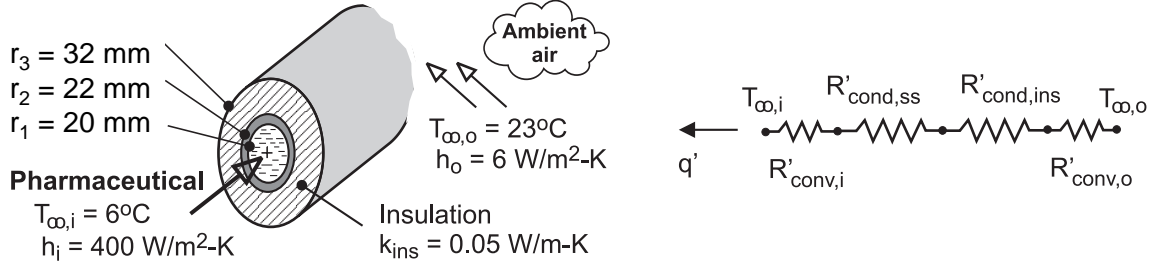


PROBLEM 3.50

KNOWN: Wall thickness and diameter of stainless steel tube. Inner and outer fluid temperatures and convection coefficients.

FIND: (a) Heat gain per unit length of tube, (b) Effect of adding a 10 mm thick layer of insulation to outer surface of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance between tube and insulation, (5) Negligible effect of radiation.

PROPERTIES: Table A-1, Ss 304 (~280K): $k_{st} = 14.2 \text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) Without the insulation, the total thermal resistance per unit length is

$$R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{conv,o} = \frac{1}{2\pi r_1 h_i} + \frac{\ln(r_2/r_1)}{2\pi k_{st}} + \frac{1}{2\pi r_2 h_o}$$

$$R'_{tot} = \frac{1}{2\pi (0.02\text{m}) 400 \text{ W/m}^2\cdot\text{K}} + \frac{\ln(22/20)}{2\pi (14.2 \text{ W/m}\cdot\text{K})} + \frac{1}{2\pi (0.022\text{m}) 6 \text{ W/m}^2\cdot\text{K}}$$

$$R'_{tot} = (0.0188 + 1.07 \times 10^{-3} + 1.21) \text{ m}\cdot\text{K/W} = 1.23 \text{ m}\cdot\text{K/W}$$

The heat gain per unit length is then

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{(23 - 6)^\circ\text{C}}{1.23 \text{ m}\cdot\text{K/W}} = 13.9 \text{ W/m} \quad <$$

(b) With the insulation, the total resistance per unit length is now $R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{cond,ins} + R'_{conv,o}$, where $R'_{conv,i}$ and $R'_{cond,st}$ remain the same. The thermal resistance of the insulation is

$$R'_{cond,ins} = \frac{\ln(r_3/r_2)}{2\pi k_{ins}} = \frac{\ln(32/22)}{2\pi (0.05 \text{ W/m}\cdot\text{K})} = 1.19 \text{ m}\cdot\text{K/W}$$

and the outer convection resistance is now

$$R'_{conv,o} = \frac{1}{2\pi r_3 h_o} = \frac{1}{2\pi (0.032\text{m}) 6 \text{ W/m}^2\cdot\text{K}} = 0.829 \text{ m}\cdot\text{K/W}$$

The total resistance is now

$$R'_{tot} = (0.0199 + 1.07 \times 10^{-3} + 1.19 + 0.829) \text{ m}\cdot\text{K/W} = 2.04 \text{ m}\cdot\text{K/W}$$

Continued ...

PROBLEM 3.50 (Cont.)

and the heat gain per unit length is

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{\text{tot}}} = \frac{17^\circ\text{C}}{2.20 \text{ m} \cdot \text{K} / \text{W}} = 8.33 \text{ W} / \text{m}$$

COMMENTS: (1) The validity of assuming negligible radiation may be assessed for the worst case condition corresponding to the bare tube. Assuming a tube outer surface temperature of $T_s = T_{\infty,i} = 279\text{K}$, large surroundings at $T_{\text{sur}} = T_{\infty,o} = 296\text{K}$, and an emissivity of $\varepsilon = 0.7$ (Table A-11), the heat gain due to net radiation exchange with the surroundings is $q'_{\text{rad}} = \varepsilon\sigma(2\pi r_2)(T_{\text{sur}}^4 - T_s^4) = 8.9 \text{ W} / \text{m}$.

Hence, the net rate of heat transfer by radiation to the tube surface is comparable to that by convection, and the assumption of negligible radiation is inappropriate.

(2) If heat transfer from the air is by natural convection, the value of h_o with the insulation would actually be less than the value for the bare tube, thereby further reducing the heat gain. Use of the insulation would also increase the outer surface temperature, thereby reducing net radiation transfer from the surroundings.

(3) The critical radius is $r_{\text{cr}} = k_{\text{ins}}/h \approx 8 \text{ mm} < r_2$. Hence, as indicated by the calculations, heat transfer is reduced by the insulation.