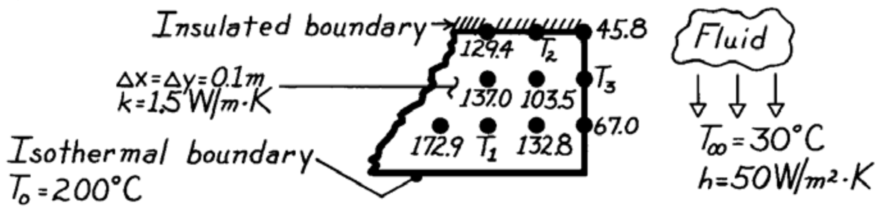
PROBLEM 4.61

KNOWN: Steady-state temperatures (°C) associated with selected nodal points in a two-dimensional system.

FIND: (a) Temperatures at nodes 1, 2 and 3, (b) Heat transfer rate per unit thickness from the system surface to the fluid.

SCHEMATIC:



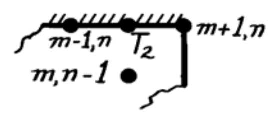
ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Using the finite-difference equations for Nodes 1, 2 and 3:

Node 1, Interior node, Eq. 4.29: $T_1 = \frac{1}{4}\cdot\sum T_{neighbors}$

$$T_1 = \frac{1}{4}\left(172.9 + 137.0 + 132.8 + 200.0\right)^\circ C = 160.7^\circ C$$ <

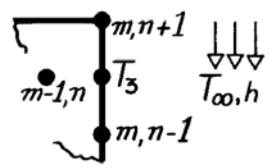
Node 2, Insulated boundary, Eq. 4.46 with h = 0, $T_{m,n} = T_2$



$$T_2 = \frac{1}{4}\left(T_{m-1,n} + T_{m+1,n} + 2T_{m,n-1}\right)$$

$$T_2 = \frac{1}{4}\left(129.4 + 45.8 + 2\times 103.5\right)^\circ C = 95.6^\circ C$$ <

Node 3, Plane surface with convection, Eq. 4.42, $T_{m,n} = T_3$



$$2\left[\frac{h\Delta x}{k} + 2\right]T_3 = \left(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}\right) + \frac{2h\Delta x}{k}T_\infty$$

$$h\Delta x/k = 50\text{W/m}^2\cdot\text{K}\times 0.1\text{m}/1.5\text{W/m}\cdot\text{K} = 3.33$$

$$2\left(3.33 + 2\right)T_3 = \left(2\times 103.5 + 45.8 + 67.0\right)^\circ C + 2\times 3.33\times 30^\circ C$$

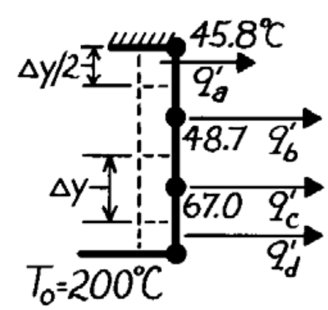
$$T_3 = \frac{1}{10.66}\left(319.80 + 199.80\right)^\circ C = 48.7^\circ C$$ <

(b) The heat rate per unit thickness from the surface to the fluid is determined from the sum of the convection rates from each control volume surface.

$$q'_{conv} = q'_a + q'_b + q'_c + q'_d$$

$$q_i = h\Delta y_i\left(T_i - T_\infty\right)$$

$$q'_{conv} = 50\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left[\frac{0.1}{2}\text{m}\ \left(45.8 - 30.0\right)^\circ C + 0.1\text{m}\ \left(48.7 - 30.0\right)^\circ C + 0.1\text{m}\ \left(67.0 - 30.0\right)^\circ C + \frac{0.1\text{m}}{2}\left(200.0 - 30.0\right)^\circ C\right]$$



$$q'_{conv} = \left(39.5 + 93.5 + 185.0 + 425\right)\ \text{W/m} = 743\ \text{W/m}.$$ <