in}}$$

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_H}{W_{net,in}}$$

$$COP_{HP} = COP_R + 1$$ for fixed values of Q_L and Q_H

11-2. THE REVERSED CARNOT CYCLE



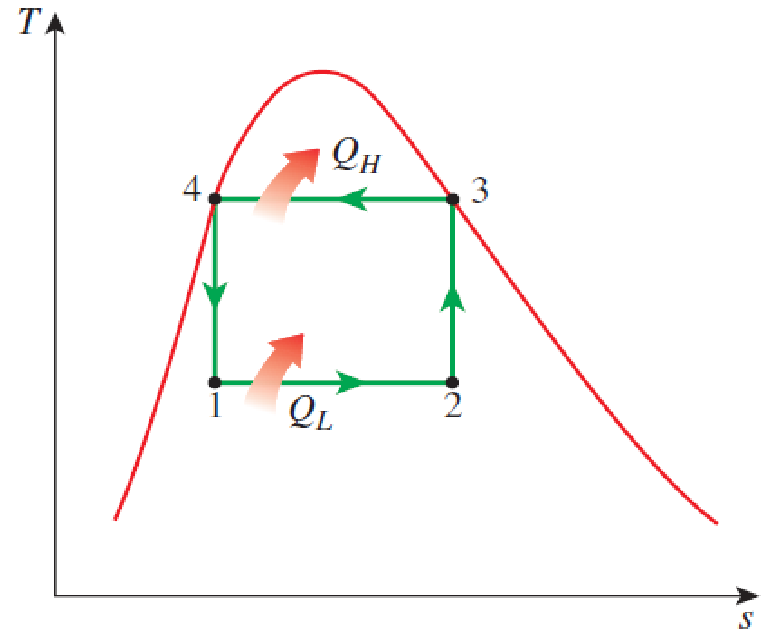


FIGURE 11–2

Schematic of a Carnot refrigerator and T-s diagram of the reversed Carnot cycle.

$$\text{COP}_{\text{R,Carnot}} = \frac{1}{T_H/T_L - 1} \qquad \text{COP}_{\text{HP,Carnot}} = \frac{1}{1 - T_L/T_H}$$

Both COPs increase as the difference between the two temperatures decreases, that is, as T_L rises or T_H falls.

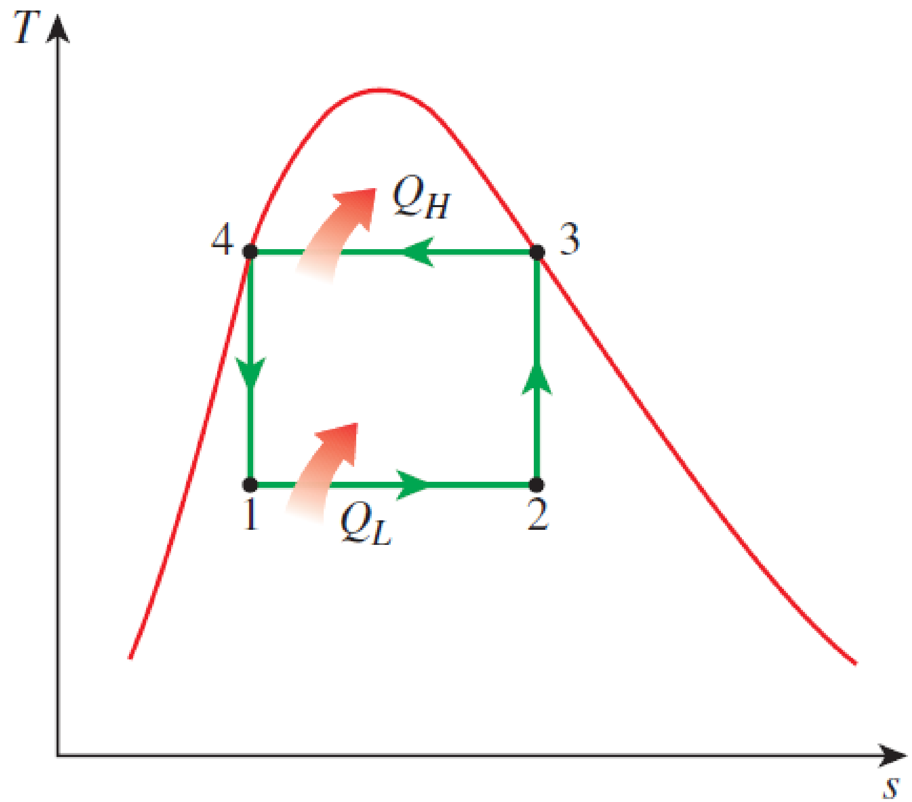
11-2. THE REVERSED CARNOT CYCLE

The reversed Carnot cycle is the *most efficient* refrigeration cycle operating between T_L *and* T_H.

It is not a suitable model for refrigeration cycles since processes 2-3 and 4-1 are not practical.

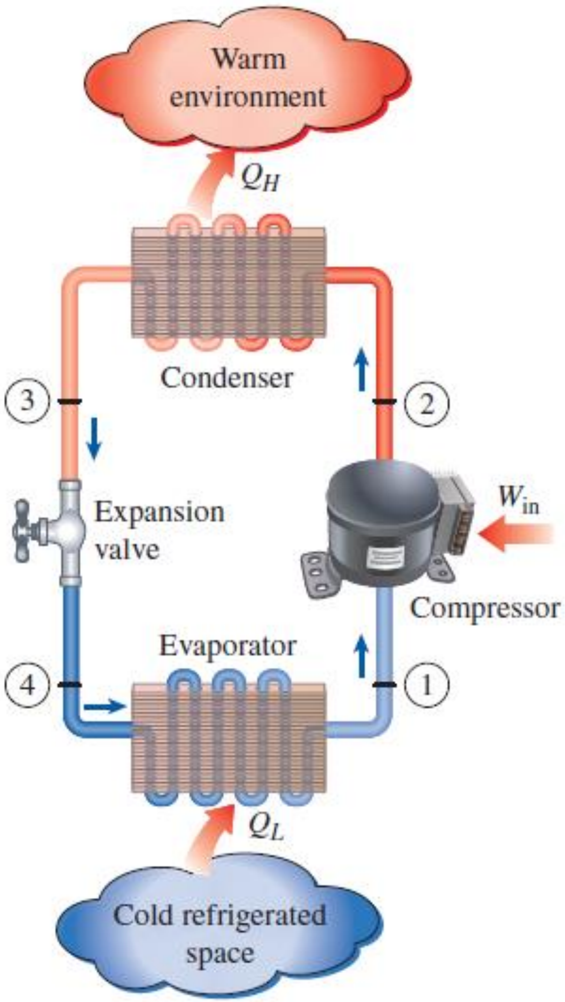
- Process 2-3 involves the compression of a liquid–vapor mixture, which requires a compressor that will handle two phases.

- Process 4-1 involves the expansion of high-moisture-content refrigerant in a turbine.

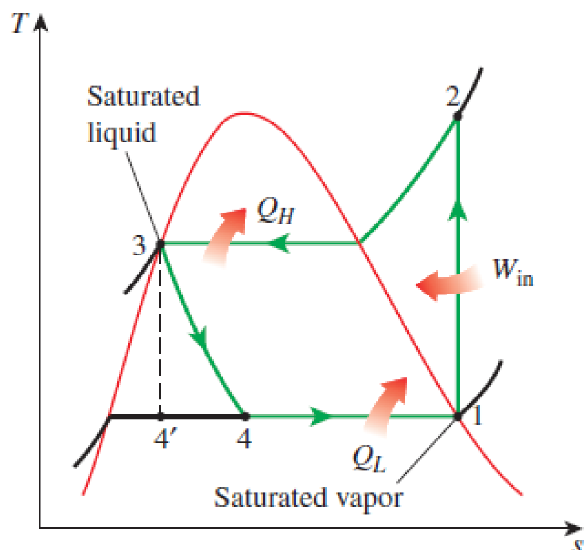


11-3. THE IDEAL VAPOR-COMPRESSION REFRIGERATION CYCLE

The **vapor-compression refrigeration cycle** is the ideal model for refrigeration systems. Unlike the reversed Carnot cycle, the refrigerant is vaporized completely before it is compressed and the turbine is replaced with a throttling device.



1-2 Isentropic compression in a compressor
2-3 Constant-pressure heat rejection in a condenser
3-4 Throttling in an expansion device
4-1 Constant-pressure heat absorption in an evaporator



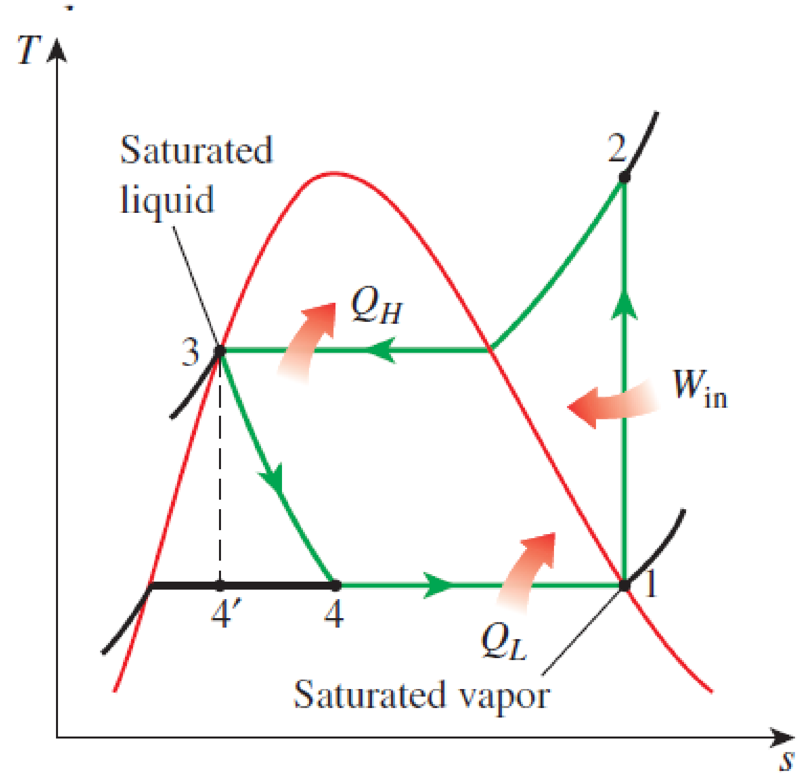
This is the most widely used cycle for refrigerators, A-C systems, and heat pumps.

Schematic and *T-s* diagram for the ideal vapor-compression refrigeration cycle.

11-3. THE IDEAL VAPOR-COMPRESSION REFRIGERATION CYCLE

Steady-flow energy balance

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e$$



$$COP_R = \frac{q_L}{w_{net,in}} = \frac{h_1 - h_4}{h_2 - h_1}$$

$$COP_{HP} = \frac{q_H}{w_{net,in}} = \frac{h_2 - h_3}{h_2 - h_1}$$

$$h_1 = h_{g\,@\,P_1}$$

$$h_3 = h_{f\,@\,P_3}$$

The ideal vapor-compression refrigeration cycle involves an irreversible (throttling) process to make it a more realistic model for the actual systems.

Replacing the expansion valve by a turbine is not practical since the added benefits cannot justify the added cost and complexity.

11-3. THE IDEAL VAPOR-COMPRESSION REFRIGERATION CYCLE

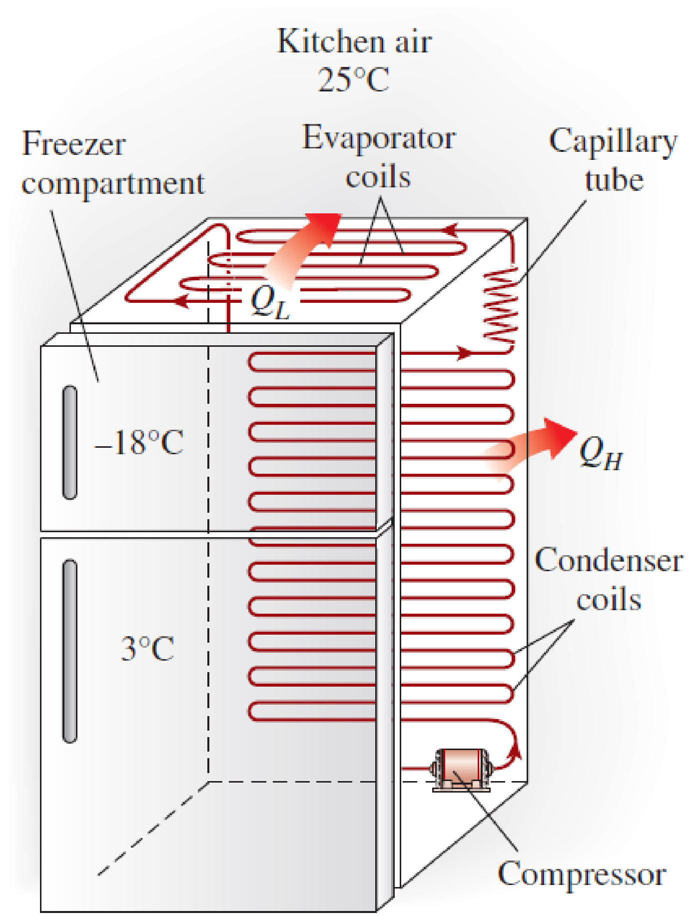


FIGURE 11–4
An ordinary household refrigerator.

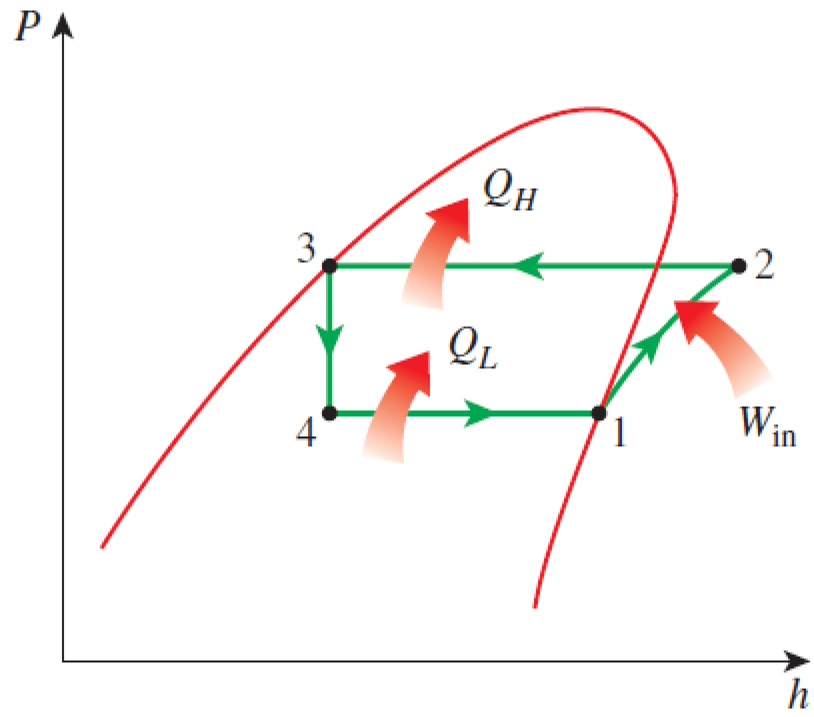


FIGURE 11–5
The *P-h* diagram of an ideal vapor-compression refrigeration cycle.

EXAMPLE 11–1 **The Ideal Vapor-Compression Refrigeration Cycle**

A refrigerator uses refrigerant-134a as the working fluid and operates on an ideal vapor-compression refrigeration cycle between 0.14 and 0.8 MPa. If the mass flow rate of the refrigerant is 0.05 kg/s, determine (*a*) the rate of heat removal from the refrigerated space and the power input to the compressor, (*b*) the rate of heat rejection to the environment, and (*c*) the COP of the refrigerator.

SOLUTION A refrigerator operates on an ideal vapor-compression refrigeration cycle between two specified pressure limits. The rate of refrigeration, the power input, the rate of heat rejection, and the COP are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis The *T-s* diagram of the refrigeration cycle is shown in Fig. 11–6. We note that this is an ideal vapor-compression refrigeration cycle, and thus the compressor is isentropic and the refrigerant leaves the condenser as a saturated liquid and enters the compressor as saturated vapor. From the refrigerant-134a tables, the enthalpies of the refrigerant at all four states are determined as follows:

$$P_1 = 0.14\text{ MPa} \longrightarrow h_1 = h_{g\,@\,0.14\text{ MPa}} = 239.19\text{ kJ/kg}$$
$$s_1 = s_{g\,@\,0.14\text{ MPa}} = 0.94467\text{ kJ/kg·K}$$

$$\left.\begin{array}{l} P_2 = 0.8\text{ MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 275.40\text{ kJ/kg}$$

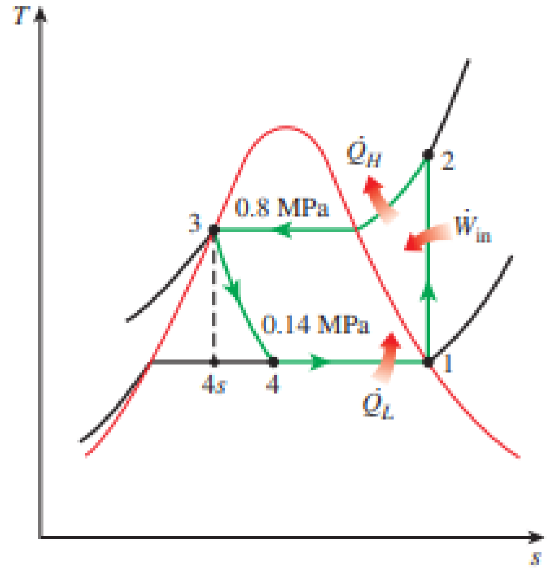
$$P_3 = 0.8\text{ MPa} \longrightarrow h_3 = h_{f\,@\,0.8\text{ MPa}} = 95.48\text{ kJ/kg}$$
$$h_4 \cong h_3\text{ (throttling)} \longrightarrow h_4 = 95.48\text{ kJ/kg}$$

(*a*) The rate of heat removal from the refrigerated space and the power input to the compressor are determined from their definitions:

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05\text{ kg/s})[(239.19 - 95.48)\text{ kJ/kg}] = \mathbf{7.19\text{ kW}}$$

and

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.05\text{ kg/s})[(275.40 - 239.19)\text{ kJ/kg}] = \mathbf{1.81\text{ kW}}$$



(*b*) The rate of heat rejection from the refrigerant to the environment is

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.05\text{ kg/s})[(275.40 - 95.48)\text{ kJ/kg}] = \mathbf{9.00\text{ kW}}$$

It could also be determined from

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 7.19 + 1.81 = 9.00\text{ kW}$$

(*c*) The coefficient of performance of the refrigerator is

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.19\text{ kW}}{1.81\text{ kW}} = \mathbf{3.97}$$

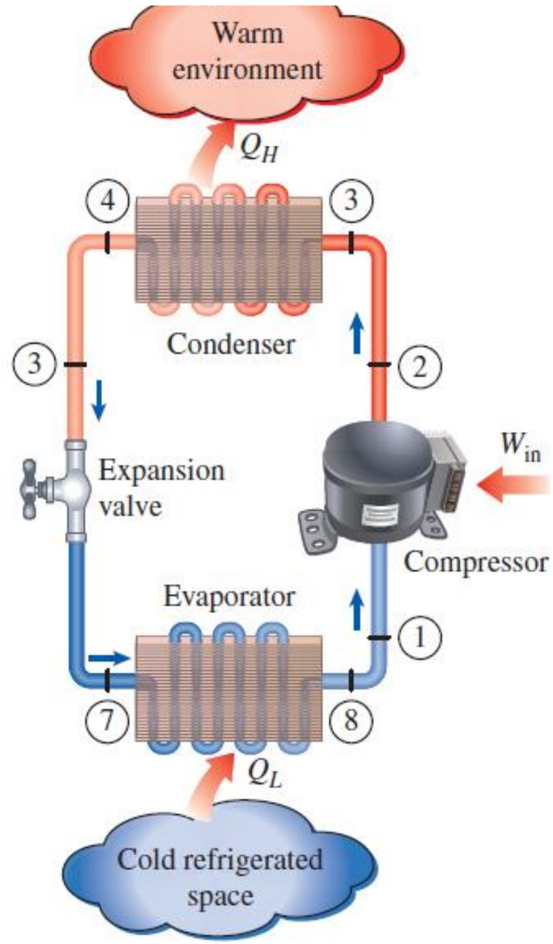
That is, this refrigerator removes about 4 units of thermal energy from the refrigerated space for each unit of electric energy it consumes.

Discussion It would be interesting to see what happens if the throttling valve were replaced by an isentropic turbine. The enthalpy at state 4*s* (the turbine exit with $P_{4s} = 0.14$ MPa, and $s_{4s} = s_3 = 0.35408$ kJ/kg·K) is 88.95 kJ/kg, and the turbine would produce 0.33 kW of power. This would decrease the power input to the refrigerator from 1.81 to 1.48 kW and increase the rate of heat removal from the refrigerated space from 7.19 to 7.51 kW. As a result, the COP of the refrigerator would increase from 3.97 to 5.07, an increase of 28 percent.

11-4. ACTUAL VAPOR-COMPRESSION REF. CYCLE

FIGURE 11–7

Schematic and T-s diagram for the actual vapor-compression refrigeration cycle.



An actual vapor-compression refrigeration cycle differs from the ideal one owing mostly to the irreversibilities that occur in various components, mainly due to fluid friction (causes pressure drops) and heat transfer to or from the surroundings.

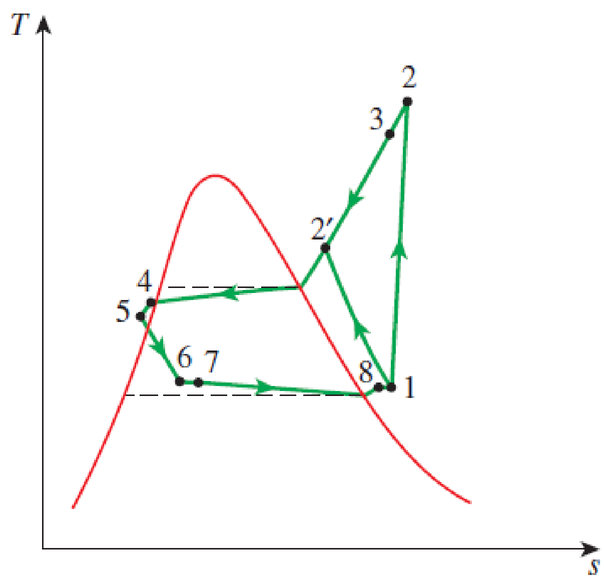
DIFFERENCES

Non-isentropic compression

Superheated vapor at evaporator exit

Subcooled liquid at condenser exit

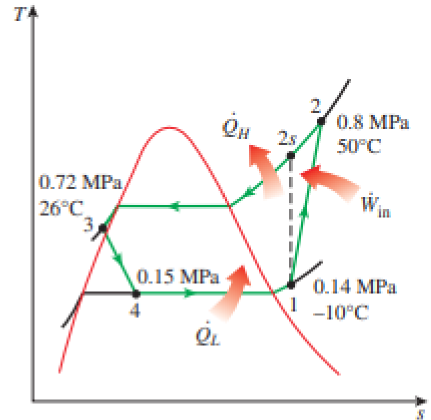
Pressure drops in condenser and evaporator



The COP decreases as a result of irreversibilities.

EXAMPLE 11–2 The Actual Vapor-Compression Refrigeration Cycle

Refrigerant-134a enters the compressor of a refrigerator as superheated vapor at 0.14 MPa and −10°C at a rate of 0.05 kg/s and leaves at 0.8 MPa and 50°C. The refrigerant is cooled in the condenser to 26°C and 0.72 MPa and is throttled to 0.15 MPa. Disregarding any heat transfer and pressure drops in the connecting lines between the components, determine (*a*) the rate of heat removal from the refrigerated space and the power input to the compressor, (*b*) the isentropic efficiency of the compressor, and (*c*) the coefficient of performance of the refrigerator.



SOLUTION A refrigerator operating on a vapor-compression cycle is considered. The rate of refrigeration, the power input, the compressor efficiency, and the COP are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis The *T-s* diagram of the refrigeration cycle is shown in Fig. 11–8. We note that the refrigerant leaves the condenser as a compressed liquid and enters the compressor as superheated vapor. The enthalpies of the refrigerant at various states are determined from the refrigerant tables to be

$$\left.\begin{aligned} P_1 &= 0.14\text{ MPa} \\ T_1 &= -10^\circ\text{C} \end{aligned}\right\} h_1 = 246.37\text{ kJ/kg}$$

$$\left.\begin{aligned} P_2 &= 0.8\text{ MPa} \\ T_2 &= 50^\circ\text{C} \end{aligned}\right\} h_2 = 286.71\text{ kJ/kg}$$

$$\left.\begin{aligned} P_3 &= 0.72\text{ MPa} \\ T_3 &= 26^\circ\text{C} \end{aligned}\right\} h_3 \cong h_{f @ 26^\circ\text{C}} = 87.83\text{ kJ/kg}$$

$$h_4 \cong h_3 \text{ (throttling)} \quad \rightarrow \quad h_4 = 87.83\text{ kJ/kg}$$

(*a*) The rate of heat removal from the refrigerated space and the power input to the compressor are determined from their definitions:

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05\text{ kg/s})[(246.37 - 87.83)\text{ kJ/kg}] = \mathbf{7.93\text{ kW}}$$

and

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.05\text{ kg/s})[(286.71 - 246.37)\text{ kJ/kg}] = \mathbf{2.02\text{ kW}}$$

(*a*) The rate of heat removal from the refrigerated space and the power input to the compressor are determined from their definitions:

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05\text{ kg/s})[(246.37 - 87.83)\text{ kJ/kg}] = \mathbf{7.93\text{ kW}}$$

and

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.05\text{ kg/s})[(286.71 - 246.37)\text{ kJ/kg}] = \mathbf{2.02\text{ kW}}$$

(*b*) The isentropic efficiency of the compressor is determined from

$$\eta_C \cong \frac{h_{2s} - h_1}{h_2 - h_1}$$

where the enthalpy at state 2*s* ($P_{2s} = 0.8$ MPa and $s_{2s} = s_1 = 0.9724$ kJ/kg·K) is 284.20 kJ/kg. Thus,

$$\eta_C = \frac{284.20 - 246.37}{286.71 - 246.37} = 0.938 \text{ or } \mathbf{93.8\%}$$

(*c*) The coefficient of performance of the refrigerator is

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.93\text{ kW}}{2.02\text{ kW}} = \mathbf{3.93}$$

Discussion This problem is identical to the one worked out in Example 11–1, except that the refrigerant is slightly superheated at the compressor inlet and subcooled at the condenser exit. Also, the compressor is not isentropic. As a result, the heat removal rate from the refrigerated space increases (by 10.3 percent), but the power input to the compressor increases even more (by 11.6 percent). Consequently, the COP of the refrigerator decreases from 3.97 to 3.93.

11-5. SECOND-LAW ANALYSIS OF VAPOR-COMPRESSION REFRIGERATION CYCLE

The maximum COP of a refrigeration cycle operating between temperature limits of T_L and T_H

$$\text{COP}_{\text{R,max}} = \text{COP}_{\text{R,rev}} = \text{COP}_{\text{R,Carnot}} = \frac{T_L}{T_H - T_L} = \frac{1}{T_H/T_L - 1}$$

Actual refrigeration cycles are less efficient than the reversed Carnot cycle because of the irreversibilities involved. But the conclusion we can draw from Carnot COP relation that the COP is inversely proportional to the temperature difference T_H - T_L is equally valid for actual refrigeration cycles.

The goal of a second-law or exergy analysis of a refrigeration system is to determine the components that can benefit the most by improvements.

This is done identifying the locations of greatest exergy destruction and the components with the lowest exergy or second-law efficiency.

Exergy destruction in a component can be determined directly from an exergy balance or by using

$$\dot{X}_{\text{dest}} = T_0 \dot{S}_{\text{gen}}$$

11-5. SECOND-LAW ANALYSIS OF VAPOR-COMPRESSION REFRIGERATION CYCLE

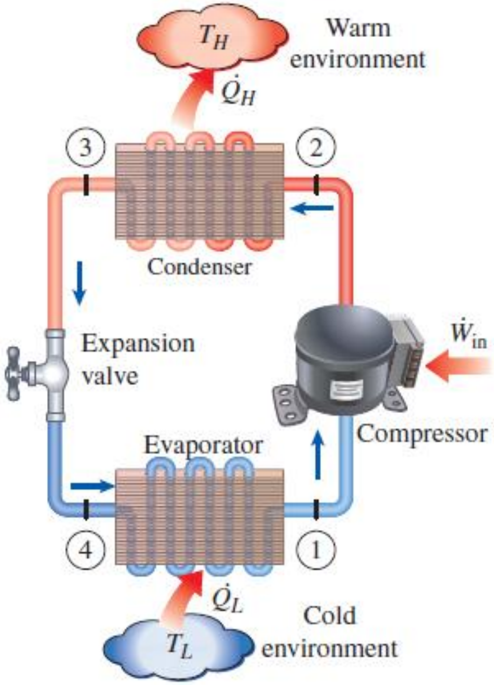


FIGURE 11–9
The vapor-compression refrigeration cycle considered in the second-law analysis.

Note that when $T_H = T_0$, which is often the case for refrigerators, $\eta_{II,cond} = 0$ since there is no recoverable exergy in this case.

Compressor:

$$\dot{X}_{dest,1-2} = T_0\dot{S}_{gen,1-2} = \dot{m}T_0(s_2 - s_1)$$

$$\eta_{II,Comp} = \frac{\dot{X}_{recovered}}{\dot{X}_{expended}} = \frac{\dot{W}_{rev}}{\dot{W}_{act,in}} = \frac{\dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]}{\dot{m}(h_2 - h_1)} = \frac{\psi_2 - \psi_1}{h_2 - h_1}$$

$$= 1 - \frac{\dot{X}_{dest,1-2}}{\dot{W}_{act,in}}$$

Condenser:

$$\dot{X}_{dest,2-3} = T_0\dot{S}_{gen,2-3} = T_0\left[\dot{m}(s_3 - s_2) + \frac{\dot{Q}_H}{T_H}\right]$$

$$\eta_{II,Cond} = \frac{\dot{X}_{recovered}}{\dot{X}_{expended}} = \frac{\dot{X}_{Q_H}}{\dot{X}_2 - \dot{X}_3} = \frac{\dot{Q}_H(1 - T_0/T_H)}{\dot{X}_2 - \dot{X}_3}$$

$$= \frac{\dot{Q}_H(1 - T_0/T_H)}{\dot{m}[h_2 - h_3 - T_0(s_2 - s_3)]} = 1 - \frac{\dot{X}_{dest,2-3}}{\dot{X}_2 - \dot{X}_3}$$

Expansion valve:

$$\dot{X}_{dest,3-4} = T_0\dot{S}_{gen,3-4} = \dot{m}T_0(s_4 - s_3)$$

$$\eta_{II,ExpValve} = \frac{\dot{X}_{recovered}}{\dot{X}_{expended}} = \frac{0}{\dot{X}_3 - \dot{X}_4} = 0 \quad \text{or}$$

$$\eta_{II,ExpValve} = 1 - \frac{\dot{X}_{dest,3-4}}{\dot{X}_{expended}} = 1 - \frac{\dot{X}_3 - \dot{X}_4}{\dot{X}_3 - \dot{X}_4} = 0$$

11-5. SECOND-LAW ANALYSIS OF VAPOR-COMPRESSION REFRIGERATION CYCLE

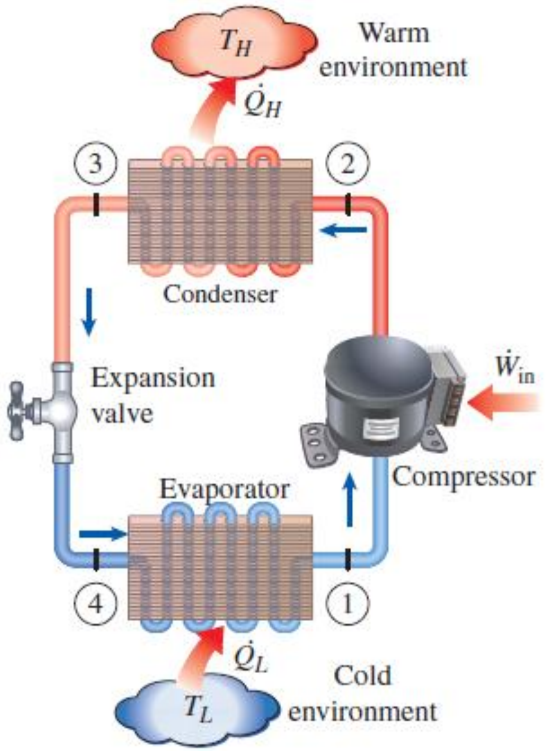


FIGURE 11–9
The vapor-compression refrigeration cycle considered in the second-law analysis.

Evaporator:

$$\dot{X}_{\text{dest},4-1} = T_0\dot{S}_{\text{gen},4-1} = T_0\left[\dot{m}(s_1 - s_4) - \frac{\dot{Q}_L}{T_L}\right]$$

$$\eta_{\text{II,Evap}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{X}_{\dot{Q}_L}}{\dot{X}_4 - \dot{X}_1} = \frac{\dot{Q}_L(T_0 - T_L)/T_L}{\dot{X}_4 - \dot{X}_1}$$

$$= \frac{\dot{Q}_L(T_0 - T_L)/T_L}{\dot{m}[h_4 - h_1 - T_0(s_4 - s_1)]} = 1 - \frac{\dot{X}_{\text{dest},4-1}}{\dot{X}_4 - \dot{X}_1}$$

$$\dot{X}_{\dot{Q}_L} = \dot{Q}_L\frac{T_0 - T_L}{T_L}$$

The exergy rate associated with the withdrawal of heat from the low-temperature medium at T_L at a rate of Q_L

This is equivalent to the power that can be produced by a Carnot heat engine receiving heat from the environment at T_0 and rejecting heat to the low temperature medium at T_L at a rate of Q_L.

$$\dot{W}_{\text{rev,in}} = \dot{W}_{\text{min,in}} = \dot{X}_{\dot{Q}_L}$$

Note that when $T_L = T_0$, which is often the case for heat pumps, $\eta_{\text{II,evap}} = 0$ since there is no recoverable exergy in this case.

11-5. SECOND-LAW ANALYSIS OF VAPOR-COMPRESSION REFRIGERATION CYCLE

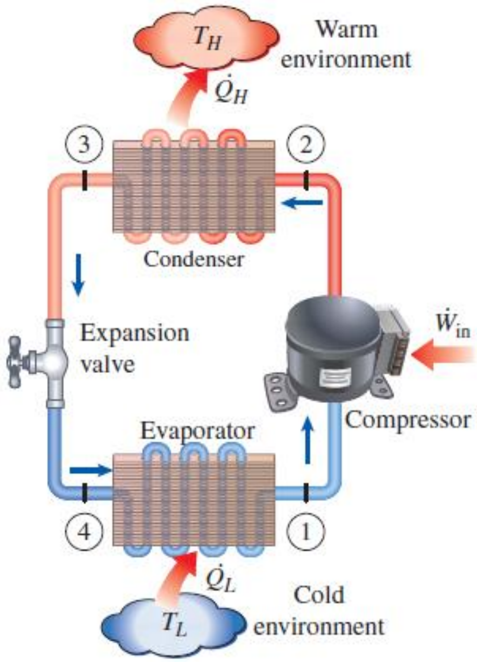


FIGURE 11–9
The vapor-compression refrigeration cycle considered in the second-law analysis.

$$\dot{X}_{dest,total} = \dot{X}_{dest,1-2} + \dot{X}_{dest,2-3} + \dot{X}_{dest,3-4} + \dot{X}_{dest,4-1}$$

$$\dot{X}_{dest,total} = \dot{W}_{in} - \dot{X}_{\dot{Q}_L}$$ Total exergy destruction

Second-law (exergy) efficiency

$$\eta_{II,cycle} = \frac{\dot{X}_{\dot{Q}_L}}{\dot{W}_{in}} = \frac{\dot{W}_{min,in}}{\dot{W}_{in}} = 1 - \frac{\dot{X}_{dest,total}}{\dot{W}_{in}}$$

$$\dot{W}_{in} = \frac{\dot{Q}_L}{COP_R} \qquad \dot{X}_{\dot{Q}_L} = \dot{Q}_L \frac{T_0 - T_L}{T_L}$$

$$\eta_{II,cycle} = \frac{\dot{X}_{\dot{Q}_L}}{\dot{W}_{in}} = \frac{\dot{Q}_L(T_0 - T_L)/T_L}{\dot{Q}_L/COP_R} = \frac{COP_R}{T_L/(T_H - T_L)} = \frac{COP_R}{COP_{R,rev}}$$

$T_0 = T_H$ for a refrigeration cycle

This second-law efficiency definition accounts for all irreversibilities associated within the refrigerator, including the heat transfers with the refrigerated space and the environment.

11-6. SELECTING THE RIGHT REFRIGERANT

Several refrigerants may be used in refrigeration systems such as chlorofluorocarbons (CFCs), HFCs, HCFCs, ammonia, hydrocarbons (propane, ethane, ethylene, etc.), carbon dioxide, air (in the air-conditioning of aircraft), and even water (in applications above the freezing point).

The industrial and heavy-commercial sectors use *ammonia* (it is toxic).

R-11 is used in large-capacity water chillers serving A-C systems in buildings.

R-134a (replaced R-12, which damages ozone layer) is used in domestic refrigerators and freezers, as well as automotive air conditioners.

11-6. SELECTING THE RIGHT REFRIGERANT

R-22 is used in window air conditioners, heat pumps, air conditioners of commercial buildings, and large industrial refrigeration systems, and offers strong competition to ammonia.

R-22 is being replaced by alternatives such as R410A and R-407C because it is ozone-depleting.

CFCs allow more ultraviolet radiation into the earth's atmosphere by destroying the protective ozone layer and thus contributing to the greenhouse effect that causes global warming. Fully halogenated CFCs (such as R-11, R-12, and R-115) do the most damage to the ozone layer.

Refrigerants that are friendly to the ozone layer have been developed.

Two important parameters that need to be considered in the selection of a refrigerant are the temperatures of the two media (the refrigerated space and the environment) with which the refrigerant exchanges heat.

11-7. HEAT PUMP SYSTEMS

The most common energy source for heat pumps is atmospheric air (air-to-air systems).

Water-source systems usually use well water and ground-source (geothermal) heat pumps use earth as the energy source. They typically have higher COPs but are more complex and more expensive to install.

Both the capacity and the efficiency of a heat pump fall significantly at low temperatures.

Therefore, most air-source heat pumps require a supplementary heating system such as electric resistance heaters or a gas furnace.

Heat pumps are most competitive in areas that have a large cooling load during the cooling season and a relatively small heating load during the heating season.

In these areas, the heat pump can meet the entire cooling and heating needs of residential or commercial buildings.

11-7. HEAT PUMP SYSTEMS

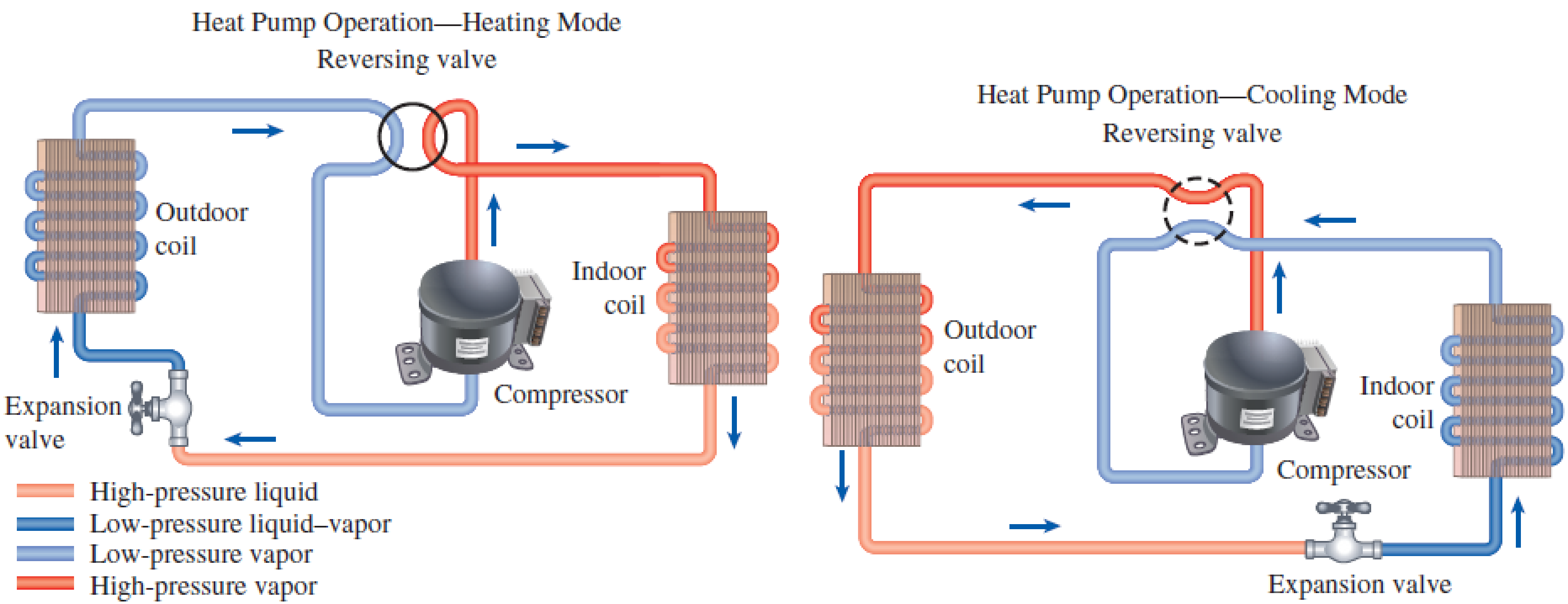


FIGURE 11–11
A heat pump can be used to heat a house in winter and to cool it in summer.

11-8. INNOVATIVE VAPOR-COMPRESSION REFRIGERATION SYSTEMS

The simple vapor-compression refrigeration cycle is the most widely used refrigeration cycle, and it is adequate for most refrigeration applications.

The ordinary vapor-compression refrigeration systems are simple, inexpensive, reliable, and practically maintenance-free.

However, for large industrial applications *efficiency*, not simplicity, is the major concern.

Also, for some applications the simple vapor-compression refrigeration cycle is inadequate and needs to be modified.

For moderately and very low temperature applications some innovative refrigeration systems are used. The following cycles will be discussed:

- Cascade refrigeration systems
- Multistage compression refrigeration systems
- Multipurpose refrigeration systems with a single compressor
- Liquefaction of gases

11-8. INNOVATIVE VAPOR-COMPRESSION REFRIGERATION SYSTEMS

Cascade Refrigeration Systems

Some industrial applications require moderately low temperatures, and the temperature range they involve may be too large for a single vapor-compression refrigeration cycle to be practical. The solution is **cascading**.

$$\dot{m}_A(h_5 - h_8) = \dot{m}_B(h_2 - h_3) \longrightarrow \frac{\dot{m}_A}{\dot{m}_B} = \frac{h_2 - h_3}{h_5 - h_8}$$

$$\mathrm{COP}_{\mathrm{R,cascade}} = \frac{\dot{Q}_L}{\dot{W}_{\mathrm{net,in}}} = \frac{\dot{m}_B(h_1 - h_4)}{\dot{m}_A(h_6 - h_5) + \dot{m}_B(h_2 - h_1)}$$

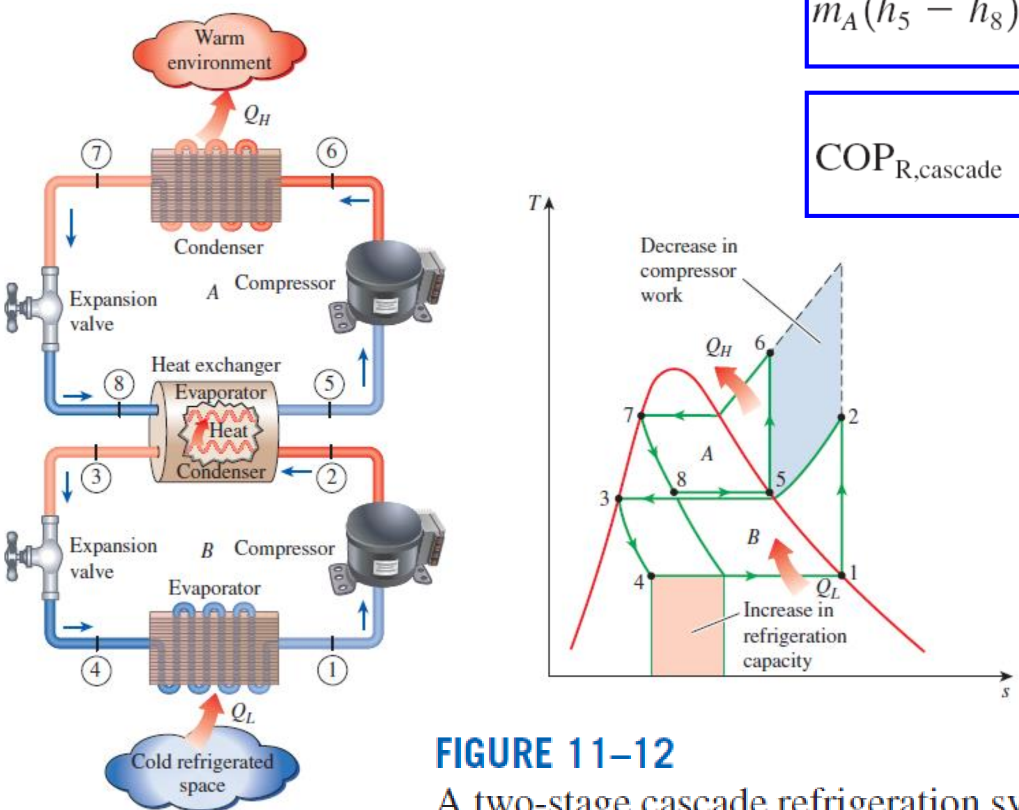


FIGURE 11–12

A two-stage cascade refrigeration system with the same refrigerant in both stages.

Cascading improves the COP of a refrigeration system.

Some systems use three or four stages of cascading.

EXAMPLE 11–4 A Two-Stage Cascade Refrigeration Cycle

Consider a two-stage cascade refrigeration system operating between the pressure limits of 0.8 and 0.14 MPa. Each stage operates on an ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid. Heat rejection from the lower cycle to the upper cycle takes place in an adiabatic counterflow heat exchanger where both streams enter at about 0.32 MPa. (In practice, the working fluid of the lower cycle is at a higher pressure and temperature in the heat exchanger for effective heat transfer.) If the mass flow rate of the refrigerant through the upper cycle is 0.05 kg/s, determine (*a*) the mass flow rate of the refrigerant through the lower cycle, (*b*) the rate of heat removal from the refrigerated space and the power input to the compressor, and (*c*) the coefficient of performance of this cascade refrigerator.

SOLUTION A cascade refrigeration system operating between the specified pressure limits is considered. The mass flow rate of the refrigerant through the lower cycle, the rate of refrigeration, the power input, and the COP are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible. **3** The heat exchanger is adiabatic.

Properties The enthalpies of the refrigerant at all eight states are determined from the refrigerant tables and are indicated on the *T-s* diagram.

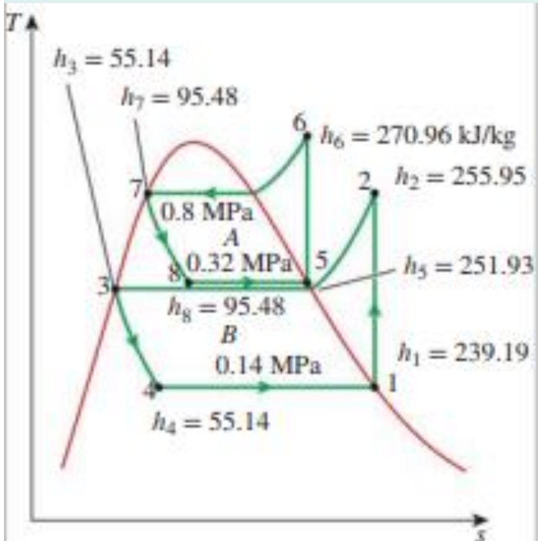


FIGURE 11–13
T-s diagram of the cascade refrigeration cycle described in Example 11–4.

Analysis The *T-s* diagram of the refrigeration cycle is shown in Fig. 11–13. The topping cycle is labeled cycle *A* and the bottoming one, cycle *B*. For both cycles, the refrigerant leaves the condenser as a saturated liquid and enters the compressor as saturated vapor.

(*a*) The mass flow rate of the refrigerant through the lower cycle is determined from the steady-flow energy balance on the adiabatic heat exchanger,

$$\dot{E}_{out} = \dot{E}_{in} \longrightarrow \dot{m}_A h_5 + \dot{m}_B h_3 = \dot{m}_A h_8 + \dot{m}_B h_2$$
$$\dot{m}_A(h_5 - h_8) = \dot{m}_B(h_2 - h_3)$$
$$(0.05 \text{ kg/s})[(251.93 - 95.48) \text{ kJ/kg}] = \dot{m}_B[(255.95 - 55.14) \text{ kJ/kg}]$$
$$\dot{m}_B = \mathbf{0.0390 \text{ kg/s}}$$

(*b*) The rate of heat removal by a cascade cycle is the rate of heat absorption in the evaporator of the lowest stage. The power input to a cascade cycle is the sum of the power inputs to all of the compressors:

$$\dot{Q}_L = \dot{m}_B(h_1 - h_4) = (0.0390 \text{ kg/s})[(239.19 - 55.14) \text{ kJ/kg}] = \mathbf{7.18 \text{ kW}}$$
$$\dot{W}_{in} = \dot{W}_{comp\ I,in} + \dot{W}_{comp\ II,in} = \dot{m}_A(h_6 - h_5) + \dot{m}_B(h_2 - h_1)$$
$$= (0.05 \text{ kg/s})[(270.96 - 251.93) \text{ kJ/kg}]$$
$$+ (0.039 \text{ kg/s})[(255.95 - 239.19) \text{ kJ/kg}]$$
$$= \mathbf{1.61 \text{ kW}}$$

(*c*) The COP of a refrigeration system is the ratio of the refrigeration rate to the net power input:

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{7.18 \text{ kW}}{1.61 \text{ kW}} = \mathbf{4.46}$$

Discussion This problem was worked out in Example 11–1 for a single-stage refrigeration system. Notice that the COP of the refrigeration system increases from 3.97 to 4.46 as a result of cascading. The COP of the system can be increased even more by increasing the number of cascade stages.

11-8. INNOVATIVE VAPOR-COMPRESSION REFRIGERATION SYSTEMS

Multistage Compression Refrigeration Systems

When the fluid used throughout the cascade refrigeration system is the same, the heat exchanger between the stages can be replaced by a mixing chamber (called a *flash chamber*) since it has better heat transfer characteristics.

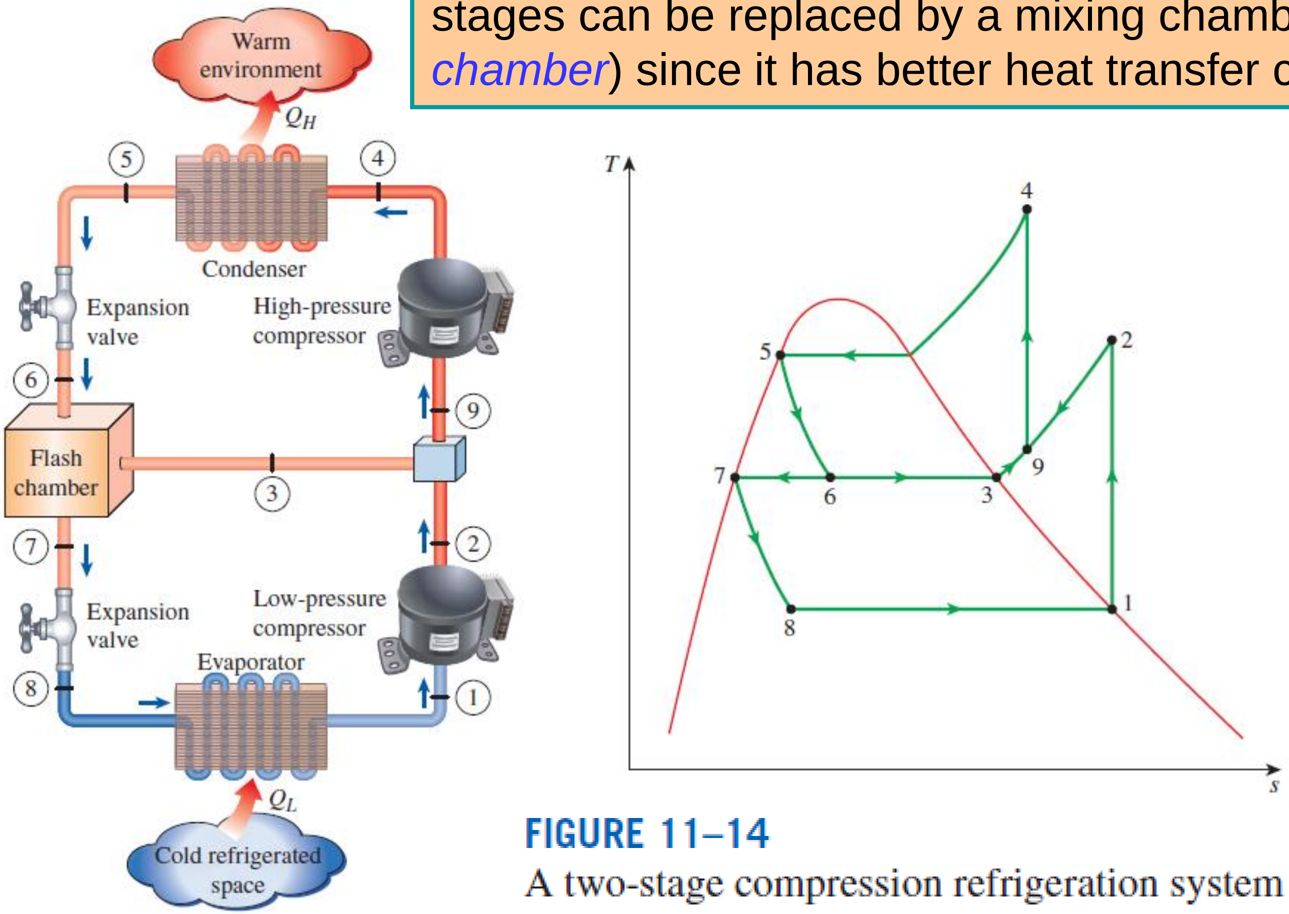


FIGURE 11–14
A two-stage compression refrigeration system with a flash chamber.

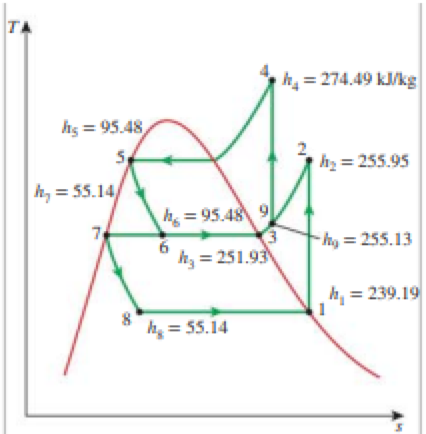


FIGURE 11–15
T-s diagram of the two-stage compression refrigeration cycle described in Example 11–5.

EXAMPLE 11–5 A Two-Stage Refrigeration Cycle with a Flash Chamber

Consider a two-stage compression refrigeration system operating between the pressure limits of 0.8 and 0.14 MPa. The working fluid is refrigerant-134a. The refrigerant leaves the condenser as a saturated liquid and is throttled to a flash chamber operating at 0.32 MPa. Part of the refrigerant evaporates during this flashing process, and this vapor is mixed with the refrigerant leaving the low-pressure compressor. The mixture is then compressed to the condenser pressure by the high-pressure compressor. The liquid in the flash chamber is throttled to the evaporator pressure and cools the refrigerated space as it vaporizes in the evaporator. Assuming the refrigerant leaves the evaporator as a saturated vapor and both compressors are isentropic, determine (*a*) the fraction of the refrigerant that evaporates as it is throttled to the flash chamber, (*b*) the amount of heat removed from the refrigerated space and the compressor work per unit mass of refrigerant flowing through the condenser, and (*c*) the coefficient of performance.

SOLUTION A two-stage compression refrigeration system operating between specified pressure limits is considered. The fraction of the refrigerant that evaporates in the flash chamber, the refrigeration and work input per unit mass, and the COP are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible. **3** The flash chamber is adiabatic.

Properties The enthalpies of the refrigerant at various states are determined from the refrigerant tables and are indicated on the *T-s* diagram.

Analysis The *T-s* diagram of the refrigeration cycle is shown in Fig. 11–15. We note that the refrigerant leaves the condenser as saturated liquid and enters the low-pressure compressor as saturated vapor.

(*a*) The fraction of the refrigerant that evaporates as it is throttled to the flash chamber is simply the quality at state 6, which is

$$x_6 = \frac{h_6 - h_f}{h_{fg}} = \frac{95.48 - 55.14}{196.78} = \mathbf{0.2050}$$

(*b*) The amount of heat removed from the refrigerated space and the compressor work input per unit mass of refrigerant flowing through the condenser are

$$q_L = (1 - x_6)(h_1 - h_8)$$
$$= (1 - 0.2050)[(239.19 - 55.14)\ \text{kJ/kg}] = \mathbf{146.3\ kJ/kg}$$

and

$$w_{\text{in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}} = (1 - x_6)(h_2 - h_1) + (1)(h_4 - h_9)$$

The enthalpy at state 9 is determined from an energy balance on the mixing chamber,

$$\dot{E}_{\text{out}} = \dot{E}_{\text{in}}$$
$$(1)h_9 = x_6 h_3 + (1 - x_6)h_2$$
$$h_9 = (0.2050)(251.93) + (1 - 0.2050)(255.95) = 255.13\ \text{kJ/kg}$$

Also, $s_9 = 0.9417$ kJ/kg·K. Thus the enthalpy at state 4 (0.8 MPa, $s_4 = s_9$) is $h_4 = 274.49$ kJ/kg. Substituting,

$$w_{\text{in}} = (1 - 0.2050)[(255.95 - 239.19)\ \text{kJ/kg}] + (274.49 - 255.13)\ \text{kJ/kg}$$
$$= \mathbf{32.68\ kJ/kg}$$

(*c*) The coefficient of performance is

$$\text{COP}_R = \frac{q_L}{w_{\text{in}}} = \frac{146.3\ \text{kJ/kg}}{32.68\ \text{kJ/kg}} = \mathbf{4.48}$$

Discussion This problem was worked out in Example 11–1 for a single-stage refrigeration system (COP = 3.97) and in Example 11–4 for a two-stage cascade refrigeration system (COP = 4.46). Notice that the COP of the refrigeration system increased considerably relative to the single-stage compression but did not change much relative to the two-stage cascade compression.

11-8. INNOVATIVE VAPOR-COMPRESSION REFRIGERATION SYSTEMS

Multipurpose Refrigeration Systems with a Single Compressor

Some applications require refrigeration at more than one temperature. A practical and economical approach is to route all the exit streams from the evaporators to a single compressor and let it handle the compression process for the entire system.

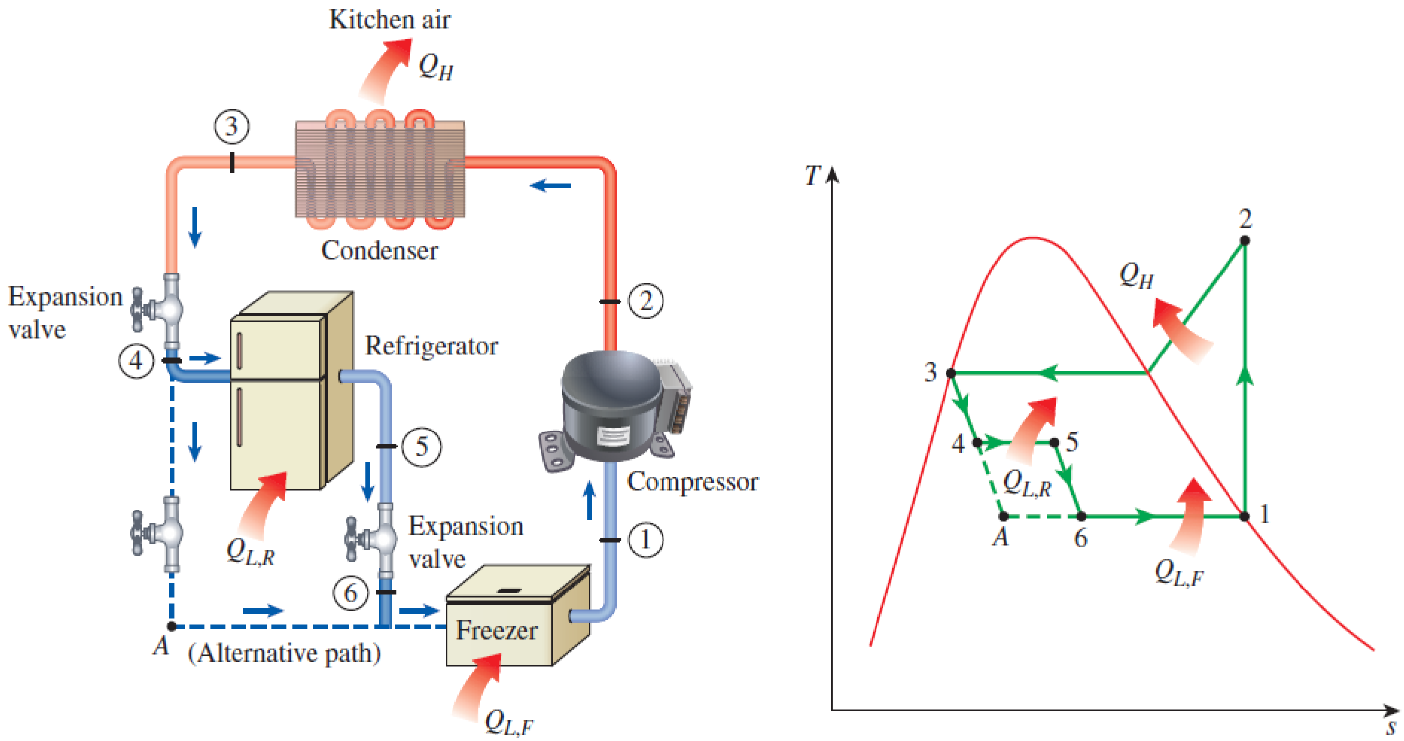


FIGURE 11–16
Schematic and *T*-*s* diagram for a refrigerator–freezer unit with one compressor.

11-8. INNOVATIVE VAPOR-COMPRESSION REFRIGERATION SYSTEMS

Liquefaction of Gases

Many important scientific and engineering processes at cryogenic temperatures (below about −100°C) depend on liquefied gases including the separation of oxygen and nitrogen from air, preparation of liquid propellants for rockets, the study of material properties at low temperatures, and the study of superconductivity.

At temperatures above the critical-point value, a substance exists in the gas phase only.

The critical temperatures of helium, hydrogen, and nitrogen (three commonly used liquefied gases) are −268, −240, and −147°C, respectively.

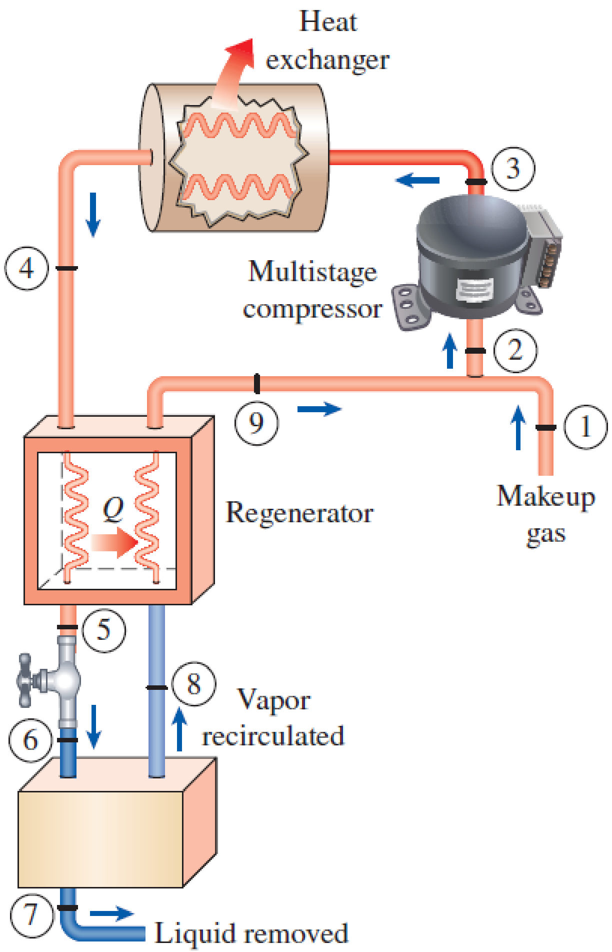
Therefore, none of these substances exist in liquid form at atmospheric conditions.

Furthermore, low temperatures of this magnitude cannot be obtained by ordinary refrigeration techniques.

The storage (i.e., hydrogen) and transportation of some gases (i.e., natural gas) are done after they are liquefied at very low temperatures.

Several innovative cycles are used for the liquefaction of gases.

11-8. INNOVATIVE VAPOR-COMPRESSION REFRIGERATION SYSTEMS



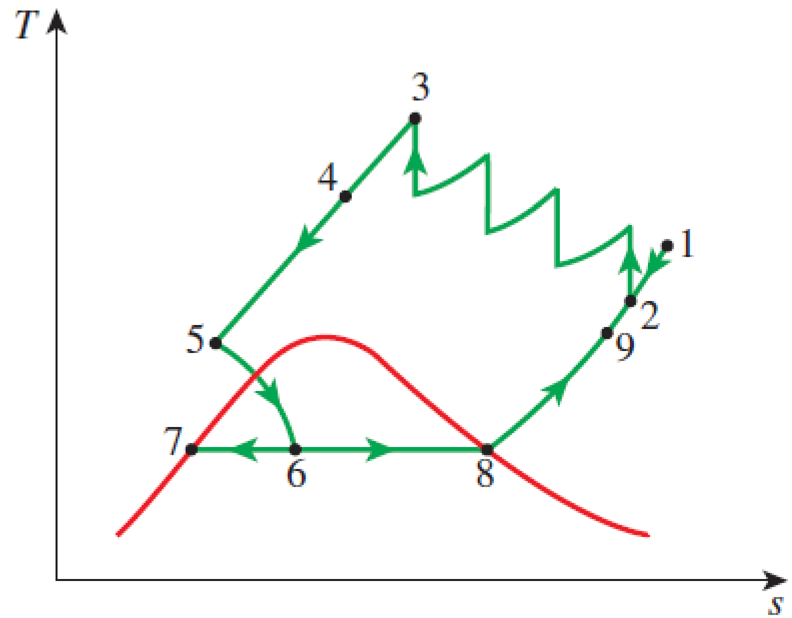
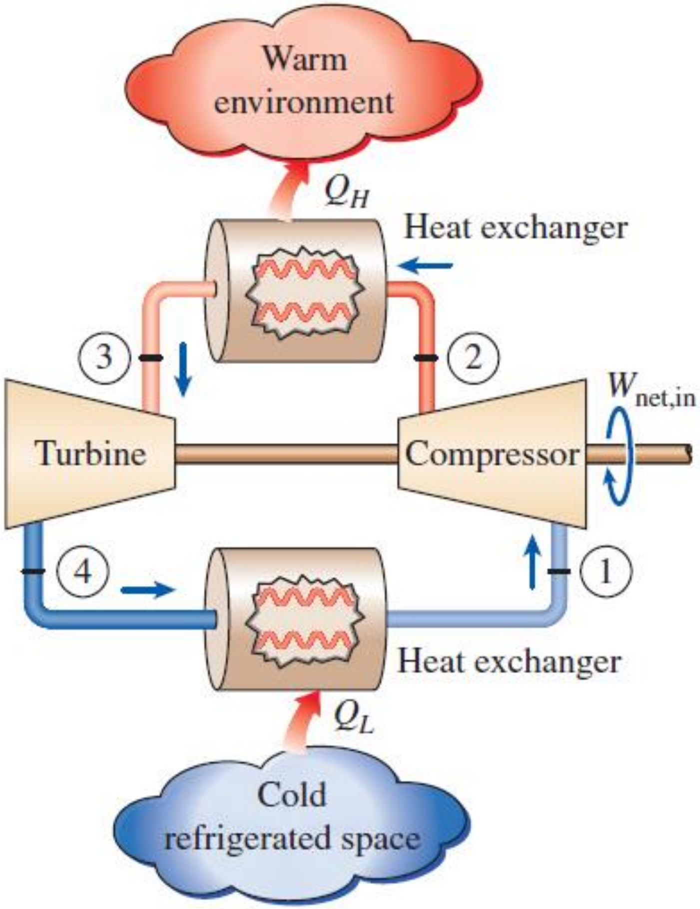


FIGURE 11–17
Linde-Hampson system for liquefying gases.

The reversed Brayton cycle (the gas refrigeration cycle) can be used for refrigeration.

$$q_L = h_1 - h_4$$
$$w_{turb,out} = h_3 - h_4$$
$$w_{comp,in} = h_2 - h_1$$

$$\mathrm{COP_R} = \frac{q_L}{w_{net,in}} = \frac{q_L}{w_{comp,in} - w_{turb,out}}$$



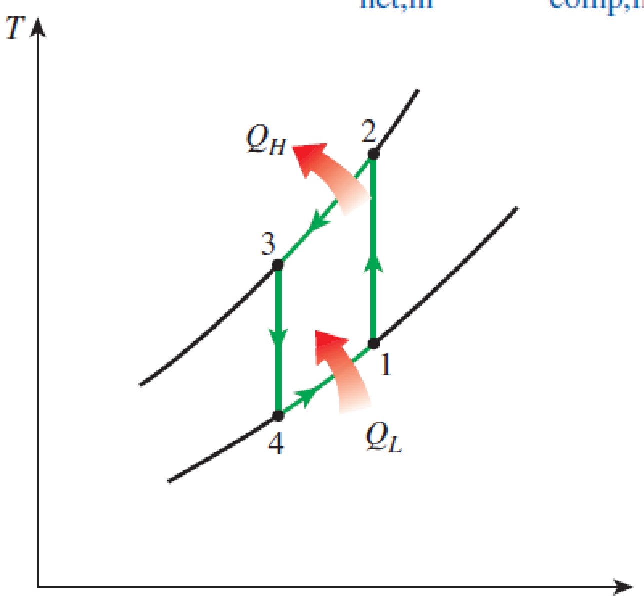


FIGURE 11–18
Simple gas refrigeration cycle.

The gas refrigeration cycles have lower COPs relative to the vapor-compression refrigeration cycles or the reversed Carnot cycle.

The reversed Carnot cycle consumes a fraction of the net work (area 1*A*3*B*) but produces a greater amount of refrigeration (triangular area under *B*1).

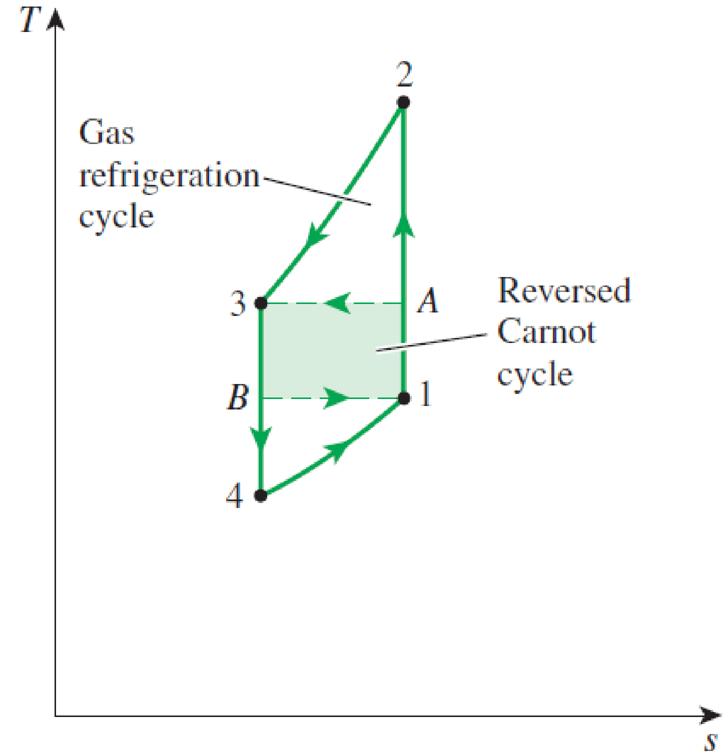


FIGURE 11–19

A reserved Carnot cycle produces more refrigeration (area under *B*1) with less work input (area 1*A*3*B*).

Despite their relatively low COPs, the gas refrigeration cycles involve simple, lighter components, which make them suitable for aircraft cooling, and they can incorporate regeneration

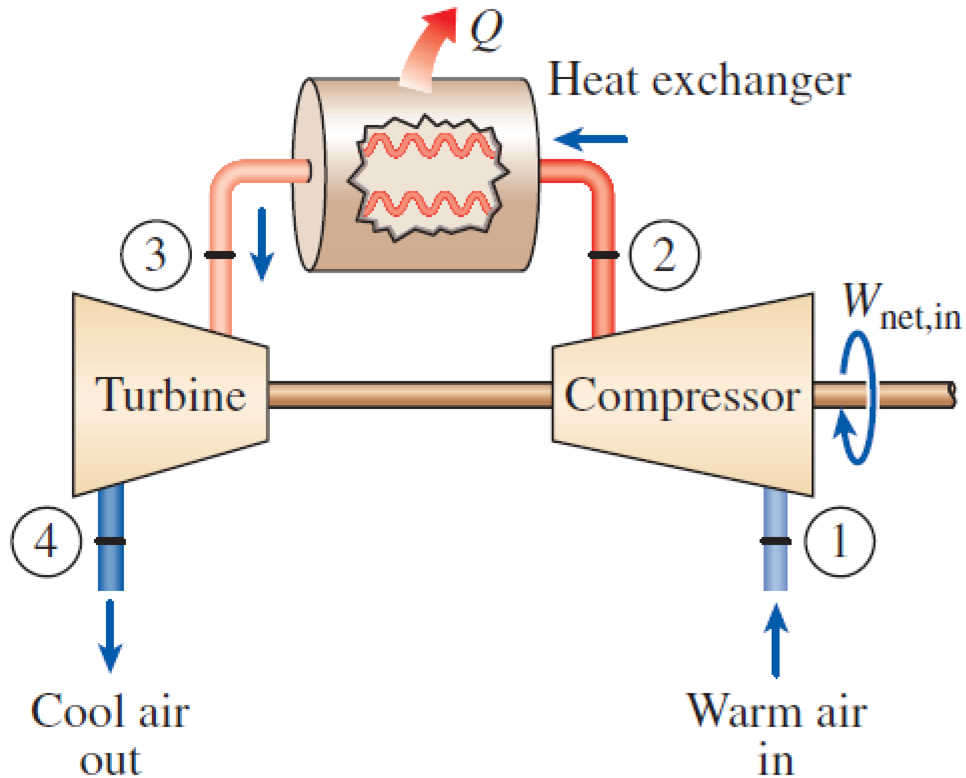


FIGURE 11–20
An open-cycle aircraft cooling system.

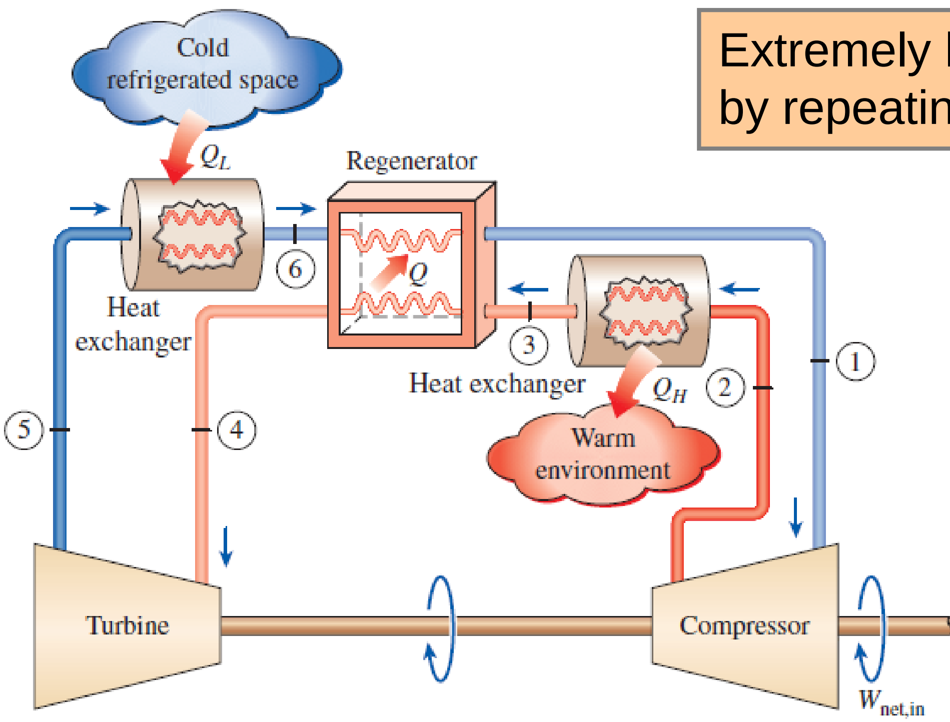
11-9. GAS REFRIGERATION CYCLES

Without regeneration, the lowest turbine inlet temperature is T_0, the temperature of the surroundings or any other cooling medium.

With regeneration, the high-pressure gas is further cooled to T_4 before expanding in the turbine.

Lowering the turbine inlet temperature automatically lowers the turbine exit temperature, which is the minimum temperature in the cycle.

Extremely low temperatures can be achieved by repeating regeneration process.



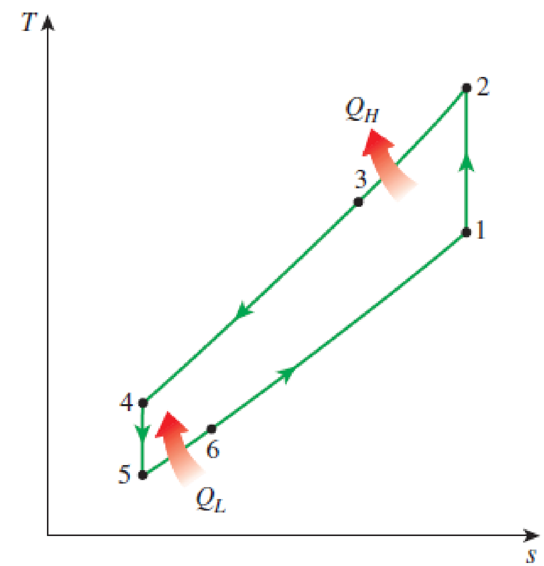


FIGURE 11–21
Gas refrigeration cycle with regeneration.

11-10. ABSORPTION REFRIGERATION SYSTEMS

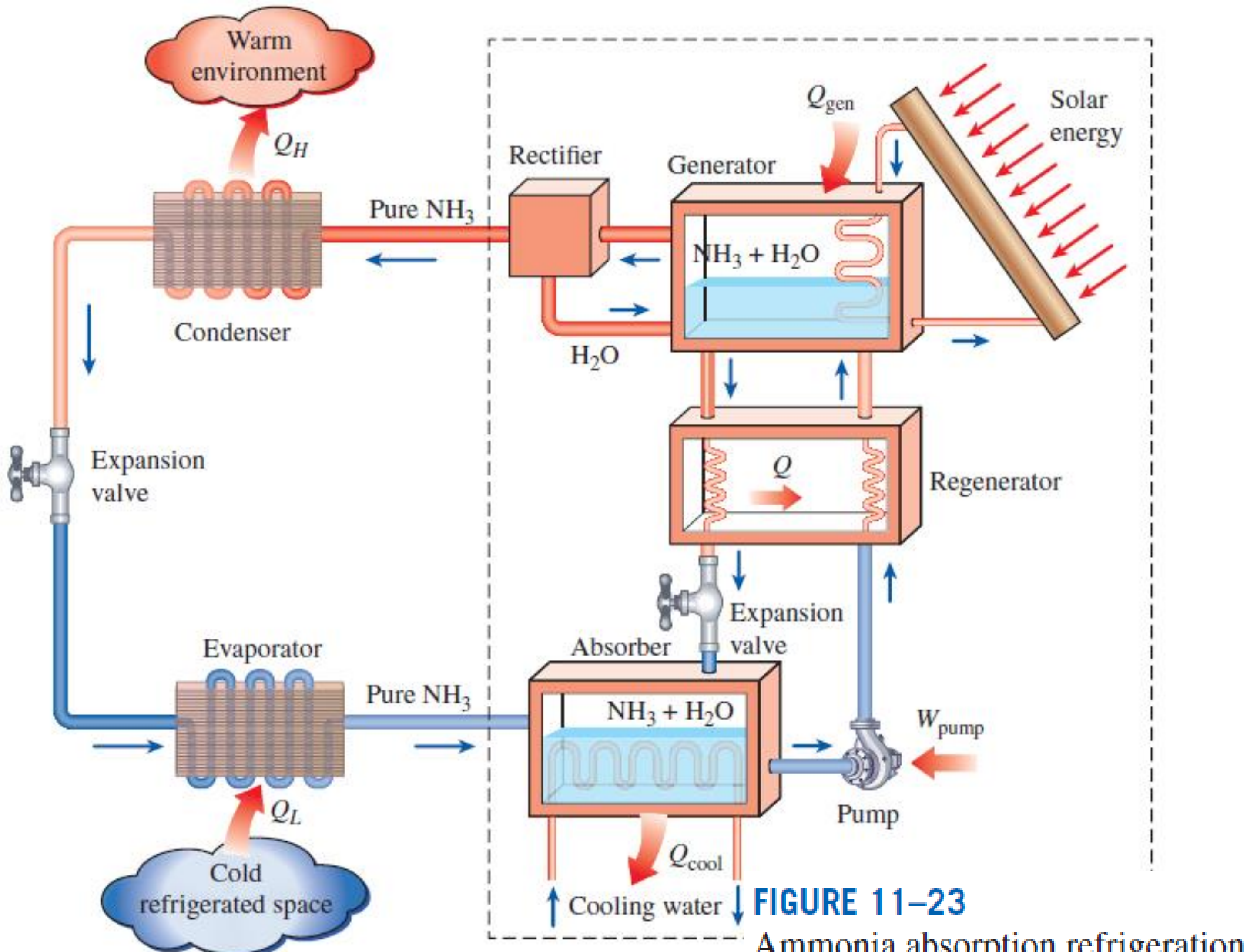


FIGURE 11–23
Ammonia absorption refrigeration cycle.

Absorption refrigeration is economic when there is a source of inexpensive thermal energy at a temperature of 100 to 200°C.

Some examples include geothermal energy, solar energy, and waste heat from cogeneration or process steam plants, and even natural gas when it is at a relatively low price.

11-10. ABSORPTION REFRIGERATION SYSTEMS

- Absorption refrigeration systems (ARS) involve the absorption of a *refrigerant* by a *transport medium*.
- The most widely used system is the ammonia–water system, where ammonia (NH_3) serves as the refrigerant and water (H_2O) as the transport medium.
- Other systems include water–lithium bromide and water–lithium chloride systems, where water serves as the refrigerant. These systems are limited to applications such as A-C where the minimum temperature is above the freezing point of water.
- Compared with vapor-compression systems, ARS have one major advantage: A liquid is compressed instead of a vapor and as a result the work input is very small (on the order of one percent of the heat supplied to the generator) and often neglected in the cycle analysis.

11-10. ABSORPTION REFRIGERATION SYSTEMS

- ARS are often classified as ***heat-driven systems***.
- ARS are much more expensive than the vapor-compression refrigeration systems.
- They are more complex and occupy more space, they are much less efficient thus requiring much larger cooling towers to reject the waste heat, and they are more difficult to service since they are less common.
- Therefore, ARS should be considered only when the unit cost of thermal energy is low and is projected to remain low relative to electricity.
- ARS are primarily used in large commercial and industrial installations.

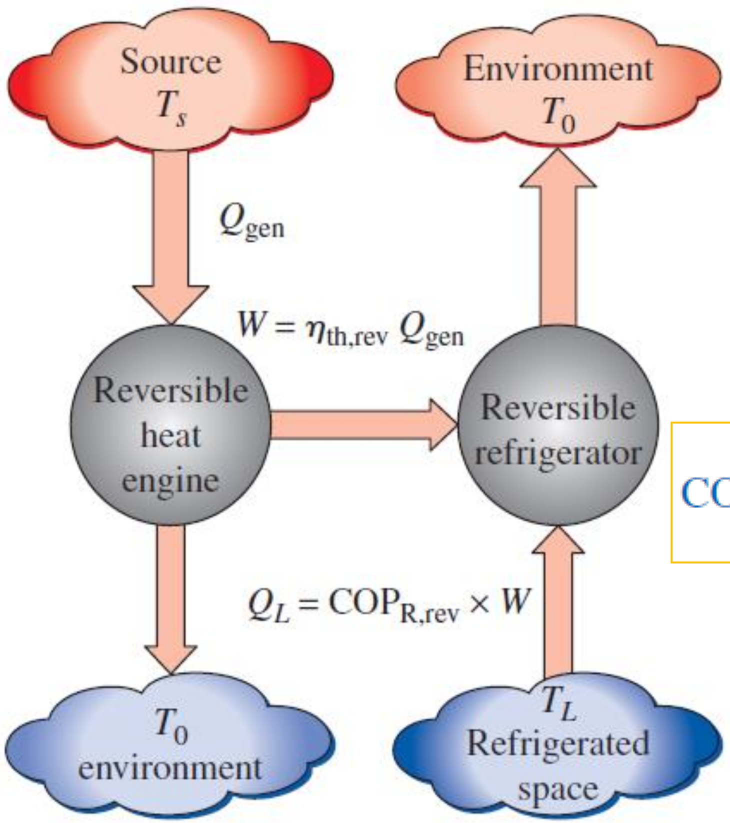
11-10. ABSORPTION REFRIGERATION SYSTEMS

$$\text{COP}_{\text{absorption}} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{Q_{\text{gen}} + W_{\text{pump}}} \cong \frac{Q_L}{Q_{\text{gen}}}$$

The COP of actual absorption refrigeration systems is usually less than 1.

Air-conditioning systems based on absorption refrigeration, called ***absorption chillers***, perform best when the heat source can supply heat at a high temperature with little temperature drop.

11-10. ABSORPTION REFRIGERATION SYSTEMS



$$W = \eta_{th,\,rev}\, Q_{gen} = \left(1 - \frac{T_0}{T_s}\right) Q_{gen}$$

$$Q_L = COP_{R,rev} W = \left(\frac{T_L}{T_0 - T_L}\right) W$$

$$COP_{rev,absorption} = \frac{Q_L}{Q_{gen}} = \left(1 - \frac{T_0}{T_s}\right)\left(\frac{T_L}{T_0 - T_L}\right)$$

$$COP_{rev,absorption} = \frac{Q_L}{Q_{gen}} = \eta_{th,rev} COP_{R,rev} = \left(1 - \frac{T_0}{T_s}\right)\left(\frac{T_L}{T_0 - T_L}\right)$$

FIGURE 11–24
Determining the maximum COP of an absorption refrigeration system.

Summary

- Refrigerators and Heat Pumps
- The Reversed Carnot Cycle
- The Ideal Vapor-Compression Refrigeration Cycle
- Actual Vapor-Compression Refrigeration Cycle
- Second-law Analysis of Vapor-Compression Refrigeration Cycle
- Selecting the Right Refrigerant
- Heat Pump Systems
- Innovative Vapor-Compression Refrigeration Systems
- Gas Refrigeration Cycles
- Absorption Refrigeration Systems