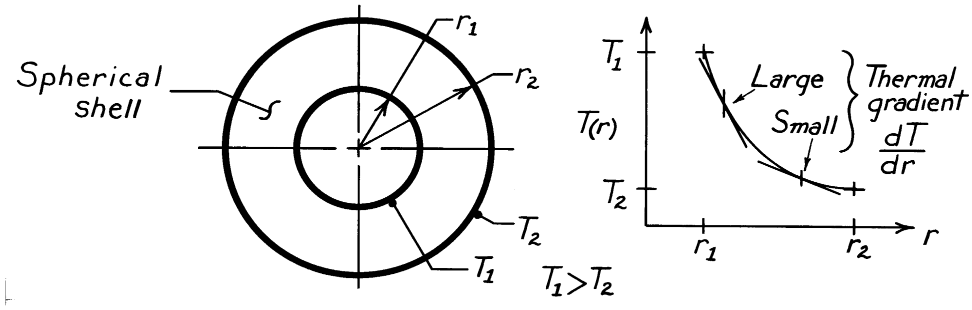
PROBLEM 2.4

KNOWN: A spherical shell with prescribed geometry and surface temperatures.

FIND: Sketch temperature distribution and explain shape of the curve.

SCHEMATIC:



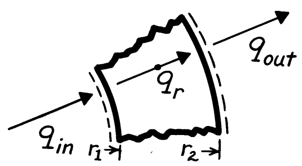
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial (spherical coordinates) direction, (3) No internal generation, (4) Constant properties.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional, radial (spherical coordinate) system has the form

$$q_r = -k\,A_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$

where A_r is the surface area of a sphere. For steady-state conditions, an energy balance on the system yields $\dot{E}_{in} = \dot{E}_{out}$, since $\dot{E}_g = \dot{E}_{st} = 0$. Hence,

$$q_{in} = q_{out} = q_r \neq q_r(r).$$



That is, q_r is a constant, independent of the radial coordinate. Since the thermal conductivity is constant, it follows that

$$r^2\left[\frac{dT}{dr}\right] = \text{Constant}.$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius squared, r^2, remains constant throughout the shell. Hence, the temperature distribution appears as shown in the sketch.

COMMENTS: Note that, for the above conditions, $q_r \neq q_r(r)$; that is, q_r is everywhere constant. How does q_r'' vary as a function of radius?