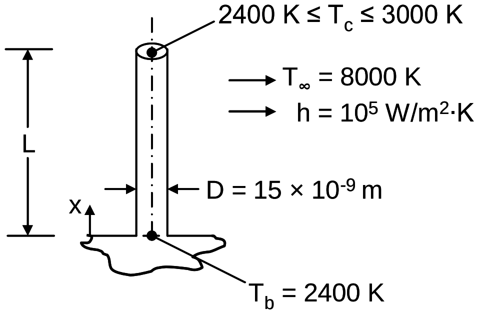
PROBLEM 3.109

KNOWN: Diameter and base temperature of a silicon carbide nanowire, required temperature of the catalyst tip.

FIND: Maximum length of a nanowire that may be grown under specified conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Nanowire stops growing when $T_c = T(x = L) = 3000$ K, (2) Constant properties, (3) One-dimensional heat transfer, (4) Convection from the tip of the nanowire, (5) Nanowire grows very slowly, (6) Negligible impact of nanoscale heat transfer effects.

PROPERTIES: Table A.2, silicon carbide (1500 K): k = 30 W/m·K.

ANALYSIS: The tip of the nanowire is initially at T = 2400 K, and increases in temperature as the nanowire becomes longer. At steady-state, the tip reaches T = 3000 K. The temperature distribution at steady-state is given by Eq. 3.75:

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL} \qquad (1)$$

where

$$m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left(\frac{4h}{kD}\right)^{1/2} = \left(\frac{4 \times 10^5 \text{ W/m}^2\cdot\text{K}}{30 \text{ W/m}\cdot\text{K} \times 15 \times 10^{-9} \text{ m}}\right)^{1/2} = 943 \times 10^3 \text{ m}^{-1}$$

and

$$\frac{h}{mk} = \frac{10^5 \text{ W/m}^2\cdot\text{K}}{943 \times 10^3 \text{ m}^{-1} \times 30 \text{ W/m}\cdot\text{K}} = 3.53 \times 10^{-3}$$

Equation 1, evaluated at x = L, is

$$\frac{\theta}{\theta_b} = \frac{(3000 - 8000)\text{ K}}{(2400 - 8000)\text{ K}} = 0.893 = \frac{1}{\cosh(943 \times 10^3 \times L) + 3.53 \times 10^{-3}\sinh(943 \times 10^3 \times L)}$$

A trial-and-error solution yields $L = 510 \times 10^{-9}$ m = 510 nm <

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PROBLEM 3.109 (Cont.)

COMMENTS: (1) The importance of radiation heat transfer may be ascertained by evaluating Eq. 1.9. Assuming large surroundings at a temperature of T_{sur} = 8000 K and an emissivity of unity, the radiation heat transfer coefficient at the fin tip is

$$h_r = \varepsilon\sigma(T(x = L) + T_{sur})\left[T^2(x{=}L) + T_{sur}^2\right]$$

$$= 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \times (3000\ \text{K} + 8000\ \text{K}) \times \left[(3000\ \text{K})^2 + (8000\ \text{K})^2\right] = 4.5 \times 10^4\ \text{W/m}^2 \cdot \text{K}$$

We see that $h_r < h$, but radiation may be important. (2) The thermal conductivity has been evaluated at 1500 K and extrapolated to a much higher temperature. More accurate values of the thermal conductivity, accounting for the high temperature and possible nanoscale heat transfer effects, are desirable. (3) If the nanowire were to grow rapidly, the transient temperature distribution within the nanowire would need to be evaluated.