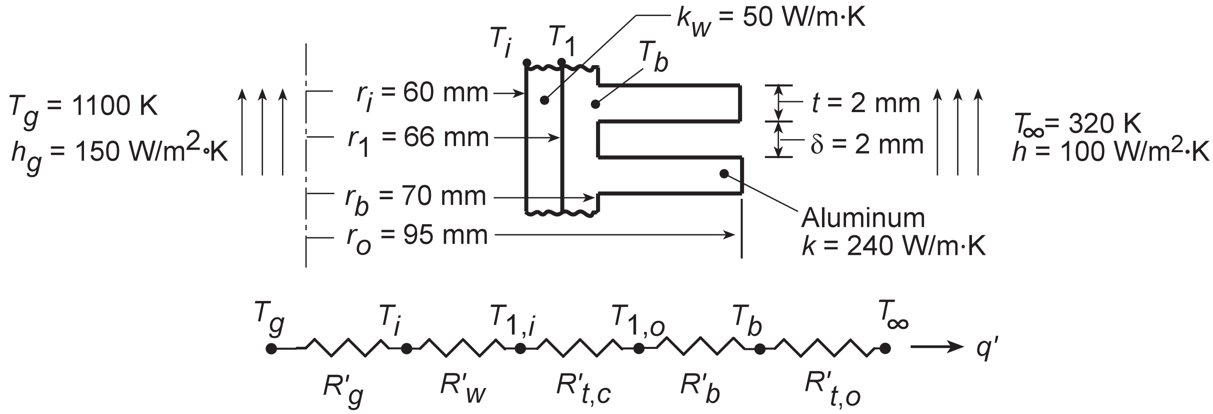
PROBLEM 3.161

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Combustion gas and ambient air conditions. Contact resistance.

FIND: (a) Heat rate per unit length and surface and interface temperatures, (b) Effect of increasing the fin thickness.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: (a) The heat rate per unit length is

$$q' = \frac{T_g - T_\infty}{R'_{tot}}$$

where $R'_{tot} = R'_g + R'_w + R'_{t,c} + R'_b + R'_{t,o}$, and

$$R'_g = \left(h_g 2\pi r_i\right)^{-1} = \left(150\,\text{W/m}^2\cdot\text{K}\times 2\pi\times 0.06\text{m}\right)^{-1} = 0.0177\,\text{m}\cdot\text{K/W},$$

$$R'_w = \frac{\ln(r_1/r_i)}{2\pi k_w} = \frac{\ln(66/60)}{2\pi(50\,\text{W/m}\cdot\text{K})} = 3.03\times10^{-4}\,\text{m}\cdot\text{K/W},$$

$$R'_{t,c} = \left(R''_{t,c}/2\pi r_1\right) = 10^{-4}\,\text{m}^2\cdot\text{K/W}\big/2\pi\times0.066\text{m} = 2.41\times10^{-4}\,\text{m}\cdot\text{K/W}$$

$$R'_b = \frac{\ln(r_b/r_1)}{2\pi k} = \frac{\ln(70/66)}{2\pi\times240\,\text{W/m}\cdot\text{K}} = 3.90\times10^{-5}\,\text{m}\cdot\text{K/W},$$

$$R_{t,o} = \left(\eta_o h A'_t\right)^{-1},$$

$$\eta_o = 1 - \frac{N'A_f}{A'_t}\left(1-\eta_f\right),$$

$$A_f = 2\pi\left(r_{oc}^2 - r_b^2\right)$$

$$A'_t = N'A_f + \left(1 - N't\right)2\pi r_b$$

$$\eta_f = \frac{(2r_b/m)}{\left(r_{oc}^2 - r_b^2\right)}\,\frac{K_1(mr_b)I_1(mr_{oc}) - I_1(mr_b)K_1(mr_{oc})}{I_0(mr_1)K_1(mr_{oc}) + K_0(mr_b)I_1(mr_{oc})}$$

$$r_{oc} = r_o + (t/2), \qquad m = \left(2h/kt\right)^{1/2}$$

Continued...

Once the heat rate is determined from the foregoing expressions, the desired interface temperatures may be obtained from

$$T_i = T_g - q'R'_g$$

$$T_{1,i} = T_g - q'\left(R'_g + R'_w\right)$$

$$T_{1,o} = T_g - q'\left(R'_g + R'_w + R'_{t,c}\right)$$

$$T_b = T_g - q'\left(R'_g + R'_w + R'_{t,c} + R'_b\right)$$

For the specified conditions we obtain A'_t = 7.00 m, η_f = 0.902, η_o = 0.906 and $R'_{t,o}$ = 0.00158 m·K/W. It follows that

$$q' = 39,300\,\text{W/m} \qquad <$$

$$T_i = 405\text{K}, \quad T_{1,i} = 393\text{K}, \quad T_{1,o} = 384\text{K}, \quad T_b = 382\text{K} \qquad <$$

(b) The *Performance Calculation, Extended Surface* Model for the *Circular Fin* Array may be used to assess the effects of fin thickness and spacing. Increasing the fin thickness to t = 3 mm, with δ = 2 mm, reduces the number of fins per unit length to 200. Hence, although the fin efficiency increases (η_f = 0.930), the reduction in the total surface area (A'_t = 5.72 m) yields an increase in the resistance of the fin array ($R'_{t,o}$ = 0.00188 m·K/W), and hence a reduction in the heat rate (q' = 38,700 W/m) and an increase in the interface temperatures (T_i = 415 K, $T_{1,i}$ = 404 K, $T_{1,o}$ = 394 K, and T_b = 393 K).

COMMENTS: Because the gas convection resistance exceeds all other resistances by at least an order of magnitude, incremental changes in $R_{t,o}$ will not have a significant effect on q' or the interface temperatures.