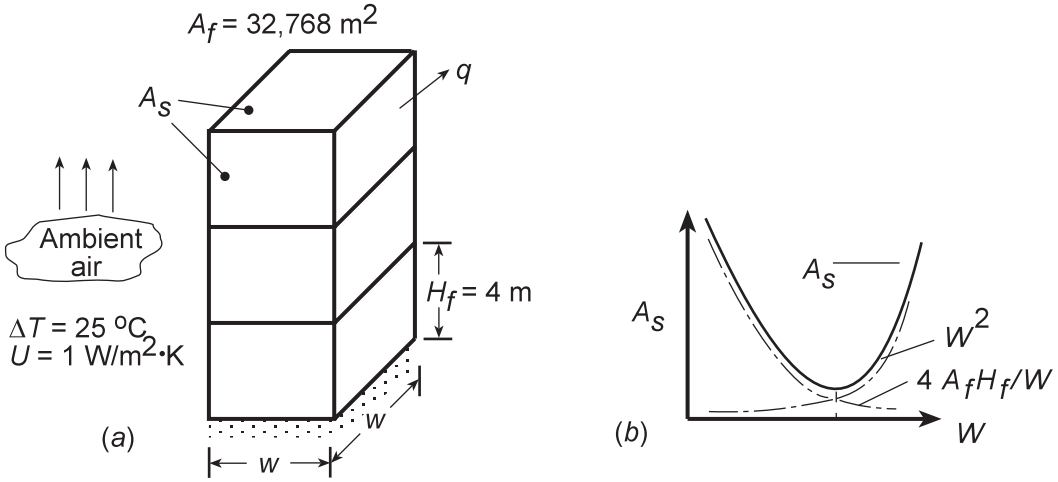
PROBLEM 3.22

KNOWN: Total floor space and vertical distance between floors for a square, flat roof building.

FIND: (a) Expression for width of building which minimizes heat loss, (b) Width and number of floors which minimize heat loss for a prescribed floor space and distance between floors. Corresponding heat loss, percent heat loss reduction from 2 floors.

SCHEMATIC:



ASSUMPTIONS: Negligible heat loss to ground.

ANALYSIS: (a) To minimize the heat loss q, the exterior surface area, A_s, must be minimized. From Fig. (a)

$$A_s = W^2 + 4WH = W^2 + 4WN_f H_f$$

where

$$N_f = A_f / W^2$$

Hence,

$$A_s = W^2 + 4WA_f H_f / W^2 = W^2 + 4A_f H_f / W$$

The optimum value of W corresponds to

$$\frac{dA_s}{dW} = 2W - \frac{4A_f H_f}{W^2} = 0$$

or

$$W_{op} = (2A_f H_f)^{1/3} \quad <$$

The competing effects of W on the areas of the roof and sidewalls, and hence the basis for an optimum, is shown schematically in Fig. (b).

(b) For A_f = 32,768 m^2 and H_f = 4 m,

$$W_{op} = \left(2 \times 32,768\,\text{m}^2 \times 4\,\text{m}\right)^{1/3} = 64\,\text{m} \quad <$$

Continued …

PROBLEM 3.22 (Cont.)

Hence,

$$N_f = \frac{A_f}{W^2} = \frac{32,768\,m^2}{(64\,m)^2} = 8 \qquad <$$

and

$$q = UA_S\Delta T = 1\,W/m^2\cdot K\left[(64\,m)^2 + \frac{4\times 32,768\,m^2\times 4\,m}{64\,m}\right]25^\circ C = 307,200\,W \qquad <$$

For $N_f = 2$,

$$W = (A_f/N_f)^{1/2} = (32,768\ m^2/2)^{1/2} = 128\ m$$

$$q = 1\,W/m^2\cdot K\left[(128\,m)^2 + \frac{4\times 32,768\,m^2\times 4\,m}{128\,m}\right]25^\circ C = 512,000\,W$$

% reduction in q = (512,000 - 307,200)/512,000 = 40% <

COMMENTS: Even the minimum heat loss is excessive and could be reduced by reducing U.