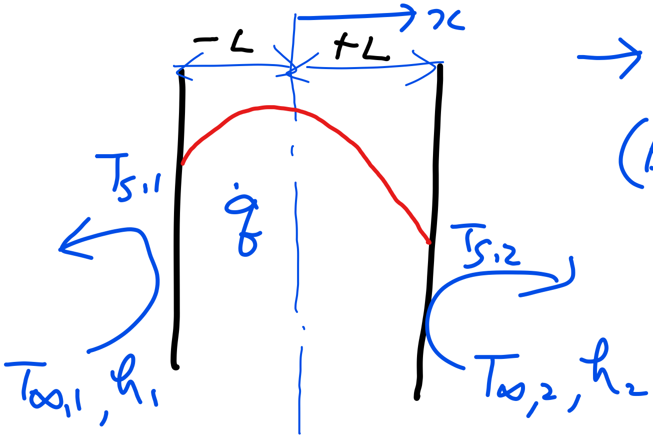
3-1. plane wall

2) 1차원. heat source 有



① Asymmetric case

$\rightarrow \dfrac{d^2T}{dx^2} + \dfrac{\dot{q}}{k} = 0$, 2번 적분하면. $T = -\dfrac{\dot{q}}{2k}x^2 + C_1 x + C_2$

(B.C) $T(-L) = T_{s,1}$: $T_{s,1} = -\dfrac{\dot{q}}{2k}L^2 - C_1 L$

$T(L) = T_{s,2}$: $T_{s,2} = -\dfrac{\dot{q}}{2k}L^2 + C_1 L$ (−

$T_{s,1} - T_{s,2} = -2C_1 L$

$C_1 = \dfrac{T_{s,2} - T_{s,1}}{2L}$, $C_2 = \dfrac{\dot{q}}{2k}L^2 + \dfrac{T_{s,1} + T_{s,2}}{2}$

$$\rightarrow \quad T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \left(\frac{x}{L}\right)^2\right) + \frac{(T_{s,2} - T_{s,1})}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2}$$

c.f. for cylinder case.

$T(r) = \dfrac{\dot{q}r_0^2}{4k}\left(1 - \left(\dfrac{r}{r_0}\right)^2\right) + T_s$

② Symetric case

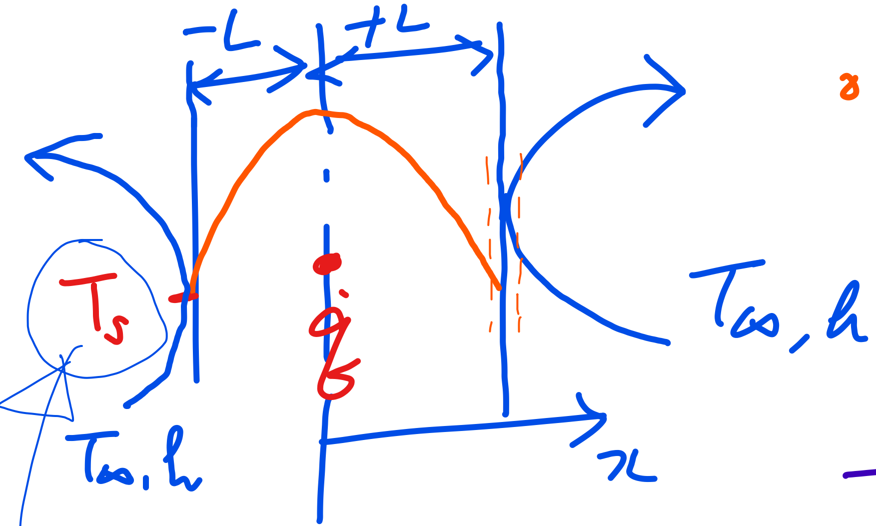
$\rightarrow T_{s,1} = T_{s,2}$, $T(x) = \dfrac{\dot{q}L^2}{2k}\left(1 - \left(\dfrac{x}{L}\right)^2\right) + T_s$

※ 최대온도는 중앙에 존재함

$T_{max} = T(0) = \dfrac{\dot{q}L^2}{2k} + T_s$

$\rightarrow \dfrac{T(x) - T_0}{T_s - T_0} = \left(\dfrac{x}{L}\right)^2$

③ T_s 와 T_∞, h 사이의 상관관계



- Surface Balance

$$\rightarrow -k\left.\frac{dT}{dx}\right|_{x=L} = h(T_s - T_\infty)$$

for symmetric case

$$\rightarrow T(x) = \frac{\dot{q}L^2}{2k}\left(1-\left(\frac{x}{L}\right)^2\right) + T_s$$

$$\frac{dT}{dx} = \frac{\dot{q}L^2}{2k}\left(1 - \frac{2x}{L^2}\right)_{x=L} = -\frac{\dot{q}L^2}{2k}\cdot\frac{2}{L} = -\frac{\dot{q}L}{k}$$

$$\therefore -k\frac{dT}{dx} = -k\left(-\frac{\dot{q}L}{k}\right) = \dot{q}L = h(T_s - T_\infty)$$

$$\therefore T_s = \frac{\dot{q}L}{h} + T_\infty$$

c.f. $T_s = \frac{\dot{q}r_o}{2h} + T_\infty$ for cylinder

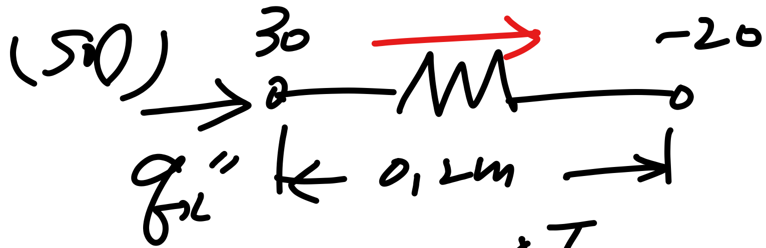
$$\longrightarrow T(x) = \frac{\dot{q}L^2}{2k}\left(1-\left(\frac{x}{L}\right)^2\right) + \left(\frac{\dot{q}L}{h} + T_\infty\right)$$

Prob.

3-1. 두께 0.2m, 벽돌($k = 0.69\ W/mK$). $T_{left} = 30^\circ C$. $T_{right} = -20^\circ C$

q_x''를 열저항개념을 구하여라.

(Sol)



$$R_{th} = \frac{L}{KA} \rightarrow R_{th}' = \frac{R_{th}}{A} = \frac{L}{K}$$

$$\Delta T = 30-(-20) = 50\ (^\circ C)$$

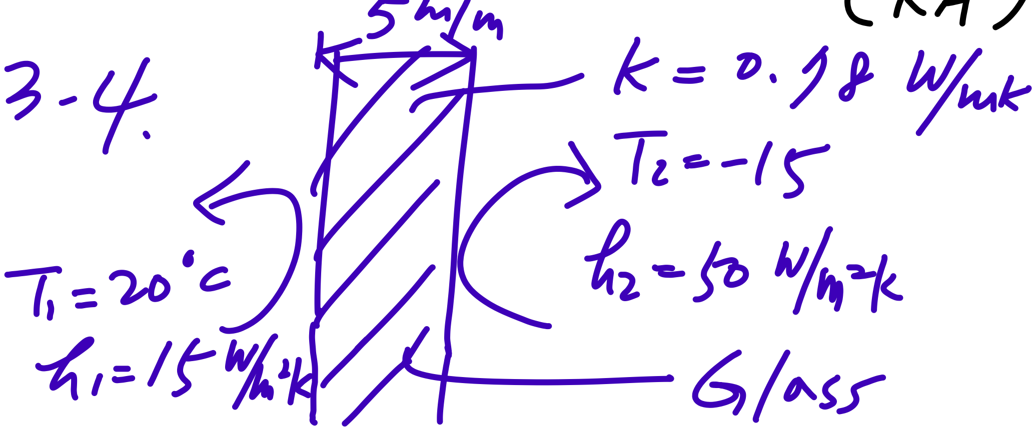
$$\therefore\ q_x'' = \frac{\Delta T}{R_{th}} = \frac{50}{\frac{0.2}{0.69}} = 172.5\ (W/m^2)$$

3-2. $k = 0.69\ W/mK$, $A = 15m^2$, 두께 0.3m. $T_1 = 16^\circ C$, $T_2 = 2^\circ C$

$\dot{Q} = ?$ by 열저항개념

(Sol) $$\dot{Q} = -KA\frac{dT}{dx} = \frac{\Delta T}{\left(\frac{\Delta L}{KA}\right)} = \frac{\Delta T}{R_{th}} = \frac{(16-2)}{\left(\frac{0.3}{0.69 \times 15}\right)} = 483\ (W)$$

3-4.

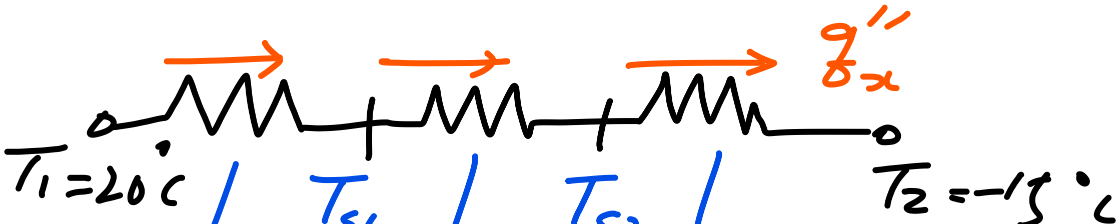


유리 내면(T_{s1})과

유리외면(T_{s2})의 온도를

열저항개념을 구하라!

(Sol) 열등가회로도 작성



$$R'_{th} = \frac{1}{h_1} + \frac{L}{K} + \frac{1}{h_2}$$

$$R'_{tot} = R_1 + R_2 + R_3$$

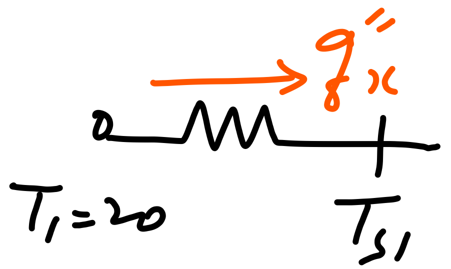
$$R_1 = \frac{1}{h_1} = \frac{1}{15} = 0.067$$

$$R_2 = \frac{L}{K_1} = \frac{0.005}{0.78} = 6.4\times10^{-3}$$

$$+ \quad R_3 = \frac{1}{h_2} = \frac{1}{50} = 0.02$$

$$R'_{tot} = 0.09241$$

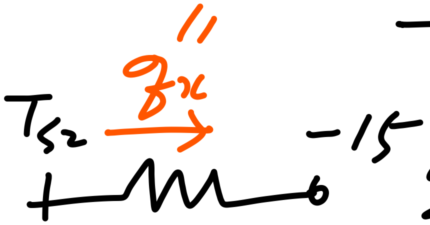
$$\therefore q''_x = \frac{\Delta T}{R'_{tot}} = \frac{20-(-15)}{0.09241} \doteq 375 (W/m^2)$$



; $q''_x = 375 = \dfrac{T_1 - T_{s1}}{R_1} = \dfrac{20 - T_{s1}}{0.067}$ 에서

$$\rightarrow T_{s1} = 20 - 375\times0.067 = -5.125^\circ C$$

Also, T_{s2}



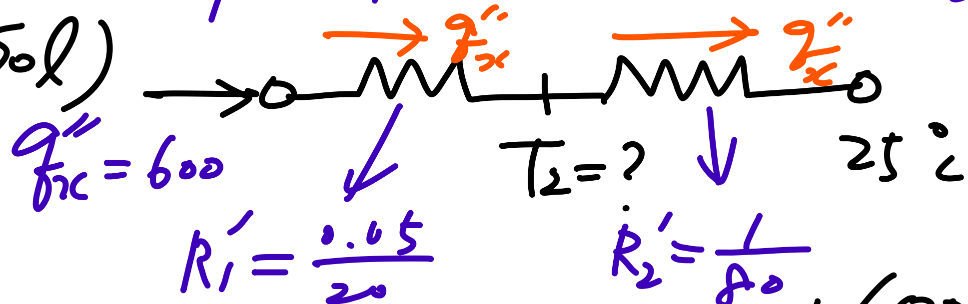
; $q''_x = 375 = \dfrac{T_{s2}-(-15)}{R_3} = \dfrac{T_{s2}+15}{0.02} = 375$

$$\rightarrow T_s = -15 + 375\times0.02 = -7.5\ (^\circ C)$$

(Ans)

3-5. 두께 5cm, $k = 20\ W/mK$, 강판 왼쪽 표면 $q'' = 600\ W/m^2$ heat flux를 받고 오른쪽은 $T_\infty = 25°C$, $h = 80\ W/m^2K$로 대류에 의해 열방출한다. 강판 오른쪽 표면 온도 $T_2 = ?$

(Sol)



$$q''_x = 600 = \frac{T_2 - 25}{R_2'}$$

$$\therefore 600 = \frac{T_2 - 25}{(1/80)} \quad \therefore T_2 = 600 \times \frac{1}{80} + 25 = 32.5°C$$

(Ans)

for 1-D, Steady state, Cylinder

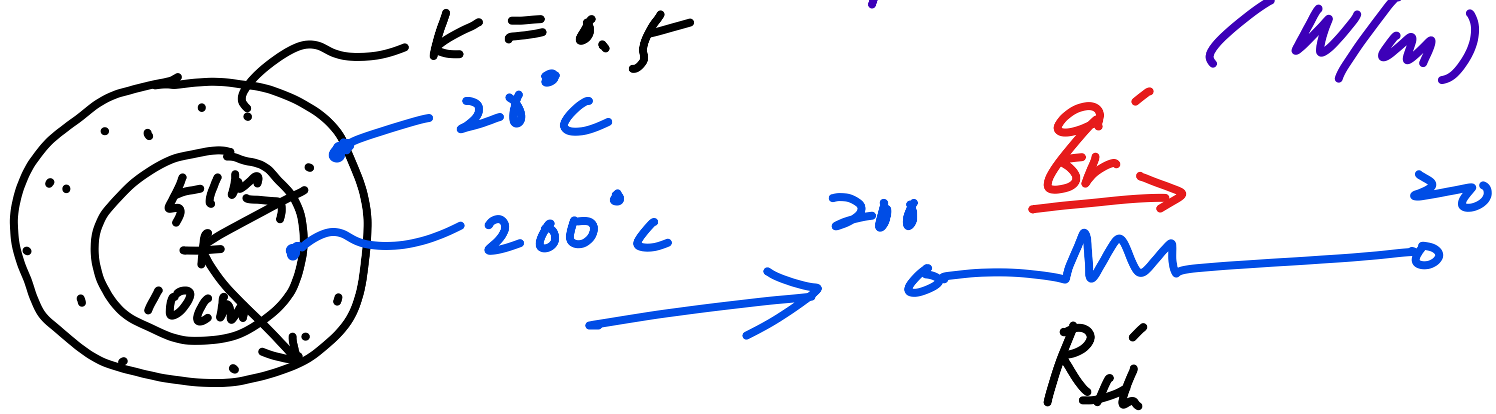
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

($\dot{q}/k \rightarrow 0$, no heat source)

$\rightarrow T = C_1 \ln r + C_2$ 에서 (B.C)으로 C_1, C_2를 구함

3-7. 튜브 단열면, $r_1 = 5cm$, $r_2 = 10cm$, $k = 0.5$
단열층 내부 표면온도 200°C, 외부 20°C 유지시
열저항개념으로 단열튜브 m당 열손실은?
(W/m)

(Sol)

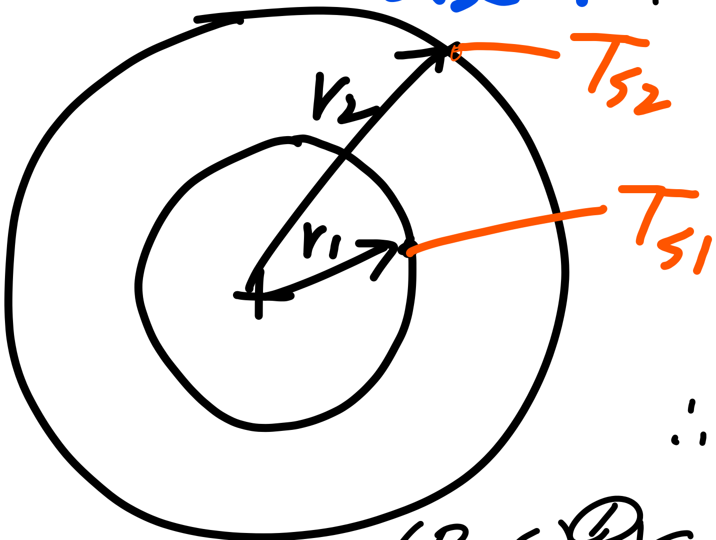


$$R_{th} = \frac{\ln(r_2/r_1)}{2\pi k L} \text{ 에서 } R'_{th} = \frac{\ln(r_2/r_1)}{2\pi k}$$

$$\therefore q'_r = \frac{\Delta T}{R'_{th}} = \frac{(200 - 20)}{\frac{\ln(10/5)}{2\pi \times 0.5}} = 815.4 \text{ (W/m)}$$

(Ans)

Case1. for no heat generation, $\dot{q} = 0$



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

$$r\frac{dT}{dr} = c_1, \quad \frac{dT}{dr} = \frac{c_1}{r}$$

$\therefore T = c_1 \ln r + c_2$ 에서

(B.C) ① $r = r_1 \rightarrow T = T_{s1}$, ② $r = r_2 \rightarrow T = T_{s2}$

$\therefore T_{s1} = c_1 \ln r_1 + c_2, \quad T_{s2} = c_1 \quad r_2 + c_2$

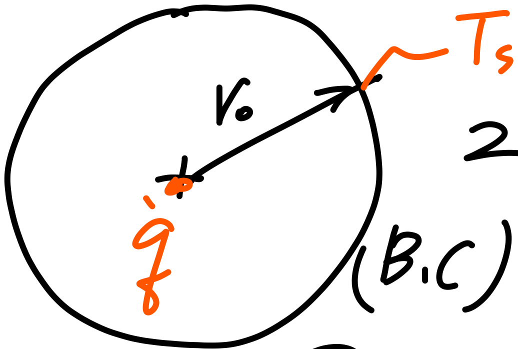
$\rightarrow$ $T(r) = \dfrac{T_{s1} - T_{s2}}{\ln(r_1/r_2)} \ln\left(\dfrac{r}{r_2}\right) + T_{s,2}$ 이다.

$\therefore \dfrac{dT}{dr} = \dfrac{T_{s1} - T_{s2}}{\ln(r_1/r_2)} \cdot \dfrac{1}{r}$ 이고 $\dot{Q} = -kA\dfrac{dT}{dr}$ 에서 $A = (2\pi r L)$ 이므로

$$\dot{Q} = -k \cdot (2\pi r L) \cdot \frac{(T_{s1} - T_{s2})}{\ln(r_1/r_2)} \cdot \frac{1}{r} = \frac{2\pi k L (T_{s1} - T_{s2})}{\ln(r_2/r_1)} = \frac{(T_{s1} - T_{s2})}{\frac{\ln(r_2/r_1)}{2\pi k L}} = \frac{\Delta T}{R_{th}}$$

$$\rightarrow R_{th} = \frac{\ln(r_2/r_1)}{2\pi k L}, \quad R'_{th} = \frac{R_{th}}{L} = \frac{\ln(r_2/r_1)}{2\pi k}$$

Case 2. for heat generation, $\dot{q} \neq 0$



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

2번 적분하면, $T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 r + C_2$

(B.C) ① $\left.\frac{dT}{dr}\right|_{r=0} = 0 \rightarrow \left.\frac{dT}{dr}\right|_{r=0} = -\frac{\dot{q}}{8k}r + C_1 = 0$ $\quad \therefore C_1 = 0$

② $T(r_o) = T_s = -\frac{\dot{q}}{4k}r_o^2 + C_2$ $\quad \therefore C_2 = T_s + \frac{\dot{q}}{4k}r_o^2$

$$\rightarrow T(r) = -\frac{\dot{q}r_o^2}{4k}\left(1 - \left(\frac{r}{r_o}\right)^2\right) + T_s$$

즉, $T(r) - T_s = -\frac{\dot{q}r_o^2}{4k}\left(1 - \left(\frac{r}{r_o}\right)^2\right)$,

Also, $T(0) = T_o = -\frac{\dot{q}r_o^2}{4k} + T_s$

즉, $T_o - T_s = -\frac{\dot{q}r_o^2}{4k}$

따라서 $$\frac{T(r) - T_s}{T_o - T_s} = 1 - \left(\frac{r}{r_o}\right)^2,$$

if, 주변온도 (T_∞)와 h가 주어지면 $T_s = T_\infty + \frac{\dot{q}r_o}{2h}$

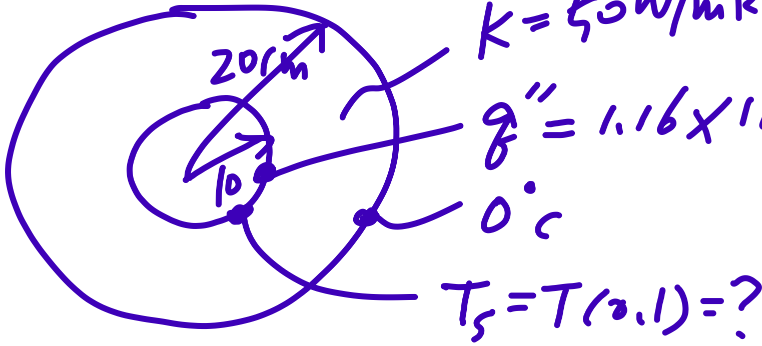
$$\dot{q} \times (\pi r_o^2 \cdot L) = h \cdot (2\pi r_o L) \cdot (T_s - T_\infty)$$

$$\therefore T_s = T_\infty + \frac{\dot{q}r_o}{2h}$$

for plate

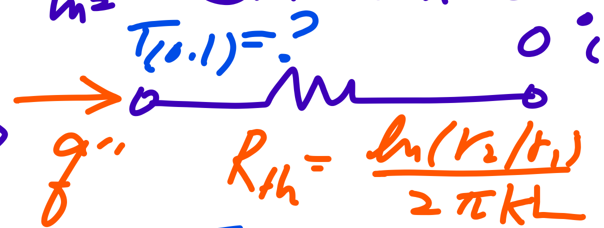
c.f $T_s = T_\infty + \frac{\dot{q}L}{h}$

3-8.



(Sol)

① Method 1.



$$\dot{Q} = A\cdot q'' = (2\pi r_1 L)\cdot q'' = \frac{\Delta T}{R_{th}} = \frac{T(0.1)-0}{\left(\frac{\ln(r_2/r_1)}{2\pi k L}\right)} = \frac{T(0.1)-0}{\frac{\ln(2)}{k}}$$

$$\rightarrow T(0.1) = r_1 \times q'' \times \ln(2) \times \frac{1}{k} = 0.1\times 1.16\times 10^5 \times \ln(2)\times\frac{1}{50} = 160.8\,(^\circ C)$$

(Ans)

② Method 2. $T(r) = c_1 \ln r + c_2$ 에서 (B.C)

- $r = r_2 = 0.2m \rightarrow T(0.2) = 0^\circ C$ ∴ $T(0.2) = c_1 \ln(0.2) + c_2 = 0$

- $r = r_1$ 에서 $\frac{dT}{dr} = \frac{c_1}{r_1}$ 이고 $q'' = -k\frac{dT}{dx}$ 이면 $q'' = -k\frac{c_1}{r_1} = 1.16\times10^5$

∴ $-50\times\frac{c_1}{0.1} = 1.16\times10^5$ 이면 $c_1 = -\frac{0.1\times1.16\times10^5}{50} = -232$

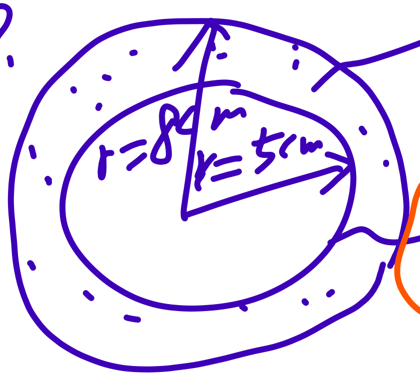
∴ $c_2 = -c_1\ln(0.2) = 232\ln(0.2) = -373.4$

∴ $T(r) = -232\ln r - 373.4$ 이므로

$r = 0.1m$ 인 내부 표면온도는 $T(0.1) = -232\ln(0.1) - 373.4$

$= 160.8\,(^\circ C)$ (Ans)

3-9.



열저항 도출 사용하여 튜브 단위길이당 열손실?

$R_1 = \frac{\ln(r_2/r_1)}{2\pi k L}$, $R_2 = \frac{1}{hA}$, $A \to 2\pi r_2 L$

$R_1' = \frac{R}{L} = \frac{\ln(r_2/r_1)}{2\pi k}$, $R_2' = \frac{R_2}{L} = \frac{1}{2\pi r_2 h}$

$$\therefore \begin{pmatrix} R_1' = \frac{\ln(r_2/r_1)}{2\pi k} \\ R_2' = \frac{1}{2\pi r_2 h} \end{pmatrix} \Rightarrow \frac{\dot{Q}}{L} = \frac{\Delta T}{R_1' + R_2'} = \frac{(130-30)}{\frac{\ln(8/5)}{2\pi \times 0.1} + \frac{1}{2\pi \times (0.08) \times 25}}$$

$$= 120.77 = \underline{120.8\,(W/m)} \quad \text{(Ans)}$$