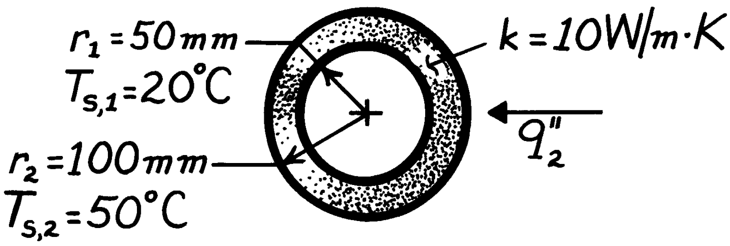
**PROBLEM 3.74**

**KNOWN:** Thermal conductivity and inner and outer radii of a hollow sphere subjected to a uniform heat flux at its outer surface and maintained at a uniform temperature on the inner surface.

**FIND:** (a) Expression for radial temperature distribution, (b) Heat flux required to maintain prescribed surface temperatures.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No generation, (4) Constant properties.

**ANALYSIS:** (a) For the assumptions, the temperature distribution may be obtained by integrating Fourier's law, Eq. 3.38. That is,

$$\frac{q_r}{4\pi}\int_{r_1}^{r}\frac{dr}{r^2} = -k\int_{T_{s,1}}^{T} dT \quad \text{or} \quad -\frac{q_r}{4\pi}\frac{1}{r}\Big|_{r_1}^{r} = -k\left(T - T_{s,1}\right).$$

Hence,

$$T(r) = T_{s,1} + \frac{q_r}{4\pi\, k}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

or, with $q''_2 \equiv q_r / 4\pi\, r_2^2$,

$$T(r) = T_{s,1} + \frac{q''_2 r_2^2}{k}\left[\frac{1}{r} - \frac{1}{r_1}\right] \qquad <$$

(b) Applying the above result at $r_2$,

$$q''_2 = \frac{k\left(T_{s,2} - T_{s,1}\right)}{r_2^2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]} = \frac{10\ \text{W/m}\cdot\text{K}\ (50-20)^\circ\text{C}}{(0.1\text{m})^2\left[\frac{1}{0.1} - \frac{1}{0.05}\right]\frac{1}{\text{m}}} = -3000\ \text{W/m}^2. \qquad <$$

**COMMENTS:** (1) The desired temperature distribution could also be obtained by solving the appropriate form of the heat equation,

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = 0$$

and applying the boundary conditions $T(r_1) = T_{s,1}$ and $-k\frac{dT}{dr}\Big]_{r_2} = q''_2$.

(2) The negative sign on $q''_2$ implies heat transfer in the negative r direction.