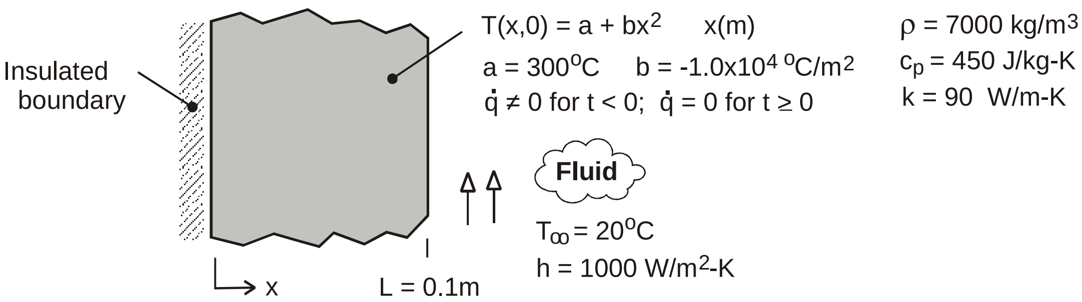
PROBLEM 2.65

KNOWN: Temperature distribution in a plane wall of thickness L experiencing uniform volumetric heating $\dot{q}$ having one surface (x = 0) insulated and the other exposed to a convection process characterized by T_∞ and h. Suddenly the volumetric heat generation is deactivated while convection continues to occur.

FIND: (a) Determine the magnitude of the volumetric energy generation rate associated with the initial condition, (b) On T-x coordinates, sketch the temperature distributions for the initial condition ($T \le 0$), the steady-state condition ($t \to \infty$), and two intermediate times; (c) On q''_x - t coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q''_x(L,t)$; calculate the corresponding value of the heat flux at t = 0; and (d) Determine the amount of energy removed from the wall per unit area (J/m^2) by the fluid stream as the wall cools from its initial to steady-state condition.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) Uniform internal volumetric heat generation for t < 0.

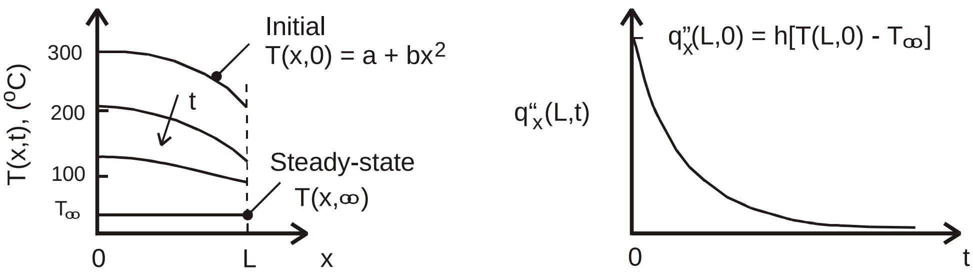
ANALYSIS: (a) The volumetric heating rate can be determined by substituting the temperature distribution for the initial condition into the appropriate form of the heat diffusion equation.

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x,0)=a+bx^2$$

$$\frac{d}{dx}(0+2bx)+\frac{\dot{q}}{k}=0=2b+\frac{\dot{q}}{k}=0$$

$$\dot{q}=-2kb=-2\times 90\,\mathrm{W/m\cdot K}\left(-1.0\times 10^{4}\,^\circ\mathrm{C/m^2}\right)=1.8\times 10^{6}\,\mathrm{W/m^3} \qquad <$$

(b) The temperature distributions are shown in the sketch below.



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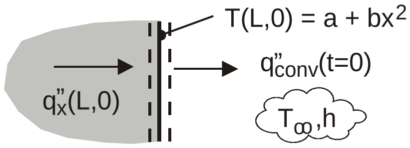
PROBLEM 2.65 (Cont.)

(c) The heat flux at the exposed surface x = L, $q''_x(L,0)$, is initially a maximum value and decreases with increasing time as shown in the sketch above. The heat flux at t = 0 is equal to the convection heat flux with the surface temperature T(L,0). See the surface energy balance represented in the schematic.

$$q''_x(L,0) = q''_{conv}(t=0) = h\left(T(L,0) - T_\infty\right) = 1000\,W/m^2\cdot K\,(200-20)\,°C = 1.80\times10^5\,W/m^2 \quad <$$

where $T(L,0) = a + bL^2 = 300°C - 1.0\times10^4 °C/m^2\,(0.1m)^2 = 200°C.$



(d) The energy removed from the wall to the fluid as it cools from its initial to steady-state condition can be determined from an energy balance on a time interval basis, Eq. 1.12b. For the initial state, the wall has the temperature distribution $T(x,0) = a + bx^2$; for the final state, the wall is at the temperature of the fluid, $T_f = T_\infty$. We have used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0$$

$$E''_{out} = \rho c_p \int_{x=0}^{x=L} \left[T(x,0) - T_\infty\right] dx$$

$$E''_{out} = \rho c_p \int_{x=0}^{x=L} \left[a + bx^2 - T_\infty\right] dx = \rho c_p \left[ax + bx^3/3 - T_\infty x\right]_0^L$$

$$E''_{out} = 7000\,kg/m^3 \times 450\,J/kg\cdot K\left[300\times0.1 - 1.0\times10^4\,(0.1)^3/3 - 20\times0.1\right]K\cdot m$$

$$E''_{out} = 7.77\times10^7\,J/m^2 \quad <$$

COMMENTS: (1) In the temperature distributions of part (a), note these features: initial condition has quadratic form with zero gradient at the adiabatic boundary; for the steady-state condition, the wall has reached the temperature of the fluid; for all distributions, the gradient at the adiabatic boundary is zero; and, the gradient at the exposed boundary decreases with increasing time.

(2) In this thermodynamic analysis, we were able to determine the energy transferred during the cooling process. However, we cannot determine the rate at which cooling of the wall occurs without solving the heat diffusion equation.