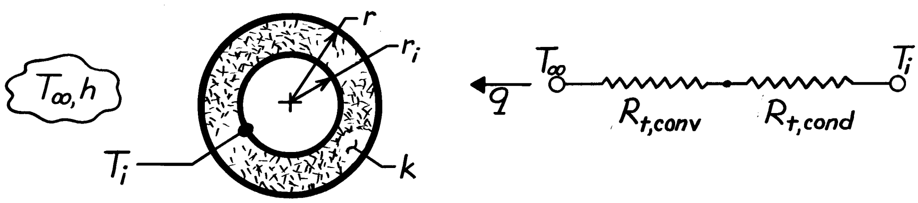
PROBLEM 3.66

KNOWN: Sphere of radius r_i, covered with insulation whose outer surface is exposed to a convection process.

FIND: Critical insulation radius, r_{cr}.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (spherical) conduction, (3) Constant properties, (4) Negligible radiation at surface.

ANALYSIS: The heat rate follows from the thermal circuit shown in the schematic,

$$q=\left(T_i - T_\infty\right)/R_{tot}$$

where $R_{tot} = R_{t,conv} + R_{t,cond}$ and

$$R_{t,conv} = \frac{1}{hA_s} = \frac{1}{4\pi\ hr^2} \qquad (3.9)$$

$$R_{t,cond} = \frac{1}{4\pi\ k}\left[\frac{1}{r_i} - \frac{1}{r}\right] \qquad (3.36)$$

If q is a maximum or minimum, we need to find the condition for which

$$\frac{d\,R_{tot}}{dr} = 0.$$

It follows that

$$\frac{d}{dr}\left[\frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r}\right] + \frac{1}{4\pi\ hr^2}\right] = \left[+\frac{1}{4\pi\ k}\frac{1}{r^2} - \frac{1}{2\pi\ h}\frac{1}{r^3}\right] = 0$$

giving

$$r_{cr} = 2\frac{k}{h}$$

The second derivative, evaluated at $r = r_{cr}$, is

$$\frac{d}{dr}\left[\frac{dR_{tot}}{dr}\right] = -\frac{1}{2\pi\ k}\ \frac{1}{r^3} + \frac{3}{2\pi\ h}\ \frac{1}{r^4}\Bigg]_{r=r_{cr}}$$

$$= \frac{1}{\left(2k/h\right)^3}\left\{-\frac{1}{2\pi\ k} + \frac{3}{2\pi\ h}\frac{1}{2k/h}\right\} = \frac{1}{\left(2k/h\right)^3}\frac{1}{2\pi\ k}\left\{-1 + \frac{3}{2}\right\} > 0$$

Hence, it follows no optimum R_{tot} exists. We refer to this condition as the critical insulation radius. See Example 3.6 which considers this situation for a cylindrical system.