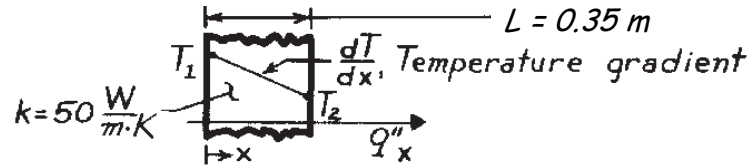


PROBLEM 2.11

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.

FIND: Unknowns for various temperature conditions and sketch distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

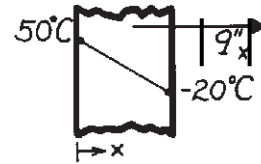
ANALYSIS: The rate equation and temperature gradient for this system are

$$q''_x = -k \frac{dT}{dx} \quad \text{and} \quad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \quad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a) $\frac{dT}{dx} = \frac{(-20 - 50) \text{ K}}{0.35 \text{ m}} = -200 \text{ K/m}$

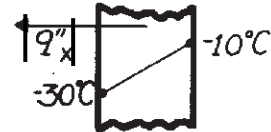
$$q''_x = -50 \frac{\text{W}}{\text{m} \cdot \text{K}} \times \left[-200 \frac{\text{K}}{\text{m}} \right] = 10.0 \text{ kW/m}^2.$$



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(b) $\frac{dT}{dx} = \frac{(-10 - (-30)) \text{ K}}{0.35 \text{ m}} = 57 \text{ K/m}$

$$q''_x = -50 \frac{\text{W}}{\text{m} \cdot \text{K}} \times \left[57 \frac{\text{K}}{\text{m}} \right] = -2.86 \text{ kW/m}^2.$$

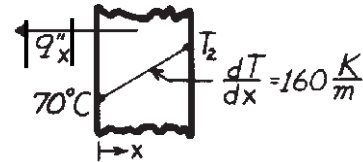


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(c) $q''_x = -50 \frac{\text{W}}{\text{m} \cdot \text{K}} \times \left[160 \frac{\text{K}}{\text{m}} \right] = -8.0 \text{ kW/m}^2$

$$T_2 = L \cdot \frac{dT}{dx} + T_1 = 0.35 \text{ m} \times \left[160 \frac{\text{K}}{\text{m}} \right] + 70^\circ \text{C}.$$

$$T_2 = 126^\circ \text{C}.$$

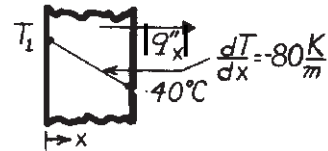


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(d) $q''_x = -50 \frac{\text{W}}{\text{m} \cdot \text{K}} \times \left[-80 \frac{\text{K}}{\text{m}} \right] = 4.0 \text{ kW/m}^2$

$$T_1 = T_2 - L \cdot \frac{dT}{dx} = 40^\circ \text{C} - 0.35 \text{ m} \times \left[-80 \frac{\text{K}}{\text{m}} \right]$$

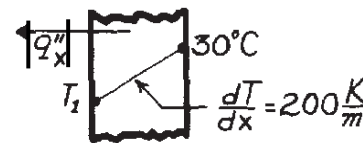
$$T_1 = 68^\circ \text{C}.$$



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(e) $q''_x = -50 \frac{\text{W}}{\text{m} \cdot \text{K}} \times \left[200 \frac{\text{K}}{\text{m}} \right] = -10.0 \text{ kW/m}^2$

$$T_1 = T_2 - L \cdot \frac{dT}{dx} = 30^\circ \text{C} - 0.35 \text{ m} \times \left[200 \frac{\text{K}}{\text{m}} \right] = -40^\circ \text{C}.$$



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