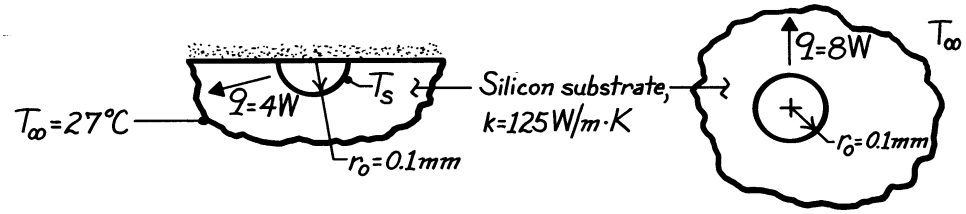
PROBLEM 3.77

KNOWN: Radius and heat dissipation of a hemispherical source embedded in a substrate of prescribed thermal conductivity. Source and substrate boundary conditions.

FIND: Substrate temperature distribution and surface temperature of heat source.

SCHEMATIC:



ASSUMPTIONS: (1) Top surface is adiabatic. Hence, hemispherical source in semi-infinite medium is equivalent to spherical source in infinite medium (with q = 8 W) and heat transfer is one-dimensional in the radial direction, (2) Steady-state conditions, (3) Constant properties, (4) No generation.

ANALYSIS: Heat equation reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \qquad r^2dT/dr=C_1$$

$$T(r)=-C_1/r+C_2.$$

Boundary conditions:

$$T(\infty)=T_\infty \qquad T(r_o)=T_s$$

Hence, $C_2 = T_\infty$ and

$$T_s=-C_1/r_o+T_\infty \quad \text{and} \quad C_1=r_o(T_\infty-T_s).$$

The temperature distribution has the form

$$T(r)=T_\infty+(T_s-T_\infty)r_o/r \qquad <$$

and the heat rate is

$$q=-kAdT/dr=-k2\pi\, r^2\left[-(T_s-T_\infty)r_o/r^2\right]=k2\pi\, r_o(T_s-T_\infty)$$

It follows that

$$T_s-T_\infty=\frac{q}{k2\pi\, r_o}=\frac{4\text{ W}}{125\text{ W/m}\cdot\text{K}\,2\pi\left(10^{-4}\text{ m}\right)}=50.9^\circ\text{C}$$

$$T_s=77.9^\circ\text{C}. \qquad <$$

COMMENTS: For the semi-infinite (or infinite) medium approximation to be valid, the substrate dimensions must be much larger than those of the transistor.