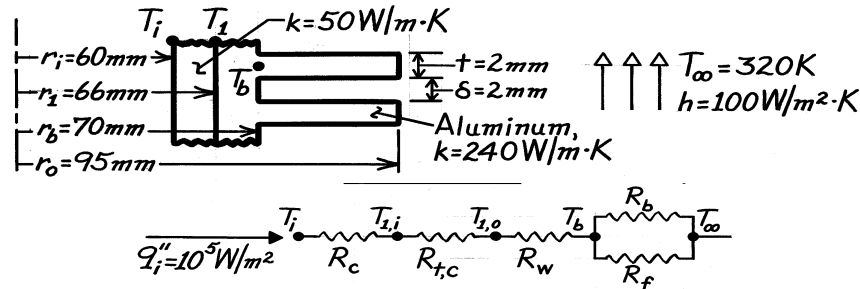


PROBLEM 3.160

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Heat flux and ambient air conditions. Contact resistance.

FIND: Surface and interface temperatures (a) without and (b) with an interface contact resistance.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: The analysis may be performed per unit length of cylinder or for a 4 mm long section. The following calculations are based on a unit length. The inner surface temperature may be obtained from

$$q' = \frac{T_i - T_\infty}{R'_{\text{tot}}} = q''_i (2\pi r_i) = 10^5 \text{ W/m}^2 \times 2\pi \times 0.06 \text{ m} = 37,700 \text{ W/m}$$

where $R'_{\text{tot}} = R'_c + R'_{t,c} + R'_w + R'_{\text{equiv}}$; $R'_{\text{equiv}} = (1/R'_f + 1/R'_b)^{-1}$.

R'_c , Conduction resistance of cylinder wall:

$$R'_c = \frac{\ln(r_1/r_i)}{2\pi k} = \frac{\ln(66/60)}{2\pi (50 \text{ W/m}\cdot\text{K})} = 3.034 \times 10^{-4} \text{ m}\cdot\text{K/W}$$

$R'_{t,c}$, Contact resistance:

$$R'_{t,c} = R''_{t,c} / 2\pi r_1 = 10^{-4} \text{ m}^2 \cdot \text{K/W} / 2\pi \times 0.066 \text{ m} = 2.411 \times 10^{-4} \text{ m}\cdot\text{K/W}$$

R'_w , Conduction resistance of aluminum base:

$$R'_w = \frac{\ln(r_b/r_1)}{2\pi k} = \frac{\ln(70/66)}{2\pi \times 240 \text{ W/m}\cdot\text{K}} = 3.902 \times 10^{-5} \text{ m}\cdot\text{K/W}$$

R'_b , Resistance of prime or unfinned surface:

$$R'_b = \frac{1}{hA'_b} = \frac{1}{100 \text{ W/m}^2 \cdot \text{K} \times 0.5 \times 2\pi (0.07 \text{ m})} = 454.7 \times 10^{-4} \text{ m}\cdot\text{K/W}$$

R'_f , Resistance of fins: The fin resistance may be determined from

$$R'_f = \frac{T_b - T_\infty}{q'_f} = \frac{1}{\eta_f h A'_f}$$

The fin efficiency may be obtained from Fig. 3.20,

$$r_{2c} = r_o + t/2 = 0.096 \text{ m} \quad L_c = L + t/2 = 0.026 \text{ m}$$

Continued ...

PROBLEM 3.160 (Cont.)

$$A_p = L_c t = 5.2 \times 10^{-5} \text{ m}^2 \quad r_{2c}/r_1 = 1.45 \quad L_c^{3/2} (h/kA_p)^{1/2} = 0.375$$

Fig. 3.20 $\rightarrow \eta_f \approx 0.88$.

The total fin surface area per meter length

$$A'_f = 250 \left[\pi (r_o^2 - r_b^2) \times 2 \right] = 250 \text{ m}^{-1} \left[2\pi (0.096^2 - 0.07^2) \right] \text{ m}^2 = 6.78 \text{ m}.$$

Hence $R'_f = \left[0.88 \times 100 \text{ W/m}^2 \cdot \text{K} \times 6.78 \text{ m} \right]^{-1} = 16.8 \times 10^{-4} \text{ m} \cdot \text{K/W}$

$$1/R'_{\text{equiv}} = \left(1/16.8 \times 10^{-4} + 1/454.7 \times 10^{-4} \right) \text{ W/m} \cdot \text{K} = 617.2 \text{ W/m} \cdot \text{K}$$

$$R'_{\text{equiv}} = 16.2 \times 10^{-4} \text{ m} \cdot \text{K/W}.$$

Neglecting the *contact resistance*,

$$R'_{\text{tot}} = (3.034 + 0.390 + 16.2) 10^{-4} \text{ m} \cdot \text{K/W} = 19.6 \times 10^{-4} \text{ m} \cdot \text{K/W}$$

$$T_i = q'R'_{\text{tot}} + T_\infty = 37,700 \text{ W/m} \times 19.6 \times 10^{-4} \text{ m} \cdot \text{K/W} + 320 \text{ K} = 393.9 \text{ K} \quad <$$

$$T_1 = T_i - q'R'_w = 393.9 \text{ K} - 37,700 \text{ W/m} \times 3.034 \times 10^{-4} \text{ m} \cdot \text{K/W} = 382.5 \text{ K} \quad <$$

$$T_b = T_1 - q'R'_b = 382.5 \text{ K} - 37,700 \text{ W/m} \times 3.902 \times 10^{-5} \text{ m} \cdot \text{K/W} = 381.0 \text{ K}. \quad <$$

Including the *contact resistance*,

$$R'_{\text{tot}} = \left(19.6 \times 10^{-4} + 2.411 \times 10^{-4} \right) \text{ m} \cdot \text{K/W} = 22.0 \times 10^{-4} \text{ m} \cdot \text{K/W}$$

$$T_i = 37,700 \text{ W/m} \times 22.0 \times 10^{-4} \text{ m} \cdot \text{K/W} + 320 \text{ K} = 402.9 \text{ K} \quad <$$

$$T_{1,i} = 402.9 \text{ K} - 37,700 \text{ W/m} \times 3.034 \times 10^{-4} \text{ m} \cdot \text{K/W} = 391.5 \text{ K} \quad <$$

$$T_{1,o} = 391.5 \text{ K} - 37,700 \text{ W/m} \times 2.411 \times 10^{-4} \text{ m} \cdot \text{K/W} = 382.4 \text{ K} \quad <$$

$$T_b = 382.4 \text{ K} - 37,700 \text{ W/m} \times 3.902 \times 10^{-5} \text{ m} \cdot \text{K/W} = 380.9 \text{ K}. \quad <$$

COMMENTS: (1) The effect of the contact resistance is small.

(2) The effect of including the aluminum fins may be determined by computing T_i without the fins. In this case $R'_{\text{tot}} = R'_c + R'_{\text{conv}}$, where

$$R'_{\text{conv}} = \frac{1}{h 2\pi r_1} = \frac{1}{100 \text{ W/m}^2 \cdot \text{K} \cdot 2\pi (0.066 \text{ m})} = 241.1 \times 10^{-4} \text{ m} \cdot \text{K/W}.$$

Hence, $R'_{\text{tot}} = 244.1 \times 10^{-4} \text{ m} \cdot \text{K/W}$, and

$$T_i = q'R'_{\text{tot}} + T_\infty = 37,700 \text{ W/m} \times 244.1 \times 10^{-4} \text{ m} \cdot \text{K/W} + 320 \text{ K} = 1240 \text{ K}.$$

Hence, the fins have a significant effect on reducing the cylinder temperature.

(3) The overall surface efficiency is

$$\eta_o = 1 - (A'_f / A'_t)(1 - \eta_f) = 1 - 6.78 \text{ m} / 7.00 \text{ m} (1 - 0.88) = 0.884.$$

It follows that $q' = \eta_o h_o A'_t \theta_b = 37,700 \text{ W/m}$, which agrees with the prescribed value.