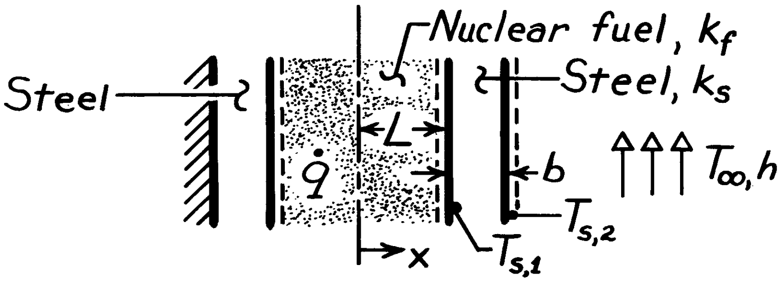
PROBLEM 3.90

KNOWN: Geometry and boundary conditions of a nuclear fuel element.

FIND: (a) Expression for the temperature distribution in the fuel, (b) Form of temperature distribution for the entire system.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance between fuel and cladding.

ANALYSIS: (a) The general solution to the heat equation, Eq. 3.44,

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k_f}=0 \qquad \left(-L \le x \le +L\right)$$

is $$T=-\frac{\dot{q}}{2k_f}x^2+C_1x+C_2.$$

The insulated wall at x = - (L+b) dictates that the heat flux at x = - L is zero (for an energy balance applied to a control volume about the wall, $\dot{E}_{in}=\dot{E}_{out}=0$). Hence

$$\left.\frac{dT}{dx}\right]_{x=-L}=-\frac{\dot{q}}{k_f}\left(-L\right)+C_1=0 \qquad \text{or} \qquad C_1=-\frac{\dot{q}L}{k_f}$$

$$T=-\frac{\dot{q}}{2k_f}x^2-\frac{\dot{q}L}{k_f}x+C_2. \qquad (1)$$

The value of $T_{s,1}$ may be determined from the energy conservation requirement that $\dot{E}_g=q_{cond}=q_{conv}$, or on a unit area basis.

$$\dot{q}\left(2L\right)=\frac{k_s}{b}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_\infty\right).$$

Hence,

$$T_{s,1}=\frac{\dot{q}\left(2\,Lb\right)}{k_s}+T_{s,2} \qquad \text{where} \qquad T_{s,2}=\frac{\dot{q}\left(2L\right)}{h}+T_\infty$$

$$T_{s,1}=\frac{\dot{q}\left(2\,Lb\right)}{k_s}+\frac{\dot{q}\left(2L\right)}{h}+T_\infty.$$

Continued …

PROBLEM 3.90 (Cont.)

Hence from Eq. (1),

$$T(L) = T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2\,L)}{h} + T_\infty = -\frac{3}{2}\,\frac{\dot{q}\left(L^2\right)}{k_f} + C_2$$

which yields

$$C_2 = T_\infty + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\,\frac{L}{k_f}\right]$$

Hence, the temperature distribution for $(-L \le x \le +L)$ is

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\,\frac{L}{k_f}\right] + T_\infty$$ <

(b) For the temperature distribution shown below,

$(-L - b) \le x \le -L$: $dT/dx = 0$, $T = T_{max}$

$-L \le x \le +L$: $|\,dT/dx\,| \uparrow$ with $\uparrow$ x

$+L \le x \le L+b$: (dT/dx) is const.