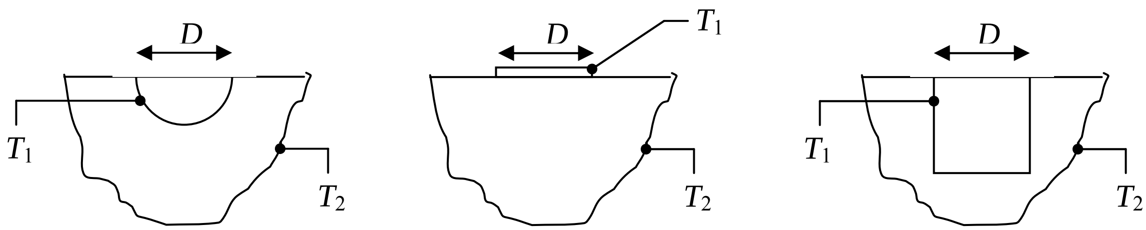
PROBLEM 4.10

KNOWN: Shape of objects at surface of semi-infinite medium.

FIND: Shape factors between object at temperature T_1 and semi-infinite medium at temperature T_2.

SCHEMATIC:



(a) (b) (c)

ASSUMPTIONS: (1) Steady-state, (2) Medium is semi-infinite, (3) Constant properties, (4) Surface of semi-infinite medium is adiabatic.

ANALYSIS: Cases 12 -15 of Table 4.1 all pertain to objects buried in an infinite medium. Since they all possess symmetry about a horizontal plane that bisects the object, they are equivalent to the cases given in this problem for which the horizontal plane is adiabatic. In particular, the heat flux is the same for the cases of this problem as for the cases of Table 4.1. Note, that when we use Table 4.1 to determine the dimensionless conduction heat rate, q_{ss}^*, we must be consistent and use the surface area of the "entire" object of Table 4.1, not the "half" object of this problem. Then

$$q'' = \frac{q}{A_s} = \frac{q_{ss}^* k(T_1 - T_2)}{L_c}$$

where $L_c = (A_s/4\pi)^{1/2}$ and A_s is the area given in Table 4.1

When we calculate the shape factors we must account for the fact that the surface areas and heat transfer rates for the objects of this problem are half as much as for the objects of Table 4.1.

$$S = \frac{q}{k(T_1 - T_2)} = \frac{q'' A_s/2}{k(T_1 - T_2)} = \frac{q_{ss}^* A_s}{2L_c} = \frac{q_{ss}^* (4\pi A_s)^{1/2}}{2}$$

where A_s is still the area in table 4.1 and the 2 in the denominator accounts for the area being halved. Thus, finally,

$$S = q_{ss}^* (\pi A_s)^{1/2}$$

(a) $S = 1 \cdot (\pi \cdot \pi D^2)^{1/2} = \pi D$ <

(b) $S = \dfrac{2\sqrt{2}}{\pi}\left(\pi \cdot \dfrac{\pi D^2}{2}\right)^{1/2} = 2D$ <

This agrees with Table 4.1a, Case 10.

(c) $S = 0.932(\pi \cdot 2D^2)^{1/2} = \sqrt{2\pi}(0.932)D = 2.34D$ <

(d) The height of the "whole object" is d = 2D. Thus

$$S = 0.961\left[\pi\left(2D^2 + 4D \cdot 2D\right)\right]^{1/2} = \sqrt{10\pi}(0.961)D = 5.39D$$ <