# PROBLEM 3.84

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where $h = 0$ on surface A.

SCHEMATIC:

$L_A = 30$ mm
$L_B = 30$ mm
$L_C = 20$ mm
$k_A = 25$ W/m·K
$k_C = 50$ W/m·K

$T_\infty = 25\ ^\circ C$
$h = 1000$ W/m²·K
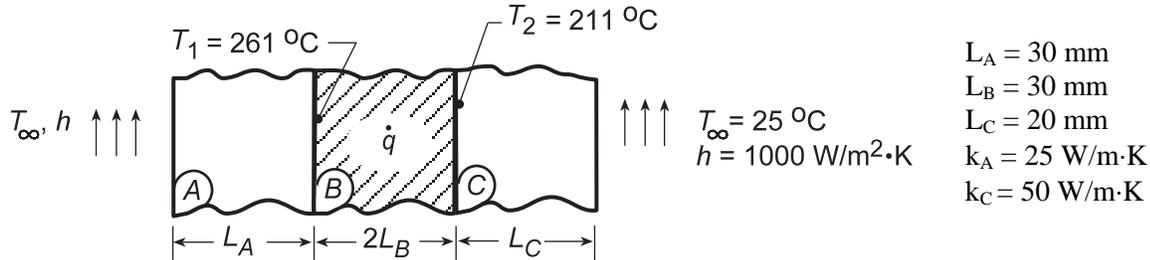
ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

**ANALYSIS:** (a) From an energy balance on wall B,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B . \qquad (1)$$

$2L_B = 60$ mm
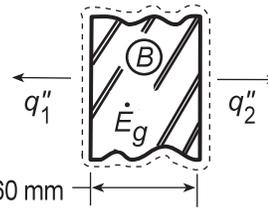
To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:
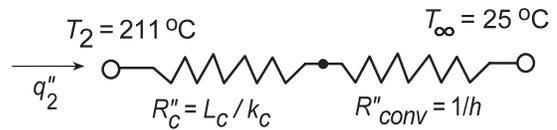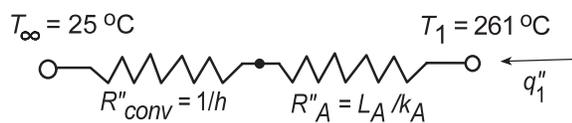
$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right) \qquad\qquad q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_1'' = \left(261 - 25\right)^\circ C\left/\left(\dfrac{1}{1000\ \text{W}/\text{m}^2 \cdot \text{K}} + \dfrac{0.030\,\text{m}}{25\ \text{W}/\text{m}\cdot\text{K}}\right)\right. \qquad q_2'' = \left(211 - 25\right)^\circ C\left/\left(\dfrac{0.020\,\text{m}}{50\ \text{W}/\text{m}\cdot\text{K}} + \dfrac{1}{1000\ \text{W}/\text{m}^2 \cdot \text{K}}\right)\right.$$

$$q_1'' = 236^\circ C\left/\left(0.001 + 0.0012\right)\text{m}^2 \cdot \text{K}/\text{W}\right. \qquad\qquad q_2'' = 186^\circ C\left/\left(0.0004 + 0.001\right)\text{m}^2 \cdot \text{K}/\text{W}\right.$$

$$q_1'' = 107,273\ \text{W}/\text{m}^2 \qquad\qquad\qquad\qquad q_2'' = 132,857\ \text{W}/\text{m}^2$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106,818 + 132,143\ \text{W}/\text{m}^2\right)\left/2 \times 0.030\,\text{m} = 4.00 \times 10^6\ \text{W}/\text{m}^3\right. . \qquad\qquad <$$

To determine k_B, use the general form of the temperature (Eq. 3.40) and heat flux distributions in wall B,

$$T(x) = -\dfrac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad\qquad q_x''(x) = -k_B\left[-\dfrac{\dot{q}_B}{k_B}x + C_1\right] \qquad\qquad (2,3)$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

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$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad\qquad \text{where } T_1 = 261°C \qquad\qquad (4)$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad\qquad \text{where } T_2 = 211°C \qquad\qquad (5)$$

$$q_x''(-L_B) = -q_1'' = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad\qquad \text{where } q_1'' = 107{,}273 \text{ W/m}^2 \qquad\qquad (6)$$

Using IHT to solve Eqs. (4), (5) and (6) simultaneously with $\dot{q}_B = 4.00 \times 10^6$ W/m^3, find

$$k_B = 15.3 \text{ W/m} \cdot \text{K} \qquad\qquad <$$

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = -L$_B$ and +L$_B$, find $q_2'' > q_1''$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

$$T_1 = 835°C \qquad\qquad T_2 = 360°C \qquad\qquad <$$

Left plot legend:
—×— T_xA, kA = 25 W/m.K
—+— T_x, kB = 15 W/m.K, qdotB = 4.00e6 W/m^3
—⊙— T_x, kC = 50 W/m.K

Loss of coolant on surface A
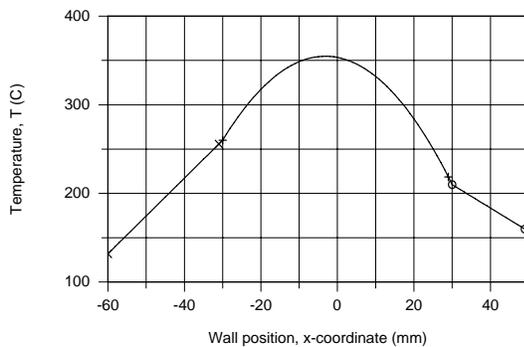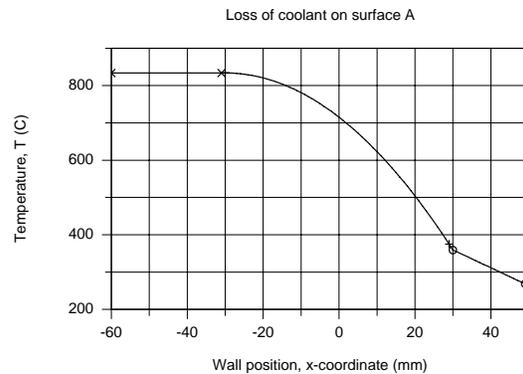
Right plot legend:
—×— T_xA, kA = 25 W/m.K; adiabatic surface
—+— T_x, kB = 15 W/m.K, qdotB = 4.00e6 W/m^3
—⊙— T_x, kC = 50 W/m.K