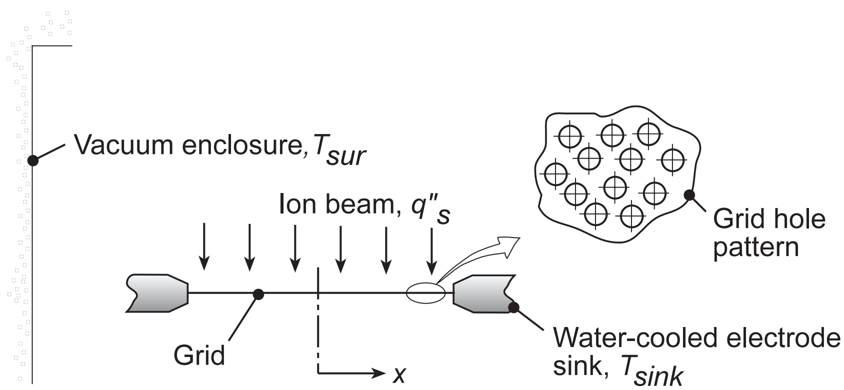
PROBLEM 4.72

KNOWN: Thin metallic foil of thickness, t, whose edges are thermally coupled to a sink at temperature T_{sink} is exposed on the top surface to an ion beam heat flux, q''_s, and experiences radiation exchange with the vacuum enclosure walls at T_{sur}.

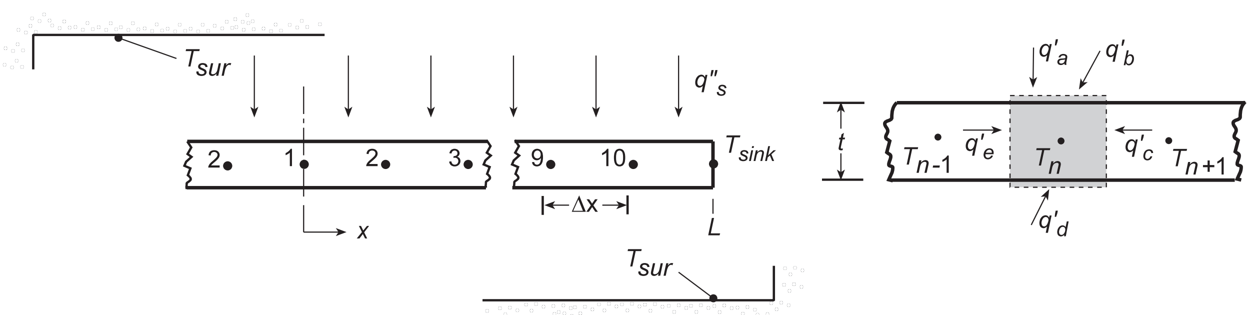
FIND: Temperature distribution across the foil.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in the foil, (2) Constant properties, (3) Upper and lower surfaces of foil experience radiation exchange, (4) Foil is of unit length normal to the page.

ANALYSIS: The 10-node network representing the foil is shown below.



From an energy balance on node n, $\dot{E}_{in} - \dot{E}_{out} = 0$, for a unit depth,

$$q'_a + q'_b + q'_c + q'_d + q'_e = 0$$

$$q''_s \Delta x + h_{r,n} \Delta x (T_{sur} - T_n) + k(t)(T_{n+1} - T_n)/\Delta x + h_{r,n} \Delta x (T_{sur} - T_n) + k(t)(T_{n-1} - T_n)/\Delta x = 0 \quad (1)$$

where the linearized radiation coefficient for node n is

$$h_{r,n} = \varepsilon\sigma (T_{sur} + T_n)\left(T_{sur}^2 + T_n^2\right) \quad (2)$$

Solving Eq. (1) for T_n find,

$$T_n = \left[(T_{n+1} + T_{n-1}) + \left(2h_{r,n}\Delta x^2 / kt\right) T_{sur} + \left(\Delta x^2 / kt\right) q''_s\right] \Big/ \left[\left(h_{r,n}\Delta x^2 / kt\right) + 2\right] \quad (3)$$

which, considering symmetry, applies also to node 1. Using IHT for Eqs. (3) and (2), the set of finite-difference equations was solved for the temperature distribution (K):

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
374.1	374.0	373.5	372.5	370.9	368.2	363.7	356.6	345.3	327.4

Continued...

COMMENTS: (1) If the temperature gradients were excessive across the foil, it would wrinkle; most likely since its edges are constrained, the foil will bow.

(2) The IHT workspace for the finite-difference analysis follows:

```
// The nodal equations:
T1 = ( (T2 + T2) + A1 * Tsur +  B *q"s ) / ( A1 + 2)
A1= 2 * hr1 * deltax^2 / (k * t)
hr1 = eps * sigma * (Tsur + T1) * (Tsur^2 + T1^2)
sigma = 5.67e-8
B = deltax^2 / (k * t)

T2 = ( (T1 + T3) + A2 * Tsur + B *q"s ) / ( A2 + 2)
A2= 2 * hr2 * deltax^2 / (k * t)
hr2 = eps * sigma * (Tsur + T2) * (Tsur^2 + T2^2)

T3 = ( (T2 + T4) + A3 * Tsur + B *q"s ) / ( A3 + 2)
A3= 2 * hr3 * deltax^2 / (k * t)
hr3 = eps * sigma * (Tsur + T3) * (Tsur^2 + T3^2)

T4 = ( (T3 + T5) + A4 * Tsur + B *q"s ) / ( A4 + 2)
A4= 2 * hr4 * deltax^2 / (k * t)
hr4 = eps * sigma * (Tsur + T4) * (Tsur^2 + T4^2)

T5 = ( (T4 + T6) + A5 * Tsur + B *q"s ) / ( A5 + 2)
A5= 2 * hr5 * deltax^2 / (k * t)
hr5 = eps * sigma * (Tsur + T5) * (Tsur^2 + T5^2)

T6 = ( (T5 + T7) + A6 * Tsur + B *q"s ) / ( A6 + 2)
A6= 2 * hr6 * deltax^2 / (k * t)
hr6 = eps * sigma * (Tsur + T6) * (Tsur^2 + T6^2)

T7 = ( (T6 + T8) + A7 * Tsur + B *q"s ) / ( A7 + 2)
A7= 2 * hr7 * deltax^2 / (k * t)
hr7 = eps * sigma * (Tsur + T7) * (Tsur^2 + T7^2)

T8 = ( (T7 + T9) + A8 * Tsur + B *q"s ) / ( A8 + 2)
A8= 2 * hr8 * deltax^2 / (k * t)
hr8 = eps * sigma * (Tsur + T8) * (Tsur^2 + T8^2)

T9 = ( (T8 + T10) + A9 * Tsur + B *q"s ) / ( A9 + 2)
A9= 2 * hr9 * deltax^2 / (k * t)
hr9 = eps * sigma * (Tsur + T9) * (Tsur^2 + T9^2)

T10 = ( (T9 + Tsink) + A10 * Tsur + B *q"s ) / ( A10 + 2)
A10= 2 * hr10 * deltax^2 / (k * t)
hr10 = eps * sigma * (Tsur + T10) * (Tsur^2 + T10^2)

// Assigned variables
deltax = L / 10                // Spatial increment, m
L = 0.150                      // Foil length, m
t = 0.00025                    // Foil thickness, m
eps = 0.45                     // Emissivity
Tsur = 300                     // Surroundings temperature, K
k = 40                         // Foil thermal conductivity, W/m.K
Tsink = 300                    // Sink temperature, K
q"s = 600                      // Ion beam heat flux, W/m^2

/* Data Browser results: Temperature distribution (K)  and linearized radiation cofficients
(W/m^2.K):

T1      T2      T3      T4      T5      T6      T7      T8      T9      T10
374.1   374     373.5   372.5   370.9   368.2   363.7   356.6   345.3   327.4

hr1     hr2     hr3     hr4     hr5     hr6     hr7     hr8     hr9     hr10
3.956   3.953   3.943   3.926   3.895   3.845   3.765   3.639   3.444   3.157 */
```