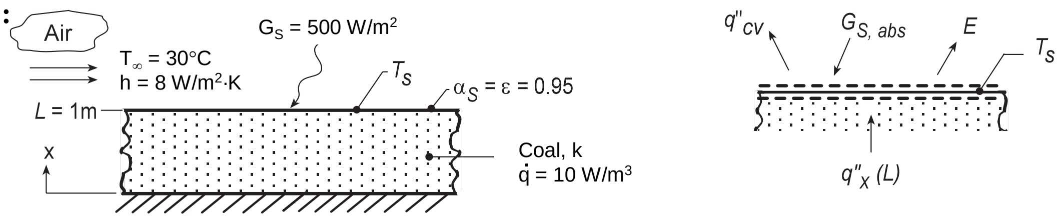
PROBLEM 2.42

KNOWN: Coal pile of prescribed depth experiencing uniform volumetric generation with convection, absorbed irradiation and emission on its upper surface.

FIND: (a) The appropriate form of the heat diffusion equation (HDE) and whether the prescribed temperature distribution satisfies this HDE; conditions at the bottom of the pile, x = 0; sketch of the temperature distribution with labeling of key features; (b) Expression for the conduction heat rate at the location x = L; expression for the surface temperature T_s based upon a surface energy balance at x = L; evaluate T_S and T(0) for the prescribed conditions; (c) Based upon typical daily averages for G_S and h, compute and plot T_S and T(0) for (1) h = 5 $W/m^2{\cdot}K$ with $50 \le G_S \le 500$ W/m^2, (2) G_S = 400 W/m^2 with $5 \le h \le 50$ $W/m^2{\cdot}K$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Uniform volumetric heat generation, (3) Constant properties, (4) Negligible irradiation from the surroundings, and (5) Steady-state conditions.

PROPERTIES: *Table A.3*, Coal (300K): k = 0.26 W/m·K

ANALYSIS: (a) For one-dimensional, steady-state conduction with uniform volumetric heat generation and constant properties the heat diffusion equation (HDE) follows from Eq. 2.22,

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot q}{k}=0 \qquad (1)$$ <

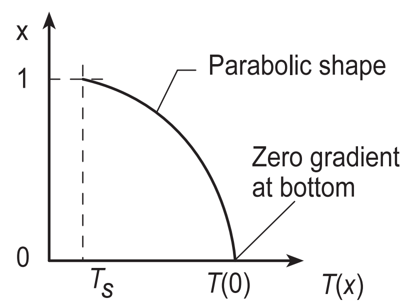
Substituting the temperature distribution into the HDE, Eq. (1),

$$T(x)=T_S+\frac{\dot q L^2}{2k}\left(1-\frac{x^2}{L^2}\right) \qquad \frac{d}{dx}\left[0+\frac{\dot q L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]+\frac{\dot q}{k}\,?=?\,0 \qquad (2,3)$$

we find that it does indeed satisfy the HDE for all values of x. <

From Eq. (2), note that the temperature distribution must be quadratic, with maximum value at x = 0. At x = 0, the heat flux is

$$q''_x(0)=-k\left.\frac{dT}{dx}\right)_{x=0}=-k\left[0+\frac{\dot q L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]_{x=0}=0$$



so that the gradient at x = 0 is zero. Hence, the bottom is insulated.

(b) From an overall energy balance on the pile, the conduction heat flux at the surface must be

$$q''_x(L)=\dot E''_g=\dot q L$$ <

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From a surface energy balance per unit area shown in the schematic above,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(L) - q''_{conv} + G_{S,abs} - E = 0$$

$$\dot{q}L - h\left(T_S - T_\infty\right) + 0.95G_S - \varepsilon\sigma T_S^4 = 0 \tag{4}$$

$$10\,\text{W}/\text{m}^3 \times\ 2\,\text{m} - 8\,\text{W}/\text{m}^2{\cdot}\text{K}\left(T_s - 303\,\text{K}\right) + 0.95\times 500\,\text{W}/\text{m}^2 - 0.95\times 5.67\times 10^{-8}\,\text{W}/\text{m}^2{\cdot}\text{K}^4 T_s^4 = 0$$

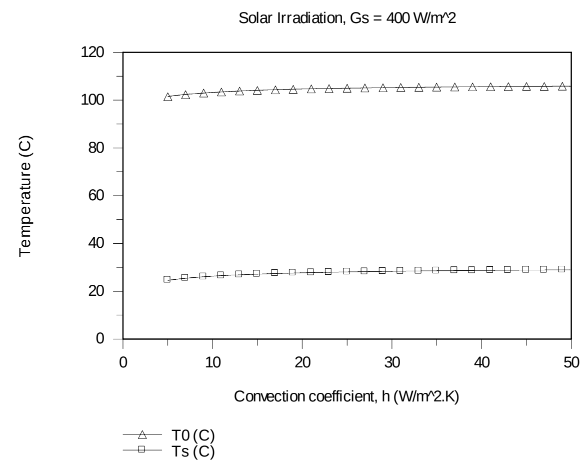
$$T_S = 305.6\ \text{K} = 32.6°\text{C}$$ <

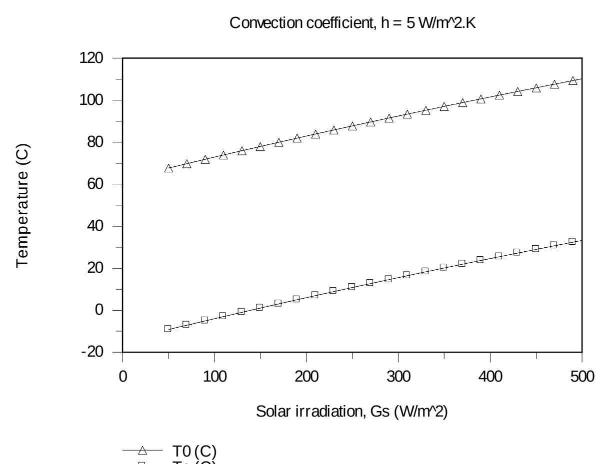
From Eq. (2) with x = 0, find

$$T(0) = T_S + \frac{\dot{q}L^2}{2k} = 32.6^\circ\text{C} + \frac{10\,\text{W}/\text{m}^3 \times (2\,\text{m})^2}{2\times 0.26\,\text{W}/\text{m}\cdot\text{K}} = 109.5^\circ\text{C} \tag{5}$$ <

where the thermal conductivity for coal was obtained from Table A.3.

(c) Two plots are generated using Eq. (4) and (5) for T_s and T(0), respectively; (1) with h = 5 W/m^2·K for $50 \le G_S \le 500$ W/m^2 and (2) with G_S = 400 W/m^2 for $5 \le h \le 50$ W/m^2·K.





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From the T vs. h plot with $G_S = 400\ W/m^2$, note that the convection coefficient does not have a major influence on the surface or bottom coal pile temperatures. From the T vs. G_S plot with $h = 5\ W/m^2 \cdot K$, note that the solar irradiation has a very significant effect on the temperatures. The fact that T_s is less than the ambient air temperature, T_∞, and, in the case of very low values of G_S, below freezing, is a consequence of the large magnitude of the emissive power E.

COMMENTS: In our analysis we ignored irradiation from the sky, an environmental radiation effect you'll consider in Chapter 12. Treated as large isothermal surroundings, $G_{sky} = \sigma T_{sky}^4$ where $T_{sky} = -30°C$ for very clear conditions and nearly air temperature for cloudy conditions. For low G_S conditions we should consider G_{sky}, the effect of which will be to predict higher values for T_s and T(0).