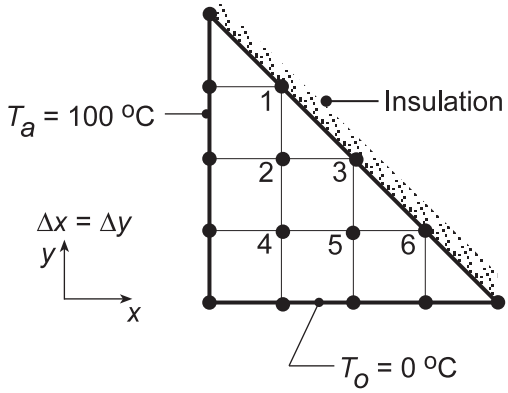
PROBLEM 4.77

KNOWN: Long triangular bar insulated on the diagonal while sides are maintained at uniform temperatures T_a and T_b.

FIND: (a) Using a nodal network with five nodes to the side, and beginning with properly defined control volumes, derive the finite-difference equations for the interior and diagonal nodes and obtain the temperature distribution; sketch the 25, 50 and 75°C isotherms and (b) Recognizing that the insulated diagonal surface can be treated as a symmetry line, show that the diagonal nodes can be treated as interior nodes, and write the finite-difference equations by inspection.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, and (3) Constant properties.

ANALYSIS: (a) For the nodal network shown above, nodes 2, 4, and 5 are interior nodes and, since $\Delta x = \Delta y$, the corresponding finite-difference equations are of the form, Equation 4.29,

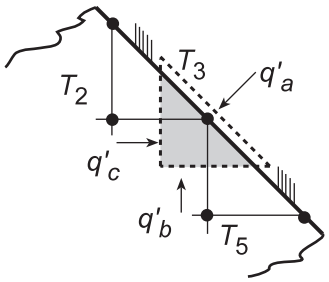
$$T_j = 1/4 \sum T_{neighbors} \tag{1}$$

For a node on the adiabatic, diagonal surface, an energy balance, $\dot{E}_{in} - \dot{E}_{out} = 0$, yields

$$q'_a + q'_b + q'_c = 0$$

$$0 + k\Delta x \frac{T_5 - T_3}{\Delta y} + k\Delta y \frac{T_2 - T_3}{\Delta x} = 0$$

$$T_3 = 1/2\left(T_2 + T_5\right) \tag{2}$$



That is, for the diagonal nodes, m,

$$T_m = 1/2 \sum T_{neighbors} \tag{3}$$

To obtain the temperature distributions, enter Eqs. (1, 2, 3) into the IHT workspace and solve for the nodal temperatures (°C), tabulated according to the nodal arrangement:

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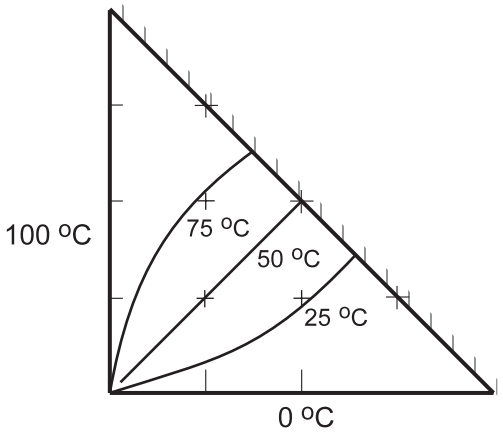
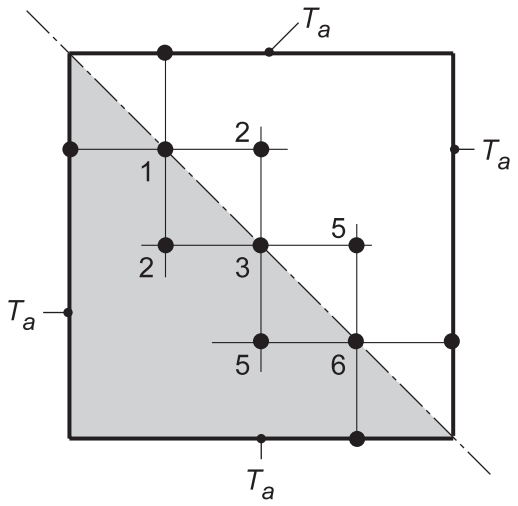
100	85.71			
100	71.43	50.00		
100	50.00	28.57	14.29	
–	0	0	0	–

The 25, 50 and 75°C isotherms are sketched below, using an interpolation scheme to scale the isotherms on the triangular bar.



(b) If we consider the insulated surface as a symmetry plane, the nodal network appears as shown. As such, the diagonal nodes can be treated as interior nodes, as Eq. (1) above applies. Recognize the form is the same as that of Eq. (2) or (3).



COMMENTS: Always look for symmetry conditions which can greatly simplify the writing of nodal equations. In this situation, the adiabatic surface can be treated as a symmetry plane such that the nodes can be treated as interior nodes, and the finite-difference equations can be written by inspection.