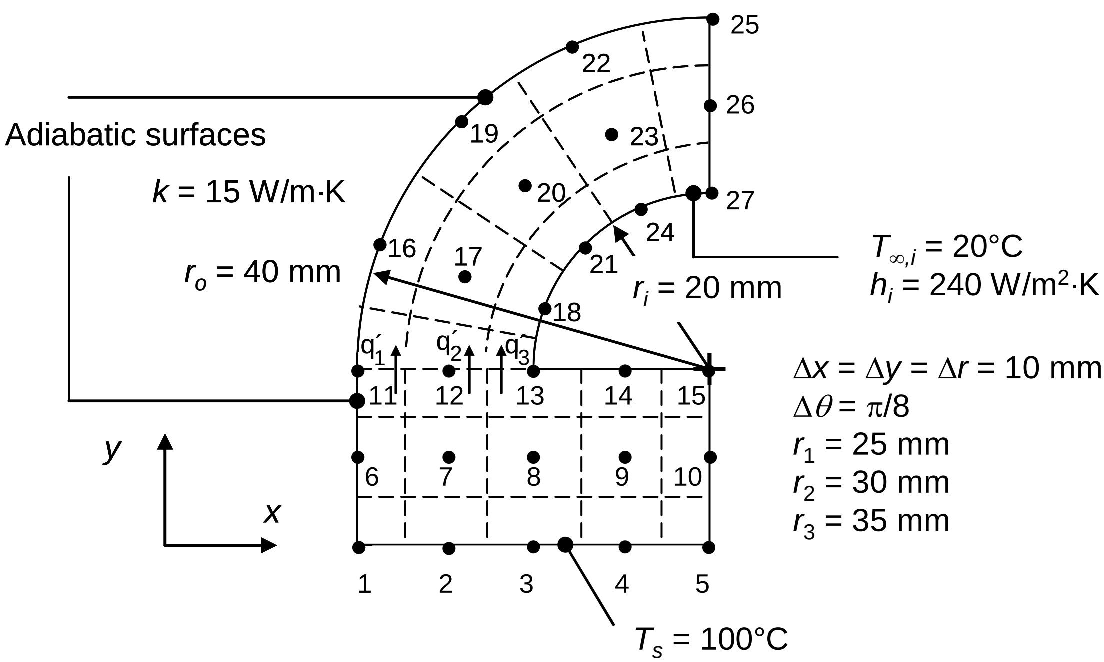
PROBLEM 4.60

KNOWN: Dimensions of a tube of non-circular cross section that can be broken into rectangular and cylindrical sub-domains. Fluid temperature and heat transfer coefficient, external surface temperature and tube wall thermal conductivity.

FIND: Heat transfer rate per unit length of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) No internal generation, (4) Two-dimensional conduction.

ANALYSIS: We may combine heat fluxes determined from Fourier's law with expressions for the size of the control surfaces of the various control volumes to determine the heat rate per unit depth into each control volume in the discretized domain. Application of conservation of energy for each control volume yields the expression $\dot{E}_{\text{in}} = 0$.

Rectangular Sub-Domain. For the rectangular sub-domain, application of Fourier's law in Cartesian coordinates along with conservation of energy yields the finite difference equations that are listed in the IHT code included in COMMENT (1).

Cylindrical Sub-Domain. For the cylindrical sub-domain, application of Fourier's law in cylindrical coordinates along with conservation of energy yields the finite difference equations that are listed in the IHT code included in COMMENT (1).

Coupling of Domains. Note that energy balances for Nodes 11, 12 and 13 are included in both the rectangular and cylindrical sub-domains. These energy balances couple the two solutions together.

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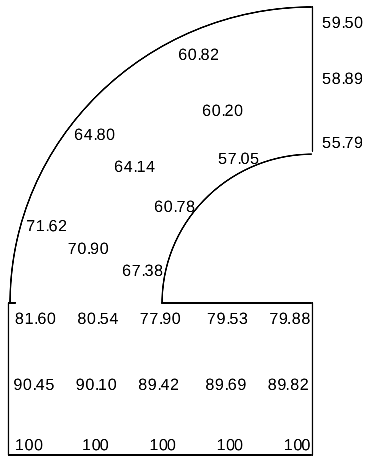
The equations are solved simultaneously to yield the following nodal temperatures in degrees Celsius.



The heat transfer rate per unit depth may be expressed as

$$q' = 2\left[\frac{k\Delta x}{\Delta y}\right]\left[(T_1 - T_6)/2 + (T_2 - T_7) + (T_3 - T_8) + (T_4 - T_9) + (T_5 - T_{10})/2\right]$$

$$= 2 \times 15\text{W/m}\cdot\text{K} \times \begin{bmatrix} (100 - 90.45)°\text{C}/2 + (100 - 90.10)°\text{C} + \\ (100 - 89.42)°\text{C} + (100 - 89.69)°\text{C} \\ +(100 - 89.82)°\text{C}/2 \end{bmatrix}$$

$= 1220$ W/m <

COMMENTS: (1) The *IHT* code is listed below. For each control volume, we note that $\dot{E}_{in} = 0$, yielding the energy balances for the rectangular and cylindrical sub-domains.

```
ri = 20/1000
ro = 40/1000
r1 = 25/1000
r2 = 30/1000
r3 = 35/1000

dx = 10/1000
dy = 10/1000
dr = 10/1000
dtheta = pi/8
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k = 15
h = 240
Tinf = 20

T1 = 100
T2 = 100
T3 = 100
T4 = 100
T5 = 100

//Rectangular Domain
//Node 6
k*(T1 - T6)*(dx/2)/dy + k*(T7 - T6)*dy/dx + k*(T11 - T6)*(dx/2)/dy = 0
//Node 7
k*(T6 - T7)*dy/dx + k*(T12 - T7)*dx/dy + k*(T8 - T7)*dy/dx + k*(T2 - T7)*dx/dy = 0
//Node 8
k*(T7 - T8)*dy/dx + k*(T13 - T8)*dx/dy + k*(T9 - T8)*dy/dx + k*(T3 - T8)*dx/dy = 0
//Node 9
k*(T8 - T9)*dy/dx + k*(T14 - T9)*dx/dy + k*(T10 - T9)*dy/dx + k*(T4 - T9)*dx/dy = 0
//Node 10
k*(T9 - T10)*dy/dx + k*(T15 - T10)*(dx/2)/dy + k*(T5 - T10)*(dx/2)/dy = 0
//Node 11
k*(T6 - T11)*(dx/2)/dy + k*(T12 - T11)*(dy/2)/dx = qprime1
//Node 12
k*(T11 - T12)*(dy/2)/dx + k*(T7 - T12)*dx/dy + k*(T13 - T12)*(dy/2)/dx = qprime2
//Node 13
k*(T12 - T13)*(dy/2)/dx + k*(T14 - T13)*(dy/2)/dx + k*(T8 - T13)*dx/dy + h*(dx/2)*(Tinf - T13) =
qprime3
//Node 14
k*(T13 - T14)*(dy/2)/dx + k*(T9 - T14)*dx/dy + k*(T15 - T14)*(dy/2)/dx + h*dx*(Tinf - T14)=0
//Node 15
k*(T14 - T15)*(dy/2)/dx + k*(T10 - T15)*(dx/2)/dy + h*(dx/2)*(Tinf - T15) = 0

//Cylindrical Domain

//Node 11
k*(T16 - T11)*(dr/2)/(ro*dtheta) + k*(T12 - T11)*(r3*dtheta/2)/dr + qprime1 = 0

//Node 12
k*(T11 - T12)*(r3*dtheta/2)/dr + k*(T17 - T12)*dr/(r2*dtheta) + k*(T13 - T12)*(r1*dtheta/2)/dr + qprime2
= 0
//Node 13
k*(T12 - T13)*(r1*dtheta/2)/dr + k*(T18 - T13)*(dr/2)/(ri*dtheta) + qprime3 + h*(ri*dtheta/2)*(Tinf - T13)
= 0
//Node 16
k*(T11 - T16)*(dr/2)/(ro*dtheta) + k*(T17 - T16)*(r3*dtheta)/dr + k*(T19 - T16)*(dr/2)/(ro*dtheta) = 0
//Node 17
k*(T16 - T17)*(r3*dtheta)/dr + k*(T20 - T17)*dr/(r2*dtheta) + k*(T18 - T17)*(r1*dtheta)/dr + k*(T12 -
T17)*dr/(r2*dtheta) = 0
//Node 18
k*(T13 - T18)*(dr/2)/(ri*dtheta) + k*(T17 - T18)*(r1*dtheta)/dr + k*(T21 - T18)*(dr/2)/(ri*dtheta) +
h*(ri*dtheta)*(Tinf - T18) = 0
//Node 19
k*(T16 - T19)*(dr/2)/(ro*dtheta) + k*(T20 - T19)*(r3*dtheta)/dr + k*(T22 - T19)*(dr/2)/(ro*dtheta) = 0
//Node 20
k*(T19 - T20)*(r3*dtheta)/dr + k*(T23 - T20)*dr/(r2*dtheta) + k*(T21 - T20)*(r1*dtheta)/dr + k*(T17 -
T20)*dr/(r2*dtheta) = 0
//Node 21
k*(T18 - T21)*(dr/2)/(ri*dtheta) + k*(T20 - T21)*(r1*dtheta)/dr + k*(T24 - T21)*(dr/2)/(ri*dtheta) +
h*(ri*dtheta)*(Tinf - T21) = 0
//Node 22
k*(T19 - T22)*(dr/2)/(ro*dtheta) + k*(T23 - T22)*(r3*dtheta)/dr + k*(T25 - T22)*(dr/2)/(ro*dtheta) = 0
//Node 23
k*(T22 - T23)*(r3*dtheta)/dr + k*(T26 - T23)*dr/(r2*dtheta) + k*(T24 - T23)*(r1*dtheta)/dr + k*(T20 -
T23)*dr/(r2*dtheta) = 0
//Node 24
k*(T21 - T24)*(dr/2)/(ri*dtheta) + k*(T23 - T24)*(r1*dtheta)/dr + k*(T27 - T24)*(dr/2)/(ri*dtheta) +
h*(ri*dtheta)*(Tinf - T24) = 0
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//Node 25
k*(T22 - T25)*(dr/2)/(ro*dtheta) + k*(T26 - T25)*(r3*dtheta/2)/dr = 0
//Node 26
k*(T25 - T26)*(r3*dtheta/2)/dr + k*(T23 - T26)*dr/(r2*dtheta) + k*(T27 - T26)*(r1*dtheta/2)/dr = 0
//Node 27
k*(T24 - T27)*(dr/2)/(ri*dtheta) + k*(T26 - T27)*(r1*dtheta/2)/dr + h*(ri*dtheta/2)*(Tinf - T24) = 0

//Heat Transfer Rate Per Unit Depth (W/m)
qprime = 2*((k/dy)*((T1 - T6)*dx/2 + (T2 - T7)*dx + (T3 - T8)*dx + (T4 - T9)*dx  + (T5 - T10)*dx/2))
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(2) For an isothermal tube at T_s = 100°C, the heat transfer per unit length is

$$q' = h(\pi r_i + 2r_i)\times(T_s - T_\infty) = 240\,\mathrm{W/m\cdot K}\times(\pi\times 20\times 10^{-3}\,\mathrm{m} + 2\times 20\times 10^{-3}\,\mathrm{m})\times(100-20)°\mathrm{C}$$

= 1975 W/m. The conduction resistance posed by the tube wall is responsible for decreasing the actual heat transfer rate below this value.