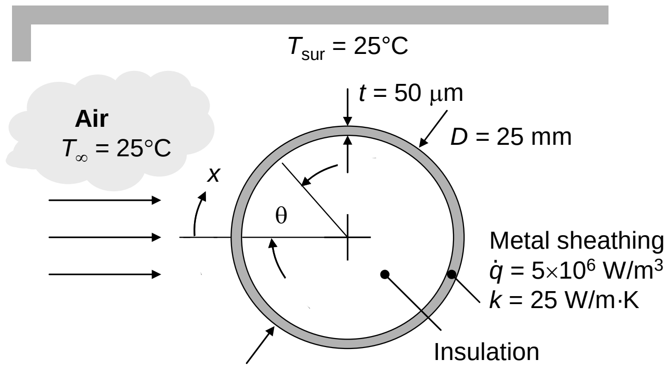
PROBLEM 4.70

KNOWN: Diameter of long cylinder, thickness of metal sheathing, volumetric generation rate within the sheathing, thermal conductivity of sheathing and convection heat transfer coefficient dependence upon angle θ. Emissivity of the sheathing.

FIND: (a) Temperature distribution within the thin sheathing accounting for convection, conduction in the sheathing, and radiation exchange with the surroundings.

SCHEMATIC:

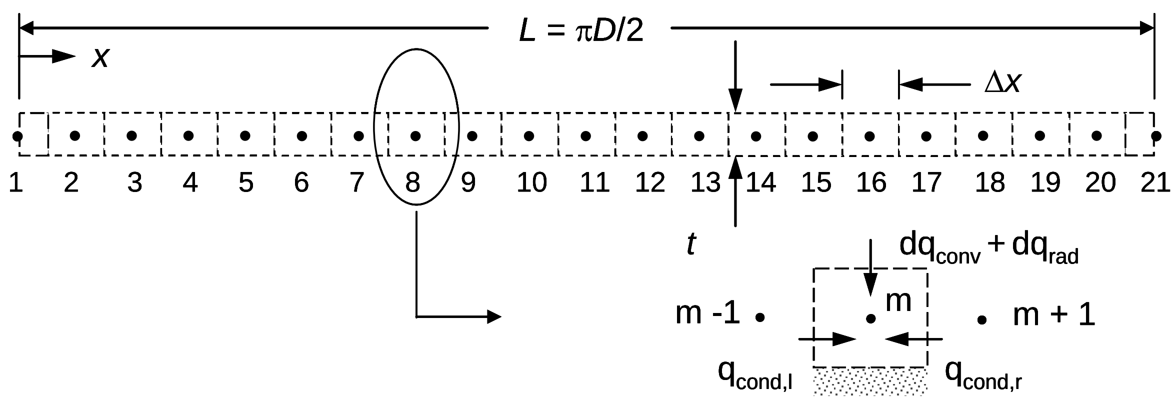


ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform internal generation, (4) Metal sheathing is very thin relative to cylinder diameter, (5) One-dimensional conduction, (6) Large surroundings.

ANALYSIS: From Problem 4.69,

$$h(\theta) = 26 + 0.637\theta - 8.92\theta^2 \text{ for } 0 \le \theta < \pi/2; \;\; h(\theta) = 5 \text{ for } \pi/2 \le \theta \le \pi$$

Since the sheathing is thin relative to the cylinder diameter, we may evaluate one-dimensional conduction in the x-direction using the Cartesian coordinate system. The finite difference equations are derived by combining expressions for heat fluxes determined from Fourier's law, Newton's law of cooling, and Eq. 1.7 along with conservation of energy for each control volume within the discretized domain. Application of conservation of energy for each control volume yields the expression $\dot{E}_{in} + \dot{E}_g = 0$. The discretized domain is shown below.



Energy balances for the control volumes are as follows.

Node 1: $$k\frac{(T_2 - T_1)}{\Delta x}t + h(\Delta x/2)(T_\infty - T_1) + \varepsilon\sigma(\Delta x/2)(T_{sur}^4 - T_1^4) + \dot{q}(\Delta x/2)t = 0$$

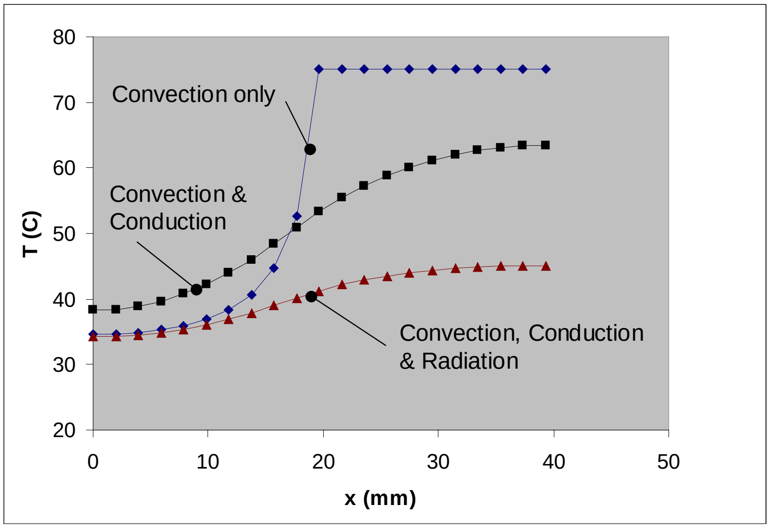
Continued…

PROBLEM 4.70 (Cont.)

Nodes 2 - 20: $k\dfrac{(T_{m-1}-T_m)}{\Delta x}t + k\dfrac{(T_{m+1}-T_m)}{\Delta x}t + h(\Delta x)(T_\infty - T_m) + \varepsilon\sigma(\Delta x)(T_{\text{sur}}^4 - T_m^4) + \dot{q}(\Delta x)t = 0$

Node 21: $k\dfrac{(T_{20}-T_{21})}{\Delta x}t + h(\Delta x/2)(T_\infty - T_{21}) + \varepsilon\sigma(\Delta x/2)(T_{\text{sur}}^4 - T_{21}^4) + \dot{q}(\Delta x/2)t = 0$

The temperature distribution is plotted below.



COMMENTS: (1) Inclusion of radiation in the analysis shows that the resulting temperatures are reduced overall, as expected. The effects of conduction and radiation on local temperatures are comparable. (2) The *IHT* code is listed below.

```
D = 25/1000                //m
L = pi*D/2                 //m
dx = L/20                  //m

qdot = 5*10^6              //W/m^3
t = 50*10^(-6)             //m
Tinf = 25 + 273            //K
k = 25                     //W/m·K
eps = 0.98                 //unitless
sigma=5.67*10^-8           //Stefan-Boltzmann constant, W/m^2·K^4
Tsur = Tinf                //K

h1 = 26                    //W/m^2·K
h2 = 25.88
h3 = 25.32
h4 = 24.32
h5 = 22.88
h6 = 21.00
h7 = 18.68
h8 = 15.92
h9 = 12.71
h10 = 9.07
h11 = 5
h12 = 5
h13 = 5
h14 = 5
h15 = 5
h16 = 5
h17 = 5
```

Continued…

```
h18 = 5
h19 = 5
h20 = 5
h21 = 5

//Node 1
k*(T2 - T1)*t/dx + h1*(dx/2)*(Tinf - T1) + eps*sigma*(dx/2)*(Tsur^4 - T1^4) + qdot*dx*t/2 = 0
//Node 2
k*(T1 - T2)*t/dx + k*(T3 - T2)*t/dx + h2*dx*(Tinf - T2) + eps*sigma*dx*(Tsur^4 - T2^4) + qdot*dx*t = 0
//Node 3
k*(T2 - T3)*t/dx + k*(T4 - T3)*t/dx + h3*dx*(Tinf - T3) + eps*sigma*dx*(Tsur^4 - T3^4) + qdot*dx*t = 0
//Node 4
k*(T3 - T4)*t/dx + k*(T5 - T4)*t/dx + h4*dx*(Tinf - T4) + eps*sigma*dx*(Tsur^4 - T4^4) + qdot*dx*t = 0
//Node 5
k*(T4 - T5)*t/dx + k*(T6 - T5)*t/dx + h5*dx*(Tinf - T5) + eps*sigma*dx*(Tsur^4 - T5^4) + qdot*dx*t = 0
//Node 6
k*(T5 - T6)*t/dx + k*(T7 - T6)*t/dx + h6*dx*(Tinf - T6) + eps*sigma*dx*(Tsur^4 - T6^4) + qdot*dx*t = 0
//Node 7
k*(T6 - T7)*t/dx + k*(T8 - T7)*t/dx + h7*dx*(Tinf - T7) + eps*sigma*dx*(Tsur^4 - T7^4) + qdot*dx*t = 0
//Node 8
k*(T7 - T8)*t/dx + k*(T9 - T8)*t/dx + h8*dx*(Tinf - T8) + eps*sigma*dx*(Tsur^4 - T8^4) + qdot*dx*t = 0
//Node 9
k*(T8 - T9)*t/dx + k*(T10 - T9)*t/dx + h9*dx*(Tinf - T9) + eps*sigma*dx*(Tsur^4 - T9^4) + qdot*dx*t  = 0
//Node 10
k*(T9 - T10)*t/dx + k*(T11 - T10)*t/dx + h10*dx*(Tinf - T10) + eps*sigma*dx*(Tsur^4 - T10^4) +
qdot*dx*t = 0
//Node 11
k*(T10 - T11)*t/dx + k*(T12 - T11)*t/dx + h11*dx*(Tinf - T11) + eps*sigma*dx*(Tsur^4 - T11^4) +
qdot*dx*t = 0
//Node 12
k*(T11 - T12)*t/dx + k*(T13 - T12)*t/dx + h12*dx*(Tinf - T12) + eps*sigma*dx*(Tsur^4 - T12^4) +
qdot*dx*t = 0
//Node 13
k*(T12 - T13)*t/dx + k*(T14 - T13)*t/dx + h13*dx*(Tinf - T13) + eps*sigma*dx*(Tsur^4 - T13^4) +
qdot*dx*t = 0
//Node 14
k*(T13 - T14)*t/dx + k*(T15 - T14)*t/dx + h14*dx*(Tinf - T14) + eps*sigma*dx*(Tsur^4 - T14^4) +
qdot*dx*t = 0
//Node 15
k*(T14 - T15)*t/dx + k*(T16 - T15)*t/dx + h15*dx*(Tinf - T15) + eps*sigma*dx*(Tsur^4 - T15^4) +
qdot*dx*t = 0
//Node 16
k*(T15 - T16)*t/dx + k*(T17 - T16)*t/dx + h16*dx*(Tinf - T16) + eps*sigma*dx*(Tsur^4 - T16^4) +
qdot*dx*t  = 0
//Node 17
k*(T16 - T17)*t/dx + k*(T18 - T17)*t/dx + h17*dx*(Tinf - T17) + eps*sigma*dx*(Tsur^4 - T17^4) +
qdot*dx *t= 0
//Node 18
k*(T17 - T18)*t/dx + k*(T19 - T18)*t/dx + h18*dx*(Tinf - T18) + eps*sigma*dx*(Tsur^4 - T18^4) +
qdot*dx*t  = 0
//Node 19
k*(T18 - T19)*t/dx + k*(T20 - T19)*t/dx + h19*dx*(Tinf - T19) + eps*sigma*dx*(Tsur^4 - T19^4) +
qdot*dx*t = 0
//Node 20
k*(T19 - T20)*t/dx + k*(T21 - T20)*t/dx + h20*dx*(Tinf - T20) + eps*sigma*dx*(Tsur^4 - T20^4) +
qdot*dx*t = 0
//Node 21
k*(T20 - T21)*t/dx +h21*(dx/2)*(Tinf - T21) + eps*sigma*(dx/2)*(Tsur^4 - T21^4) + qdot*dx*t/2 = 0
```