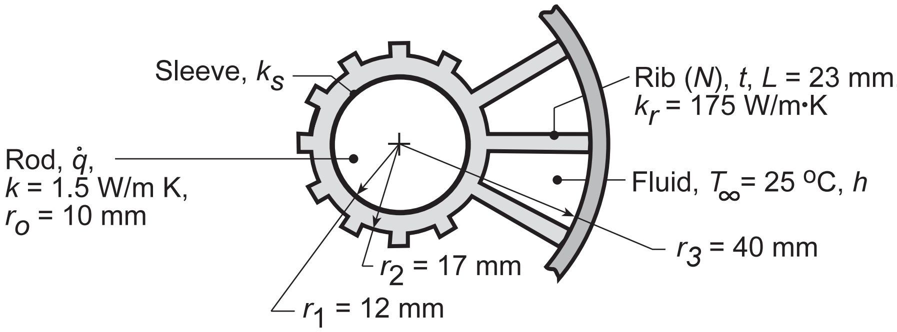
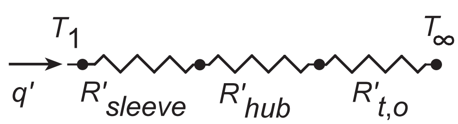
PROBLEM 3.153

KNOWN: Long rod with internal volumetric generation covered by an electrically insulating sleeve and supported with a ribbed spider.

FIND: Combination of convection coefficient, spider design, and sleeve thermal conductivity which enhances volumetric heating subject to a maximum centerline temperature of 100°C.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial heat transfer in rod, sleeve and hub, (3) Negligible interfacial contact resistances, (4) Constant properties, (5) Adiabatic outer surface.

ANALYSIS: The system heat rate per unit length may be expressed as

$$q' = \dot{q}\left(\pi r_o^2\right) = \frac{T_1 - T_\infty}{R'_{sleeve} + R'_{hub} + R'_{t,o}}$$

where

$$R'_{sleeve} = \frac{\ln\left(r_1/r_o\right)}{2\pi k_s}, \quad R'_{hub} = \frac{\ln\left(r_2/r_1\right)}{2\pi k_r} = 3.168\times10^{-4}\,\text{m}\cdot\text{K/W}, \quad R'_{t,o} = \frac{1}{\eta_o h A'_t},$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1-\eta_f\right), \qquad A'_f = 2\left(r_3 - r_2\right), \qquad A'_t = NA'_f + \left(2\pi r_3 - Nt\right),$$

$$\eta_f = \frac{\tanh m\left(r_3 - r_2\right)}{m\left(r_3 - r_2\right)}, \qquad m = \left(2h/k_r t\right)^{1/2}.$$

The rod centerline temperature is related to T_1 through

$$T_o = T(0) = T_1 + \frac{\dot{q} r_o^2}{4k}$$

Calculations may be expedited by using the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array*. For base case conditions of k_s = 0.5 W/m·K, h = 20 W/m^2·K, t = 4 mm and N = 12, R'_{sleeve} = 0.0580 m·K/W, $R'_{t,o}$ = 0.0826 m·K/W, η_f = 0.990, q' = 387 W/m, and $\dot{q}$ = 1.23 × 10^6 W/m^3. As shown below, $\dot{q}$ may be increased by increasing h, where h = 250 W/m^2·K represents a reasonable upper limit for airflow. However, a more than 10-fold increase in h yields only a 63% increase in $\dot{q}$.

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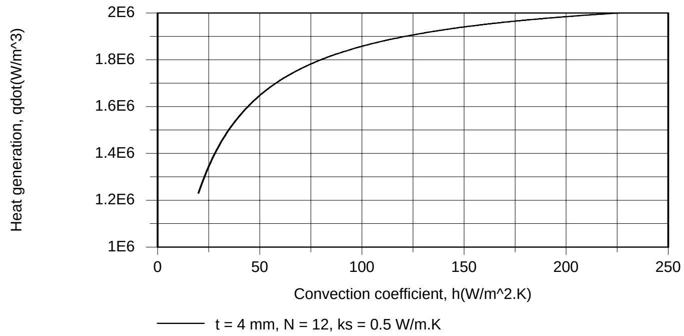
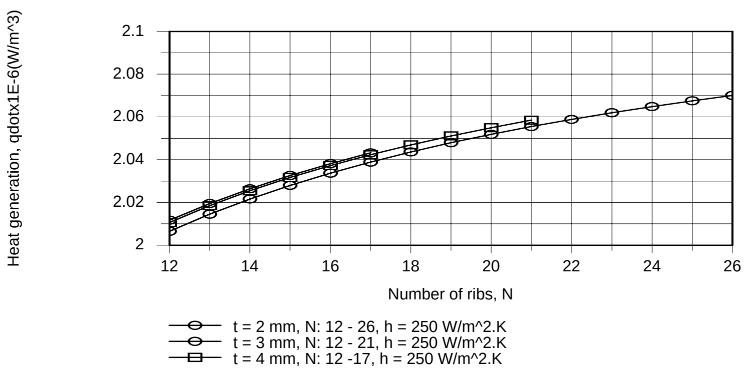
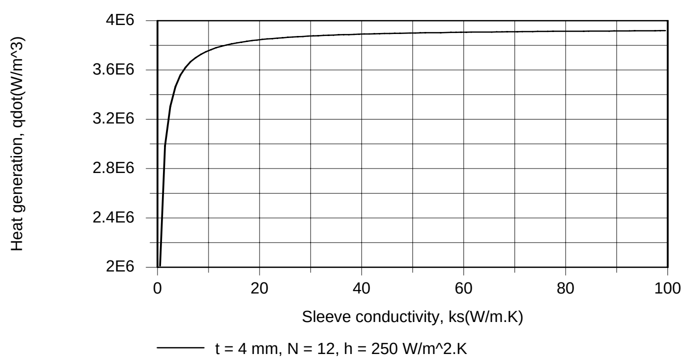


The difficulty is that, by significantly increasing h, the thermal resistance of the fin array is reduced to 0.00727 m·K/W, rendering the sleeve the dominant contributor to the total resistance.

Similar results are obtained when N and t are varied. For values of t = 2, 3 and 4 mm, variations of N in the respective ranges $12 \le N \le 26$, $12 \le N \le 21$ and $12 \le N \le 17$ were considered. The upper limit on N was fixed by requiring that $(S - t) \ge 2$ mm to avoid an excessive resistance to airflow between the ribs. As shown below, the effect of increasing N is small, and there is little difference between results for the three values of t.



In contrast, significant improvement is associated with changing the sleeve material, and it is only necessary to have $k_s \approx 25$ W/m·K (e.g. a boron sleeve) to approach an upper limit to the influence of k_s.



For h = 250 W/m^2·K and k_s = 25 W/m·K, only a slight improvement is obtained by increasing N. Hence, the recommended conditions are:

$$h = 250\,\mathrm{W/m^2 \cdot K}, \qquad k_s = 25\,\mathrm{W/m \cdot K}, \qquad N = 12, \qquad t = 4\mathrm{mm} \qquad <$$

COMMENTS: The upper limit to $\dot{q}$ is reached as the total thermal resistance approaches zero, in which case $T_1 \to T_\infty$. Hence $\dot{q}_{max} = 4k\left(T_o - T_\infty\right)/r_o^2 = 4.5\times10^6\ \mathrm{W/m^3}$.