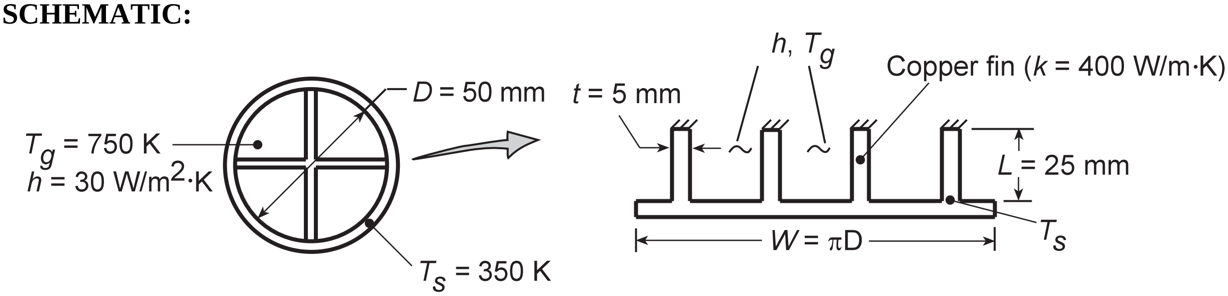
PROBLEM 3.149

KNOWN: Diameter and internal fin configuration of copper tubes submerged in water. Tube wall temperature and temperature and convection coefficient of gas flow through the tube.

FIND: Rate of heat transfer per tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional fin conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Tube wall may be unfolded and represented as a plane wall with four straight, rectangular fins, each with an adiabatic tip (since, by symmetry, there can be no heat flow along the fins where they cross).

ANALYSIS: The rate of heat transfer per unit tube length is:

$$q_t' = \eta_o h A_t' \left(T_g - T_s\right)$$

$$\eta_o = 1 - \frac{NA_f'}{A_t'}\left(1 - \eta_f\right)$$

$$NA_f' = 4 \times 2L = 8\left(0.025\text{m}\right) = 0.20\text{m}$$

$$A_t' = NA_f' + A_b' = 0.20\text{m} + \left(\pi D - 4t\right) = 0.20\text{m} + \left(\pi \times 0.05\text{m} - 4 \times 0.005\text{m}\right) = 0.337\text{m}$$

For an adiabatic fin tip,

$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{h\left(2L \cdot 1\right)\left(T_g - T_s\right)}$$

$$M = \left[h2\left(1\text{m} + t\right)k\left(1\text{m} \times t\right)\right]^{1/2}\left(T_g - T_s\right) \approx \left[30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)400\,\text{W/m}\cdot\text{K}\left(0.005\text{m}^2\right)\right]^{1/2}\left(400\text{K}\right) = 4382\text{W}$$

$$mL = \left\{\left[h2\left(1\text{m} + t\right)\right]/\left[k\left(1\text{m} \times t\right)\right]\right\}^{1/2} L \approx \left[\frac{30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)}{400\,\text{W/m}\cdot\text{K}\left(0.005\text{m}^2\right)}\right]^{1/2} 0.025\text{m} = 0.137$$

Hence, tanh mL = 0.136, and

$$\eta_f = \frac{4382\text{W}\left(0.136\right)}{30\,\text{W/m}^2 \cdot \text{K}\left(0.05\text{m}^2\right)\left(400\text{K}\right)} = \frac{595\text{W}}{600\text{W}} = 0.992$$

$$\eta_o = 1 - \frac{0.20}{0.337}\left(1 - 0.992\right) = 0.995$$

$$q_t' = 0.995\left(30\,\text{W/m}^2 \cdot \text{K}\right)0.337\text{m}\left(400\text{K}\right) = 4025\,\text{W/m}$$

COMMENTS: Alternatively, $q_t' = 4q_f' + h\left(A_t' - A_f'\right)\left(T_g - T_s\right)$. Hence, q' = 4(595 W/m) + 30 W/m^2·K (0.137 m)(400 K) = (2380 + 1644) W/m = 4024 W/m.