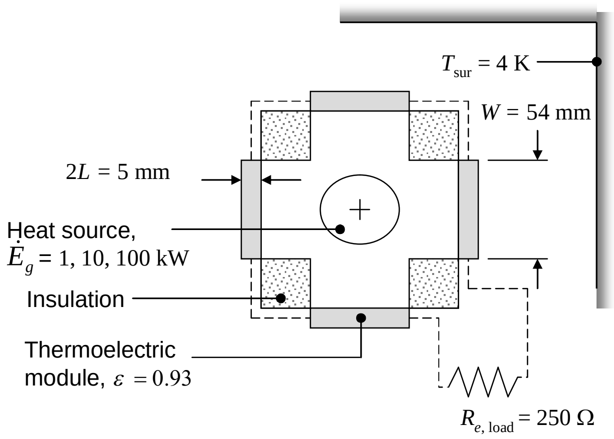
PROBLEM 3.171

KNOWN: Thermal energy generation rate. Dimensions of thermoelectric modules and total number of modules. Thermoelectric module performance parameters, load electrical resistance, emissivity of the exposed surface of the thermoelectric modules, deep space temperature.

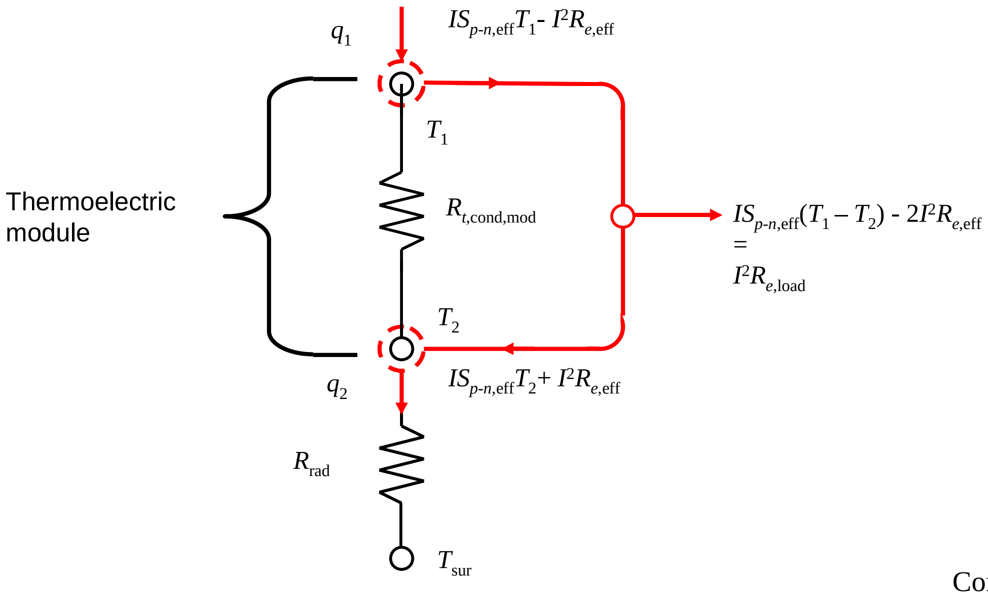
FIND: Electrical power generated by the device. Surface temperatures of the modules.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) Large surroundings.

ANALYSIS: The portion of the equivalent thermal circuit that describes the thermoelectric module is the same as shown in Figure 3.24*b*. The energy generated in the uranium is known, and under steady-state conditions $q_1 = \dot{E}_g / M$. As a consequence, knowledge of the thermal resistance between the uranium and the inner surface of the TEMs is not needed. The low temperature side of the TEMs exchanges heat with the surroundings through radiation. Thus, the equivalent thermal circuit is as shown below. Note that $q_{conv,1}$ and $q_{conv,2}$ have been replaced with the more general terms q_1 and q_2.



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The analysis proceeds as in Example 3.13. The conduction resistance of one module is the same as in the example, namely

$$R_{t,\text{cond,mod}} = \frac{L}{NA_{c,s}k_s} = \frac{2.5 \times 10^{-3}\ \text{m}}{100 \times 1.2 \times 10^{-5}\ \text{m}^2 \times 1.2\ \text{W/m}\cdot\text{K}} = 1.736\ \text{K/W}$$

From Equations 3.125 and 3.126,

$$q_1 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_1 - I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736\ \text{K/W}} + I \times 0.1435\ \text{V/K} \times T_1 - I^2 \times 4\ \Omega \qquad (1)$$

$$q_2 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_2 + I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736\ \text{K/W}} + I \times 0.1435\ \text{V/K} \times T_2 + I^2 \times 4\ \Omega \qquad (2)$$

An additional relationship can be written by considering heat transfer by radiation to deep space.

$$q_2 = h_rW^2(T_2 - T_{\text{sur}}) = h_r \times (0.054\ \text{m})^2 \times (T_2 - 4\ \text{K}) \qquad (3)$$

where

$$h_r = \varepsilon\sigma(T_2 + T_{\text{sur}})(T_2^2 + T_{\text{sur}}^2) = 0.93 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K} \times (T_2 + 4\ \text{K}) \times (T_2^2 + (4\ \text{K})^2) \qquad (4)$$

The radiation heat transfer coefficient, h_r, depends on the unknown TEM surface temperature, T_2. This can be left as an unknown in solving the simultaneous equations.

The electric power produced by all 80 modules, P_{tot}, is equal to the electric power dissipated in the load resistance. Making use of Equation 3.127 and equating the total electrical power generated in the M modules to the electric power dissipated in the load gives

$$P_{\text{tot}} = MP_N = I^2R_{e,\text{load}}$$

$$M\left[IS_{p\text{-}n,\text{eff}}(T_1 - T_2) - 2I^2R_{e,\text{eff}}\right] = I^2R_{e,\text{load}}$$

$$80\left[I \times 0.1435\ \text{V/K} \times (T_1 - T_2) - 2I^2 \times 4\ \Omega\right] = I^2 \times 250\ \Omega \qquad (5)$$

With q_1 known from $q_1 = \dot{E}_g / M$, Equations 1 through 5 can be solved for the five unknowns, T_1, T_2, I, q_2, and h_r. Solving the equations numerically using IHT yields the following results for the three different values of $\dot{E}_g$:

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PROBLEM 3.171 (Cont.)

$\dot{E}_g$ (kW)	I (A)	P_{tot} (W)	T_2 (K)	$\eta = P_N/q_1$
1	0.10	2.63	534	0.0026
10	0.67	114	947	0.011
100	3.99	3990	1671	0.040

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COMMENTS: (1) The temperature for the highest thermal energy generation rate is unacceptably high. (2) The electrical energy generated by the device is relatively high, but the efficiency is quite low. The efficiency increases as a function of the thermal generation rate because of larger temperature differences across the module, which are ΔT = 8, 52, and 310 K for the low, medium and high energy generation rates. (3) Numerical solution of the equations requires a good initial guess. One can be obtained by assuming that the current is zero, resulting in $q_1 = q_2$ and enabling direct calculation of the temperatures due to conduction across the TEM and radiation to the surroundings. (4) In this application, thermal generation can occur continuously for many years, providing reliable electrical power to the satellite over its lifetime. (5) What steps could be taken to increase the electrical power generated for each thermal energy generation rate?