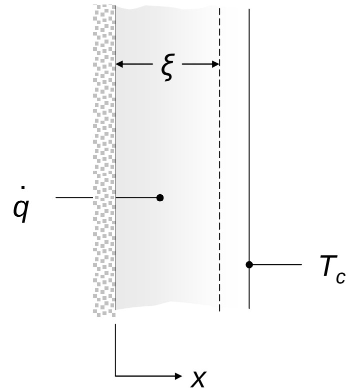
PROBLEM 2.29

KNOWN: Plane wall with constant properties and uniform volumetric energy generation. Insulated left face and isothermal right face.

FIND: (a) Expression for the heat flux distribution based upon the heat equation. (b) Expression for the heat flux distribution based upon a finite control volume.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform volumetric generation, (4) One-dimensional conduction.

ANALYSIS: (a) The appropriate form of the heat equation is Eq. 2.19 which may be written as

$$\frac{d^2T}{dx^2} = \frac{d}{dx}\left(\frac{dT}{dx}\right) = -\frac{\dot{q}}{k}$$

The heat equation may be integrated once to yield

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1 \quad \text{and, since } \left.\frac{dT}{dx}\right|_{x=0} = 0,\ C_1 = 0. \text{ Therefore, } -k\frac{dT}{dx} = q''(x) = -\dot{q}x \qquad <$$

(b) For the finite control volume, $\dot{E}_g = \dot{E}_{\text{out}}$, and for a unit cross-sectional area,

$$\dot{q}(\xi) = -k\left.\frac{dT}{dx}\right|_{x=\xi} \quad \text{which may be re-arranged to yield } -k\frac{dT}{dx} = q''(x) = -\dot{q}\xi = -\dot{q}x \qquad <$$

The expressions for the local heat flux are identical.

COMMENTS: (1) Although the two methods yield identical results, as they must, the heat equation is more general and can be used to determine temperature and heat flux distributions in more complex situations. (2) The value of the right face temperature is not needed to solve the problem. Is the value of T_c needed to determine the temperature distribution?