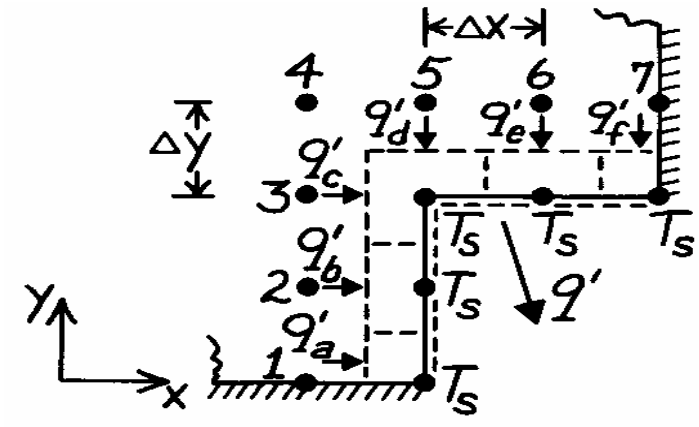
PROBLEM 4.48

KNOWN: Two-dimensional grid for a system with no internal volumetric generation.

FIND: Expression for heat rate per unit length normal to page crossing the isothermal boundary.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, (3) Constant properties.

ANALYSIS: Identify the surface nodes (T_s) and draw control volumes about these nodes. Since there is no heat transfer in the direction parallel to the isothermal surfaces, the heat rate out of the constant temperature surface boundary is

$$q' = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$$

For each q'_i, use Fourier's law and pay particular attention to the manner in which the cross-sectional area and gradients are specified.

$$q' = k(\Delta y/2)\frac{T_1 - T_s}{\Delta x} + k(\Delta y)\frac{T_2 - T_s}{\Delta x} + k(\Delta y)\frac{T_3 - T_s}{\Delta x}$$
$$+ k(\Delta x)\frac{T_5 - T_s}{\Delta y} + k(\Delta x)\frac{T_6 - T_s}{\Delta y} + k(\Delta x/2)\frac{T_7 - T_s}{\Delta y}$$

Regrouping with $\Delta x = \Delta y$, find

$$q' = k\left[0.5T_1 + T_2 + T_3 + T_5 + T_6 + 0.5T_7 - 5T_s\right]. \quad <$$

COMMENTS: Looking at the corner node, it is important to recognize the areas associated with q'_c and q'_d (Δy and Δx, respectively).