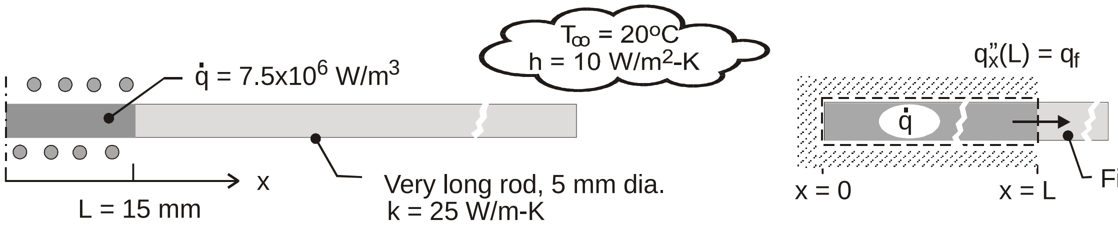
PROBLEM 3.124

KNOWN: Very long rod (D, k) subjected to induction heating experiences uniform volumetric generation $(\dot{q})$ over the center, 30-mm long portion. The unheated portions experience convection (T_∞, h).

FIND: Calculate the temperature of the rod at the mid-point of the heated portion within the coil, T_o, and at the edge of the heated portion, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform $\dot{q}$ in portion of rod within the coil; no convection from lateral surface of rod, (3) Exposed portions of rod behave as infinitely long fins, and (4) Constant properties, (5) Neglect radiation.

ANALYSIS: The portion of the rod within the coil, $0 \le x \le +L$, experiences one-dimensional conduction with uniform generation. From Eq. 3.48,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \qquad (1)$$

The portion of the rod beyond the coil, $L \le x \le \infty$, behaves as an infinitely long fin for which the heat rate from Eq. 3.85 is

$$q_f = q_x(L) = \left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right) \qquad (2)$$

where $P = \pi D$ and $A_c = \pi D^2/4$. From an overall energy balance on the imbedded portion of the rod as illustrated in the schematic above, find the heat rate as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_f + \dot{q}A_cL = 0$$

$$q_f = \dot{q}A_cL \qquad (3)$$

Combining Eqs. (1-3),

$$T_b = T_\infty + \dot{q}A_c^{1/2}L\left(hPk\right)^{-1/2} \qquad (4)$$

$$T_o = T_\infty + \frac{\dot{q}L^2}{2k} + \dot{q}A_c^{1/2}L\left(hPk\right)^{-1/2} \qquad (5)$$

and substituting numerical values find

$$T_o = 305°C \qquad T_b = 272°C$$ <

COMMENT: Assuming $\varepsilon = 0.8$ and $T_{sur} = T_\infty = 20°C$, $h_{rad} = 14.6\ W/m^2 \cdot K$. Hence, radiation is significant and would serve to substantially reduce both T_o and T_b.