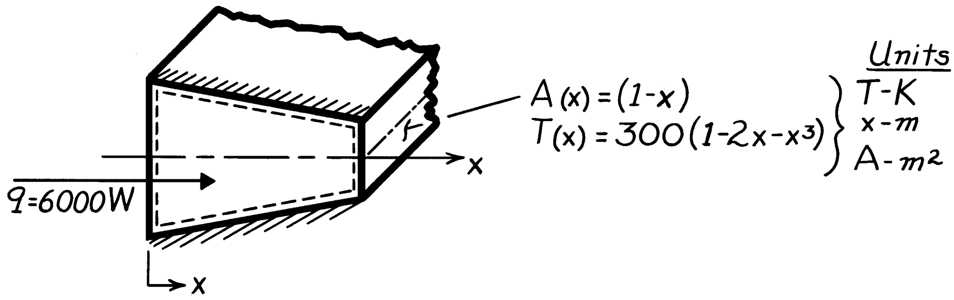
PROBLEM 2.5

KNOWN: Symmetric shape with prescribed variation in cross-sectional area, temperature distribution and heat rate.

FIND: Expression for the thermal conductivity, k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation.

ANALYSIS: Applying the energy balance, Eq. 1.12c, to the system, it follows that, since $\dot{E}_{in} = \dot{E}_{out}$,

$$q_x = \text{Constant} \neq f(x).$$

Using Fourier's law, Eq. 2.1, with appropriate expressions for A_x and T, yields

$$q_x = -k\, A_x \frac{dT}{dx}$$

$$6000\text{W}=-k \cdot (1\text{-x})\text{m}^2 \cdot \frac{d}{dx}\left[300\left(1-2x\text{-}x^3\right)\right]\frac{\text{K}}{\text{m}}.$$

Solving for k and recognizing its units are W/m·K,

$$k=\frac{-6000}{(1\text{-x})\left[300\left(-2-3x^2\right)\right]}=\frac{20}{(1-x)\left(2+3x^2\right)}.$$ <

COMMENTS: (1) At x = 0, k = 10W/m·K and k → ∞ as x → 1. (2) Recognize that the 1-D assumption is an approximation which becomes more inappropriate as the area change with x, and hence two-dimensional effects, become more pronounced.