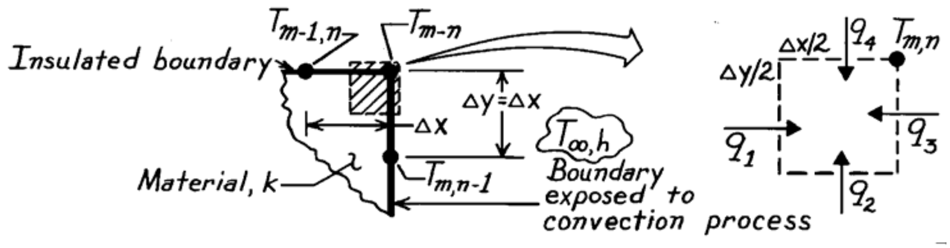
PROBLEM 4.40

KNOWN: External corner of a two-dimensional system whose boundaries are subjected to prescribed conditions.

FIND: Finite-difference equations for these situations: (a) Upper boundary is perfectly insulated and side boundary is subjected to a convection process, (b) Both boundaries are perfectly insulated; compare result with Eq. 4.43.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) No internal generation.

ANALYSIS: Consider the nodal point configuration shown in the schematic and also as Case 4, Table 4.2. The control volume about the node – shaded area above of unit thickness normal to the page – has dimensions, $(\Delta x/2)(\Delta y/2)\cdot 1$. The heat transfer processes at the surface of the CV are identified as q_1, q_2 $\cdots$. Perform an energy balance wherein the processes are expressed using the appropriate rate equations.

(a) With the upper boundary insulated and the side boundary subjected to a convection process, the energy balance has the form

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_3 + q_4 = 0 \tag{1,2}$$

$$k\left[\frac{\Delta y}{2}\cdot 1\right]\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + h\left[\frac{\Delta y}{2}\cdot 1\right]\left(T_{\infty} - T_{m,n}\right) + 0 = 0.$$

Letting $\Delta x = \Delta y$, and regrouping, find

$$T_{m,n-1} + T_{m-1,n} + \frac{h\Delta x}{k}T_{\infty} - 2\left[\frac{1}{2}\frac{h\Delta x}{k} + 1\right]T_{m,n} = 0. \tag{3}$$ <

(b) With both boundaries insulated, the energy balance of Eq. (2) would have $q_3 = q_4 = 0$. The same result would be obtained by letting $h = 0$ in the finite-difference equation, Eq. (3). The result is

$$T_{m,n-1} + T_{m-1,n} - 2T_{m,n} = 0.$$ <

Note that this expression is identical to Eq. 4.43 when $h = 0$, in which case both boundaries are insulated.

COMMENTS: Note the convenience resulting from formulating the energy balance by *assuming* that all the heat flow is *into the node.*