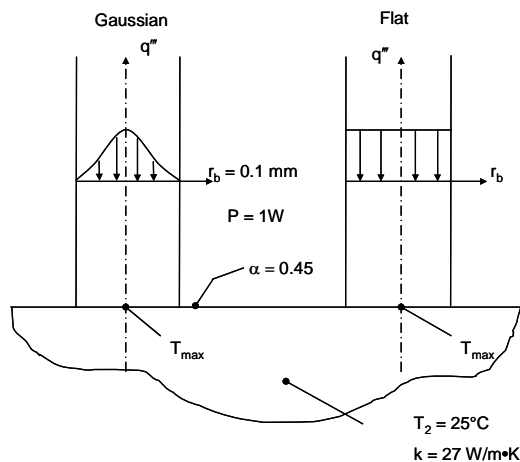


PROBLEM 4.18

KNOWN: Power, size and shape of laser beam. Material properties.

FIND: Maximum surface temperature for a Gaussian beam, maximum temperature for a flat beam, and average temperature for a flat beam.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Semi-infinite medium, (4) Negligible heat loss from the top surface.

ANALYSIS: The shape factor is defined in Eq. 4.20 and is $q = Sk\Delta T_{1-2}$ (1)

From the problem statement and Section 4.3, the shape factors for the three cases are:

Beam Shape	Shape Factor	$T_{1,\text{avg}}$ or $T_{1,\text{max}}$
Gaussian	$2\sqrt{\pi}r_b$	$T_{1,\text{max}}$
Flat	πr_b	$T_{1,\text{max}}$
Flat	$3\pi^2 r_b / 8$	$T_{1,\text{avg}}$

For the Gaussian beam, $S_1 = 2\sqrt{\pi} \times 0.1 \times 10^{-3} \text{ m} = 354 \times 10^{-6} \text{ m}$

For the flat beam (max. temperature), $S_2 = \pi \times 0.1 \times 10^{-3} \text{ m} = 314 \times 10^{-6} \text{ m}$

For the flat beam (avg. temperature), $S_3 = (3/8) \times \pi^2 \times 0.1 \times 10^{-3} \text{ m} = 370 \times 10^{-6} \text{ m}$

The temperature at the heated surface for the three cases is evaluated from Eq. (1) as

$$T_1 = T_2 + q/Sk = T_2 + P\alpha/Sk$$

For the Gaussian beam, $T_{1,\text{max}} = 25^\circ\text{C} + 1 \text{ W} \times 0.45 / (354 \times 10^{-6} \text{ m} \times 27 \text{ W/m}\cdot\text{K}) = 72.1^\circ\text{C} <$

For the flat beam (T_{\max}), $T_{1,\text{max}} = 25^\circ\text{C} + 1 \text{ W} \times 0.45 / (314 \times 10^{-6} \text{ m} \times 27 \text{ W/m}\cdot\text{K}) = 78.1^\circ\text{C} <$

For the flat beam (T_{avg}), $T_{1,\text{avg}} = 25^\circ\text{C} + 1 \text{ W} \times 0.45 / (370 \times 10^{-6} \text{ m} \times 27 \text{ W/m}\cdot\text{K}) = 70.0^\circ\text{C} <$

COMMENTS: (1) The maximum temperature occurs at $r = 0$ for all cases. For the flat beam, the maximum temperature exceeds the average temperature by $78.1 - 70.0 = 8.1$ degrees Celsius.