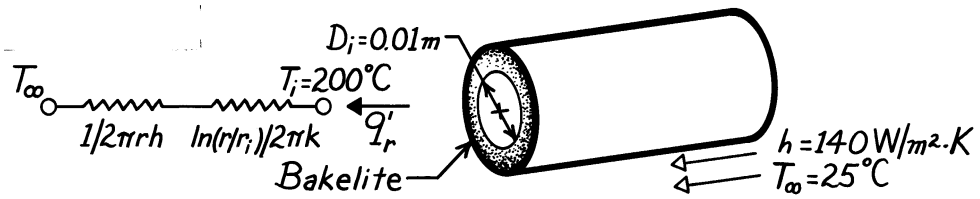
PROBLEM 3.62

KNOWN: Surface temperature of a circular rod coated with Bakelite and adjoining fluid conditions.

FIND: (a) Critical insulation radius, (b) Heat transfer per unit length for bare rod and for insulation at critical radius, (c) Insulation thickness needed for 25% heat rate reduction.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties, (4) Negligible radiation and contact resistance.

PROPERTIES: *Table A-3*, Bakelite (300K): k = 1.4 W/m·K.

ANALYSIS: (a) From Example 3.6, the critical radius is

$$r_{cr} = \frac{k}{h} = \frac{1.4\ \text{W/m}\cdot\text{K}}{140\ \text{W/m}^2\cdot\text{K}} = 0.01\text{m}. \qquad <$$

(b) For the bare rod,

$$q' = h(\pi\ D_i)\ (T_i - T_\infty)$$

$$q' = 140\frac{\text{W}}{\text{m}^2\cdot\text{K}}(\pi\times 0.01\text{m})\ (200-25)^\circ\text{C} = 770\ \text{W/m} \qquad <$$

For the critical insulation thickness,

$$q' = \frac{T_i - T_\infty}{\frac{1}{2\pi\ r_{cr}h} + \frac{\ln(r_{cr}/r_i)}{2\pi\ k}} = \frac{(200-25)^\circ\text{C}}{\frac{1}{2\pi\times(0.01\text{m})\times 140\ \text{W/m}^2\cdot\text{K}} + \frac{\ln(0.01\text{m}/0.005\text{m})}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}}$$

$$q' = \frac{175^\circ\text{C}}{(0.1137+0.0788)\ \text{m}\cdot\text{K/W}} = 909\ \text{W/m} \qquad <$$

(c) The insulation thickness needed to reduce the heat rate to 577 W/m is obtained from

$$q' = \frac{T_i - T_\infty}{\frac{1}{2\pi\ rh} + \frac{\ln(r/r_i)}{2\pi\ k}} = \frac{(200-25)^\circ\text{C}}{\frac{1}{2\pi(r)140\ \text{W/m}^2\cdot\text{K}} + \frac{\ln(r/0.005\text{m})}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}} = 577\frac{\text{W}}{\text{m}}$$

From a trial-and-error solution, find

$r \approx 0.06$ m.

The desired insulation thickness is then

$$\delta = (r - r_i) \approx (0.06 - 0.005)\text{m} = 55\ \text{mm}. \qquad <$$