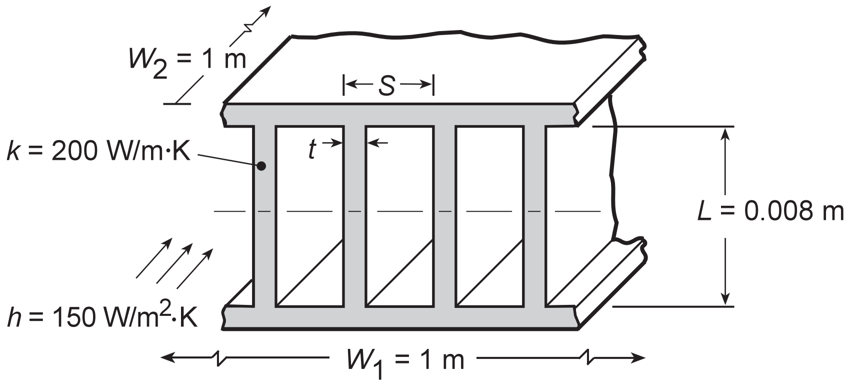
PROBLEM 3.143

KNOWN: Conditions associated with an array of straight rectangular fins.

FIND: Thermal resistance of the array.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Uniform convection coefficient, (3) Symmetry about midplane.

ANALYSIS: (a) Considering a one-half section of the array, the corresponding resistance is

$$R_{t,o} = \left(\eta_o h A_t\right)^{-1}$$

where $A_t = NA_f + A_b$. With S = 4 mm and t = 1 mm, it follows that N = W_1/S = 250, A_f = $2(L/2)W_2 = 0.008\ m^2$, $A_b = W_2(W_1 - Nt) = 0.75\ m^2$, and $A_t = 2.75\ m^2$.
The overall surface efficiency is

$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right)$$

where the fin efficiency is

$$\eta_f = \frac{\tanh m(L/2)}{m(L/2)} \quad \text{and} \quad m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left[\frac{h\left(2t + 2W_2\right)}{ktW_2}\right]^{1/2} \approx \left(\frac{2h}{kt}\right)^{1/2} = 38.7\text{m}^{-1}$$

With m(L/2) = 0.155, it follows that η_f = 0.992 and η_o = 0.994. Hence

$$R_{t,o} = \left(0.994 \times 150\text{W/m}^2 \cdot \text{K} \times 2.75\text{m}^2\right)^{-1} = 2.44 \times 10^{-3}\text{K/W}$$ <

(b) The requirements that t ≥ 0.5 m and (S - t) > 2 mm are based on manufacturing and flow passage restriction constraints. Repeating the foregoing calculations for representative values of t and (S - t), we obtain

S (mm)	N	t (mm)	$R_{t,o}$ (K/W)
2.5	400	0.5	0.00169
3	333	0.5	0.00193
3	333	1	0.00202
4	250	0.5	0.00234
4	250	2	0.00268
5	200	0.5	0.00269
5	200	3	0.00334

COMMENTS: Clearly, the thermal performance of the fin array improves ($R_{t,o}$ decreases) with increasing N. Because $\eta_f \approx 1$ for the entire range of conditions, there is a slight degradation in performance ($R_{t,o}$ increases) with increasing t and fixed N. The reduced performance is associated with the reduction in surface area of the exposed base. Note that the overall thermal resistance for the entire fin array (top and bottom) is $R_{t,o}/2 = 1.22 \times 10^{-2}$ K/W.