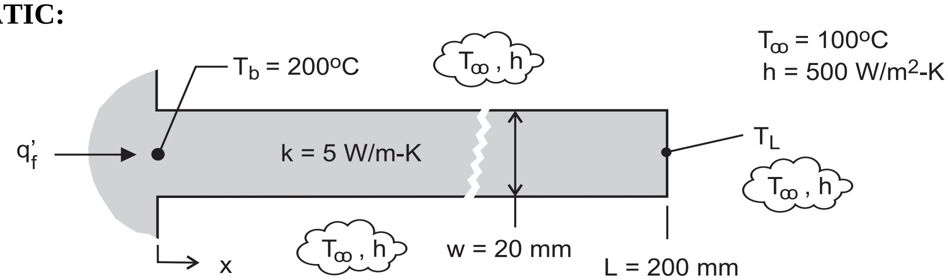
PROBLEM 4.87

KNOWN: Straight fin of uniform cross section with prescribed thermal conditions and geometry; tip condition allows for convection.

FIND: (a) Calculate the fin heat rate, q'_f, and tip temperature, T_L, assuming one-dimensional heat transfer in the fin; calculate the Biot number to determine whether the one-dimensional assumption is valid, (b) Using the finite-element software FEHT, perform a two-dimensional analysis to determine the fin heat rate and the tip temperature; display the isotherms; describe the temperature field and the heat flow pattern inferred from the display, and (c) Validate your FEHT code against the 1-D analytical solution for a fin using a thermal conductivity of 50 and 500 W/m·K.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conduction with constant properties, (2) Negligible radiation exchange, (3) Uniform convection coefficient.

ANALYSIS: (a) Assuming one-dimensional conduction, q'_L and T_L can be determined using Eqs. 3.77 and 3.75, respectively, from Table 3.4, Case A. Alternatively, use the IHT *Model | Extended Surfaces | Temperature Distribution and Heat Rate | Straight Fin | Rectangular*. These results are tabulated below and labeled as "1-D." The Biot number for the fin is

$$Bi = \frac{h(t/2)}{k} = \frac{500\ W/m^2 \cdot K\ (0.020\ m/2)}{5\ W/m \cdot K} = 1$$

(b, c) The fin can be drawn as a two-dimensional outline in FEHT with convection boundary conditions on the exposed surfaces, and with a uniform temperature on the base. Using a fine mesh (at least 1280 elements), solve for the temperature distribution and use the *View | Temperature Contours* command to view the isotherms and the *Heat Flow* command to determine the heat rate into the fin base. The results of the analysis are summarized in the table below.

k (W/m·K)	Bi	Tip temperature, T_L (°C)		Fin heat rate, q'_f (W/m)		Difference* (%)
		1-D	2-D	1-D	2-D	
5	1	100	100	1010	805	20
50	0.1	100.3	100	3194	2990	6.4
500	0.01	123.8	124	9812	9563	2.5

* Difference = $\left(q'_{f,1D} - q'_{f,2D}\right) \times 100 / q'_{f,1D}$

COMMENTS: (1) From part (a), since $Bi = 1 > 0.1$, the internal conduction resistance is not negligible. Therefore significant transverse temperature gradients exist, and the one-dimensional conduction assumption in the fin is a poor one.

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(2) From the table, with k = 5 W/m·K (Bi = 1), the 2-D fin heat rate obtained from the FEA analysis is 20% lower than that for the 1-D analytical analysis. This is as expected since the 2-D model accounts for transverse thermal resistance to heat flow. Note, however, that analyses predict the same tip temperature, a consequence of the fin approximating an infinitely long fin ($mL = 20.2 \gg 2.56$; see Ex. 3.8 Comments).

(3) For the k = 5 W/m·K case, the FEHT isotherms show considerable curvature in the region near the fin base. For example, at x = 10 and 20 mm, the difference between the centerline and surface temperatures are 15 and 7°C.

(4) From the table, with increasing thermal conductivity, note that Bi decreases, and the one-dimensional heat transfer assumption becomes more appropriate. The difference for the case when k = 500 W/m·K is mostly due to the approximate manner in which the heat rate is calculated in the FEA software.