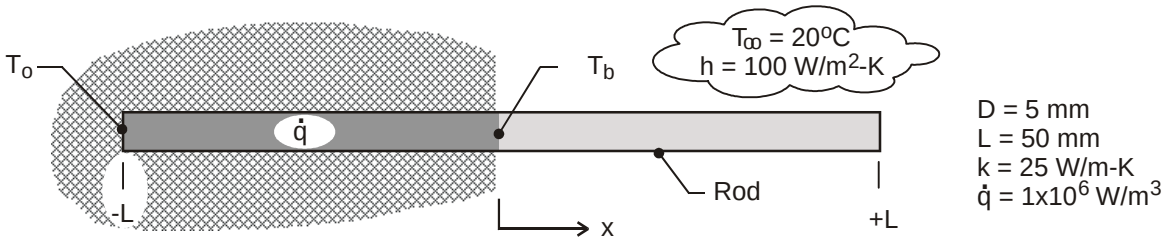
PROBLEM 3.123

KNOWN: Rod (D, k, 2L) inserted into a perfectly insulating wall, exposing one-half of its length to an airstream (T_∞, h). An electromagnetic field induces a uniform volumetric energy generation $(\dot{q})$ in the imbedded portion.

FIND: (a) Derive an expression for T_b at the base of the exposed half of the rod; the exposed region may be approximated as a very long fin; (b) Derive an expression for T_o at the end of the imbedded half of the rod, and (c) Using numerical values, plot the temperature distribution in the rod and describe its key features. Does the rod behave as a very long fin?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in imbedded portion of rod, (3) Imbedded portion of rod is perfectly insulated, (4) Exposed portion of rod behaves as an infinitely long fin, and (5) Constant properties.

ANALYSIS: (a) Since the exposed portion of the rod ($0 \le x \le +L$) behaves as an infinite fin, the fin heat rate using Eq. 3.85 is

$$q_x(0) = q_f = M = (hPkA_c)^{1/2}(T_b - T_\infty) \tag{1}$$

From an energy balance on the imbedded portion of the rod,

$$q_f = \dot{q} A_c L \tag{2}$$

Combining Eqs. (1) and (2), with $P = \pi D$ and $A_c = \pi D^2/4$, find

$$T_b = T_\infty + q_f (hPkA_c)^{-1/2} = T_\infty + \dot{q} A_c^{1/2} L (hPk)^{-1/2} \tag{3}$$ <

(b) The imbedded portion of the rod ($-L \le x \le 0$) experiences one-dimensional heat transfer with uniform $\dot{q}$. From Eq. 3.48,

$$T_o = \frac{\dot{q} L^2}{2k} + T_b$$ <

(c) The temperature distribution T(x) for the rod is piecewise parabolic and exponential,

$$T(x) - T_b = \frac{\dot{q} L^2}{2k}\left(\frac{x}{L}\right)^2 \qquad -L \le x \le 0$$

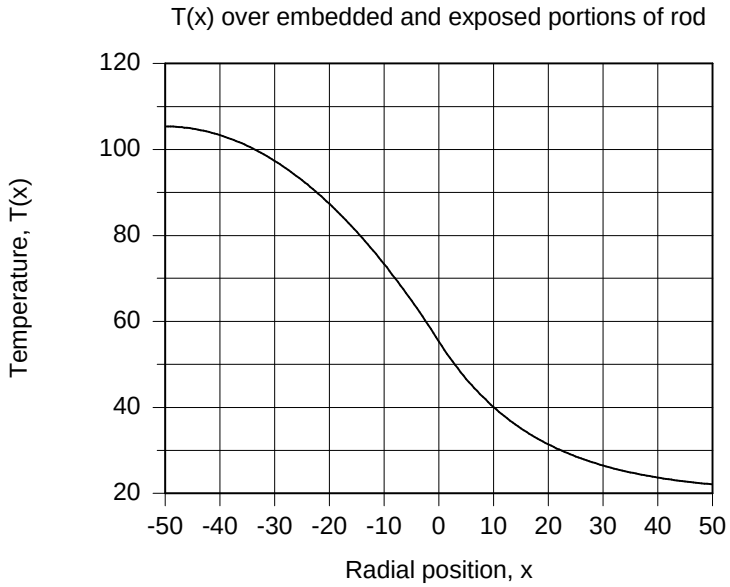
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \exp(-mx) \qquad 0 \le x \le +L$$

where $m = (hP/kA_c)^{1/2}$.

Continued …

PROBLEM 3.123 (Cont.)

The gradient at x = 0 will be continuous since we used this condition in evaluating T_b. The distribution is shown below with T_o = 105.4°C and T_b = 55.4°C.



COMMENTS: The assumption that the rod behaves as an infinitely long fin is accurate; we see from the figure above that the temperature approaches the ambient temperature near the end of the rod.