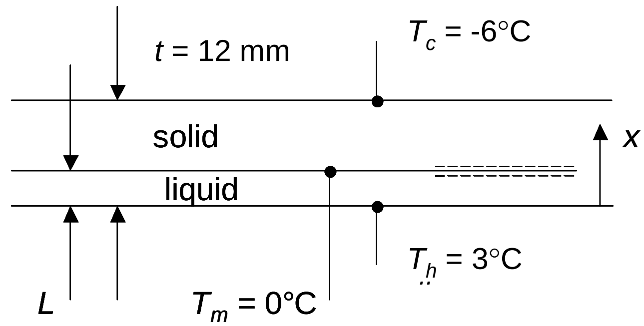
PROBLEM 3.8

KNOWN: Top and bottom temperatures applied to a water layer of known thickness.

FIND: Steady-state location of the solid-liquid interface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible radiation, (4) Negligible convection in the liquid.

PROPERTIES: Table A.6, liquid water (T = 273 K): k_f = 0.569 W/m·K; Table A.3, ice (T = 0 K), k_s = 1.88 W/m·K.

ANALYSIS: An energy balance at the control surface shown in the schematic yields

$$k_f(T_h - T_m)/L = k_s(T_m - T_c)/(t - L)$$

or

$$L = \frac{t}{\left(\dfrac{k_s(T_m - T_c)}{k_f(T_h - T_m)} + 1\right)} = \frac{12\times10^{-3}\text{m}}{\left(\dfrac{1.88\text{W/m}\cdot\text{K}(0°\text{C} - (-6°\text{C})}{0.569\text{W/m}\cdot\text{K}(3°\text{C} - 0°\text{C})} + 1\right)} = 1.58\times10^{-3}\text{m} = 1.58 \text{ mm} \qquad <$$

COMMENTS: (1) Liquid water is opaque to thermal radiation, but ice is semi-transparent. A more detailed analysis would account for the effects of radiation. (2) Free convection in the liquid is negligible because the density of liquid water at T_h = 3°C is greater than the density at T_m = 0°C. Water is one of only several liquids that experiences such a *density inversion*.