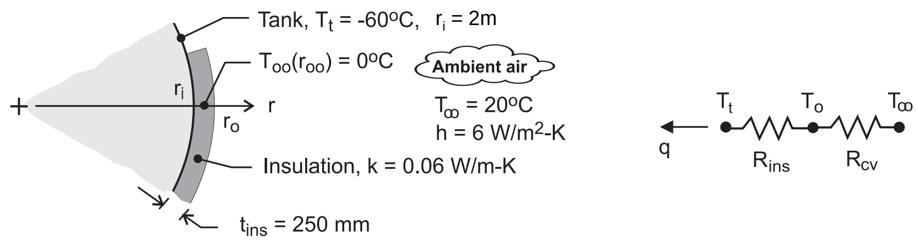
PROBLEM 3.76

KNOWN: Spherical tank of 4-m diameter containing LP gas at -60°C with 250 mm thickness of insulation having thermal conductivity of 0.06 W/m·K. Ambient air temperature and convection coefficient on the outer surface are 20°C and 6 $W/m^2 \cdot K$, respectively.

FIND: (a) Determine the radial position in the insulation at which the temperature is 0°C and (b) If the insulation is pervious to moisture, what conclusions can be reached about ice formation? What effect will ice formation have on the heat gain? How can this situation be avoided?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings, (4) Inner surface of insulation at T_t.

ANALYSIS: (a) The heat transfer situation can be represented by the thermal circuit shown above. The heat gain to the tank is

$$q = \frac{T_\infty - T_t}{R_{ins} + R_{cv}} = \frac{\left[20-(-60)\right]K}{\left(0.0737 + 2.62\times10^{-3}\right)K/W} = 1048\ W$$

where the thermal resistances for the insulation (see Table 3.3) and the convection process on the outer surface are, respectively,

$$R_{ins} = \frac{1/r_i - 1/r_o}{4\pi k} = \frac{(1/2.0 - 1/2.25)m^{-1}}{4\pi\times0.06\ W/m\cdot K} = 0.0737\ K/W$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{h4\pi r_o^2} = \frac{1}{6\ W/m^2\cdot K\times4\pi(2.25\ m)^2} = 2.62\times10^{-3}K/W$$

To determine the location within the insulation where T_{oo} (r_{oo}) = 0°C, use the conduction rate equation, Eq. 3.41,

$$q = \frac{4\pi k\left(T_{oo} - T_t\right)}{\left(1/r_i - 1/r_{oo}\right)} \qquad r_{oo} = \left[\frac{1}{r_i} - \frac{4\pi k\left(T_{oo} - T_t\right)}{q}\right]^{-1}$$

and substituting numerical values, find

$$r_{oo} = \left[\frac{1}{2.0\ m} - \frac{4\pi\times0.06\ W/m\cdot K\left(0-(-60)\right)K}{1048\ W}\right]^{-1} = 2.189\ m$$ <

(b) With r_{oo} = 2.189 m, we'd expect the region of the insulation $r_i \le r \le r_{oo}$ to be filled with ice formations if the insulation is pervious to water vapor. The effect of the ice formation is to substantially increase the heat gain since k_{ice} is nearly twice that of k_{ins}, and the ice region is of thickness (2.189 – 2.0)m = 189 mm. To avoid ice formation, a vapor barrier should be installed at a radius larger than r_{oo}.