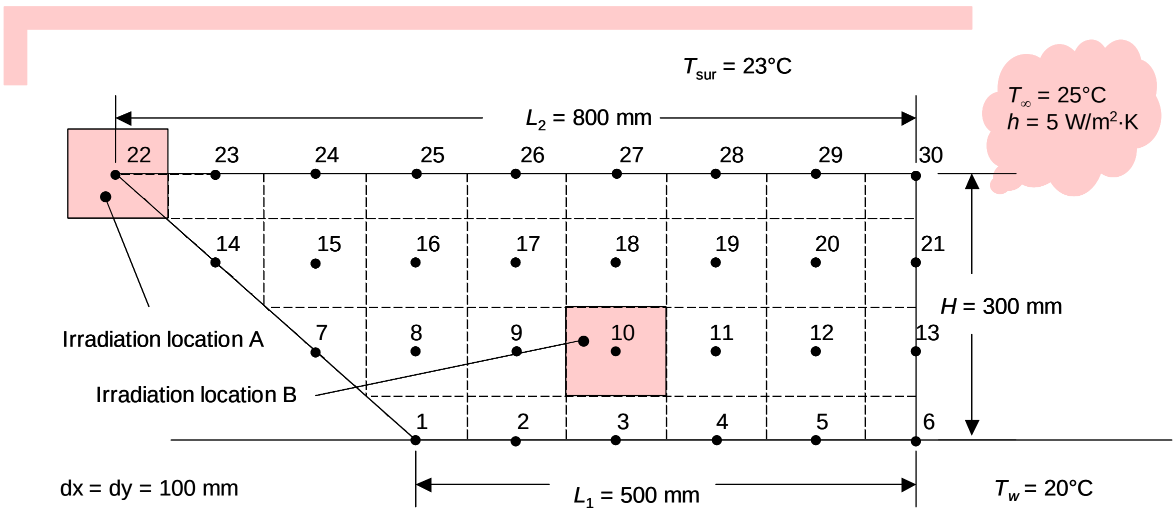
PROBLEM 4.51

KNOWN: Dimensions of mockup, absorbed irradiation in 100 mm × 100 mm area, thermal conductivity and emissivity of plywood and stainless steel, temperature of water, ambient air, and surroundings. Convection heat transfer coefficient.

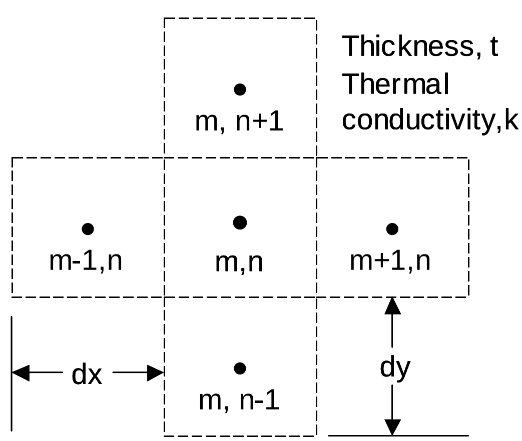
FIND: (a) Maximum steady-state temperature for plywood at locations A and B, (c) Maximum steady-state temperature for stainless steel at locations A and B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Two-dimensional heat transfer, (4) Uniform irradiation, (5) Large surroundings, (6) Submerged section of mockup at water temperature, (7) Negligible temperature gradients through thickness of mockup.

ANALYSIS: We apply Newton's law of cooling, Fourier's law and Eq. 1.7 to a general control volume within the mockup and apply the following general finite-difference formula. Note that radiation and convective losses occur from both the front and back surfaces of the mockup through an area of 2*dxdy*.



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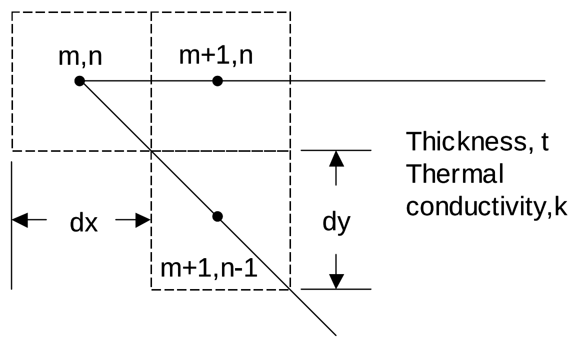
$$\frac{k(T_{m-1,n}-T_{m,n})}{dx}(tdy)+\frac{k(T_{m,n+1}-T_{m,n})}{dy}(tdx)+\frac{k(T_{m+1,n}-T_{m,n})}{dx}(tdy)+\frac{k(T_{m,n-1}-T_{m,n})}{dy}(tdx)$$
$$+G_{s,N}(dx\cdot dy)+h(2dx\cdot dy)(T_\infty-T_{m,n})+\varepsilon\sigma(2dx\cdot dy)(T_{sur}^4-T_{m,n}^4)=0$$

Likewise, for the control volume at the tip of the bow,



$$\frac{k(T_{m+1,n}-T_{m,n})}{dx}(tdy/2)+G_{s,N}(dx\cdot dy/8)+h(dx\cdot dy/4)(T_\infty-T_{m,n})+\varepsilon\sigma(dx\cdot dy/4)(T_{sur}^4-T_{m,n}^4)=0$$

Additional finite difference energy balances are included in the IHT code available in Comment (1). The node numbers are keyed to the schematic.

(a) For plywood with k = 0.8 W/m·K and ε = 0.9, the steady-state temperature at location A is T_{22} = 613.6°C. For irradiation at location B, the steady-state temperature is T_{10} = 613.6°C. Therefore, it does not matter where the irradiation is directed. The temperatures are very high and combustion is likely to occur. <

(b) For stainless steel with k = 15 W/m·K and ε = 0.2, the steady-state temperature at location A is T_{22} = 804.7°C. For irradiation at location B, the steady-state temperature is T_{10} = 767.3°C. Therefore, it is preferable to direct the beam to the tip of the bow of the ship (location A). <

COMMENTS: (1) The IHT code is shown below. Note that this version of the code is associated with irradiation at location A for stainless steel. Appropriate modification of the thermal conductivity and emissivity, as well as revision of the energy balances at Nodes 22 and 10 are necessary to simulate the thermal response of stainless steel or irradiation at location B, respectively.

```
dx = 0.1            //m
dy = 0.1            //m
t = 10/1000         //m
k = 0.8             //W/mK
GsN = 700*100       //W/m^2
eps = 0.2
sigma = 5.67e-8     //Stefan-Boltzmann constant, W/m^2K^4
h = 5               //W/m^2K
Twater = 20 + 273   //K
Tinf = 25 + 273     //K
Tsur = 23 + 273     //K
```

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PROBLEM 4.51 (Cont.)

```
//Nodes 1 through 6
T1 = Twater
T2 = Twater
T3 = Twater
T4 = Twater
T5 = Twater
T6 = Twater

//Node 7
k*t*dx*(T15 - T7)/dy + k*t*dy*(T8 - T7)/dx + 2*h*(dx*dy/2)*(Tinf - T7) + 2*eps*sigma*(dx*dy/2)*(Tsur^4
- T7^4) = 0

//Node 8
k*t*dy*(T7 - T8)/dx + k*t*dx*(T16 - T8)/dy + k*t*dy*(T9 - T8)/dx +k*t*dx*(T1 - T8)/dy+ 2*h*(dx*dy)*(Tinf
- T8) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T8^4) = 0

//Node 9
k*t*dy*(T8 - T9)/dx + k*t*dx*(T17 - T9)/dy + k*t*dy*(T10 - T9)/dx +k*t*dx*(T2 - T9)/dy+
2*h*(dx*dy)*(Tinf - T9) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T9^4) = 0

//Node 10
k*t*dy*(T9 - T10)/dx + k*t*dx*(T18 - T10)/dy + k*t*dy*(T11 - T10)/dx +k*t*dx*(T3 - T10)/dy+
2*h*(dx*dy)*(Tinf - T10) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T10^4)  = 0

//Node 11
k*t*dy*(T10 - T11)/dx + k*t*dx*(T19 - T11)/dy + k*t*dy*(T12 - T11)/dx +k*t*dx*(T4 - T11)/dy+
2*h*(dx*dy)*(Tinf - T11) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T11^4) = 0

//Node 12
k*t*dy*(T11 - T12)/dx + k*t*dx*(T20 - T12)/dy + k*t*dy*(T13 - T12)/dx +k*t*dx*(T5 - T12)/dy+
2*h*(dx*dy)*(Tinf - T12) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T12^4) = 0

//Node 13
k*t*dy*(T12 - T13)/dx + k*t*(dx/2)*(T21 - T13)/dy + k*t*(dx/2)*(T6 - T13)/dy+ 2*h*(dx*dy/2)*(Tinf - T13)
+ 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T13^4) = 0

//Node 14
k*t*dx*(T23 - T4)/dy + k*t*dy*(T15 - T14)/dx + 2*h*(dx*dy/2)*(Tinf - T14) +
2*eps*sigma*(dx*dy/2)*(Tsur^4 - T14^4) = 0

//Node 15
k*t*dy*(T14 - T15)/dx + k*t*dx*(T7 - T15)/dy + k*t*dy*(T16 - T15)/dx +k*t*dx*(T24 - T15)/dy+
2*h*(dx*dy)*(Tinf - T15) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T15^4) = 0

//Node 16
k*t*dy*(T15 - T16)/dx + k*t*dx*(T8 - T16)/dy + k*t*dy*(T17 - T16)/dx +k*t*dx*(T25 - T16)/dy+
2*h*(dx*dy)*(Tinf - T16) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T16^4) = 0

//Node 17
k*t*dy*(T16 - T17)/dx + k*t*dx*(T9 - T17)/dy + k*t*dy*(T18 - T17)/dx +k*t*dx*(T26 - T17)/dy+
2*h*(dx*dy)*(Tinf - T17) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T17^4) = 0

//Node 18
k*t*dy*(T17 - T18)/dx + k*t*dx*(T10 - T18)/dy + k*t*dy*(T19 - T18)/dx +k*t*dx*(T27 - T18)/dy+
2*h*(dx*dy)*(Tinf - T18) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T18^4)  = 0

//Node 19
k*t*dy*(T18 - T19)/dx + k*t*dx*(T11 - T19)/dy + k*t*dy*(T20 - T19)/dx +k*t*dx*(T28 - T19)/dy+
2*h*(dx*dy)*(Tinf - T19) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T19^4) = 0

//Node 20
k*t*dy*(T19 - T20)/dx + k*t*dx*(T12 - T20)/dy + k*t*dy*(T21 - T20)/dx +k*t*dx*(T29 - T20)/dy+
2*h*(dx*dy)*(Tinf - T20) + 2*eps*sigma*(dx*dy)*(Tsur^4 - T20^4) = 0

//Node 21
k*t*dy*(T20 - T21)/dx + k*t*(dx/2)*(T13 - T21)/dy + k*t*(dx/2)*(T30 - T21)/dy+ 2*h*(dx*dy/2)*(Tinf - T21)
+ 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T21^4) = 0
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PROBLEM 4.51 (Cont.)

```
//Node 22
k*t*(dy/2)*(T23 - T22)/dx + 2*(dx*dy/8)*h*(Tinf - T22) + 2*(dx*dy/8)*eps*sigma*(Tsur^4 - T22^4) +
GsN*(dx*dy/8)  = 0

//Node 23
k*t*(dy/2)*(T22 - T23)/dx + k*t*dx*(T14 - T23)/dy + k*t*(dy/2)*(T24 - T23)/dx + 2*h*(dx*dy/2)*(Tinf -
T23) + 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T23^4) = 0

//Node 24
k*t*(dy/2)*(T23 - T24)/dx + k*t*dx*(T15 - T24)/dy + k*t*(dy/2)*(T25 - T24)/dx + 2*h*(dx*dy/2)*(Tinf -
T24) + 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T24^4) = 0

//Node 25
k*t*(dy/2)*(T24 - T25)/dx + k*t*dx*(T16 - T25)/dy + k*t*(dy/2)*(T26 - T25)/dx + 2*h*(dx*dy/2)*(Tinf -
T25) + 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T25^4) = 0

//Node 26
k*t*(dy/2)*(T25 - T26)/dx + k*t*dx*(T17 - T26)/dy + k*t*(dy/2)*(T27 - T26)/dx + 2*h*(dx*dy/2)*(Tinf -
T26) + 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T26^4) = 0

//Node 27
k*t*(dy/2)*(T26 - T27)/dx + k*t*dx*(T18 - T27)/dy + k*t*(dy/2)*(T28 - T27)/dx + 2*h*(dx*dy/2)*(Tinf -
T27) + 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T27^4) = 0

//Node 28
k*t*(dy/2)*(T27 - T28)/dx + k*t*dx*(T19 - T28)/dy + k*t*(dy/2)*(T29 - T28)/dx + 2*h*(dx*dy/2)*(Tinf -
T28) + 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T28^4) = 0

//Node 29
k*t*(dy/2)*(T28 - T29)/dx + k*t*dx*(T20 - T29)/dy + k*t*(dy/2)*(T30 - T29)/dx + 2*h*(dx*dy/2)*(Tinf -
T29) + 2*eps*sigma*(dx*dy/2)*(Tsur^4 - T29^4) = 0

//Node 30
k*t*(dy/2)*(T29 - T30)/dx + k*t*(dx/2)*(T21 - T30)/dy +  2*h*(dx*dy/4)*(Tinf - T30) +
2*eps*sigma*(dx*dy/4)*(Tsur^4 - T30^4) = 0
```

(2) For irradiation at location A of plywood, steady-state temperatures in the vicinity of T_{22} are T_{23} = 42.99°C and T_{14} = 25.62°C. Temperatures at locations far-removed from the irradiation site are at T = 23.97°C. For irradiation at location B, steady-state temperatures in the vicinity of T_{10} are T_{18} = 42.99°C and $T_9 = T_{11}$ = 42.80°C. The thermal conductivity and thickness of the plywood are small, and conduction from the irradiated area to neighboring nodes is negligible. In fact, setting the thermal conductivity to k = 0 W/m·K yields a steady-state temperature of T_{22} = 619.8°C, only a few degrees higher than when conduction is accounted for. (3) For irradiation at location A of stainless steel, steady-state temperatures in the vicinity of T_{22} are T_{23} = 274.9°C and T_{14} = 237.2°C. Temperatures at locations far-removed from the irradiation site are at T = 24.37°C. For irradiation at location B, steady-state temperatures in the vicinity of T_{10} are T_{18} = 233.4°C, T_9 = 202.0°C and T_{11} = 201.6°C. The thermal conductivity and thickness of the stainless steel are sufficiently large so that conduction from the irradiated area to neighboring nodes is significant. Setting the thermal conductivity to k = 0 W/m·K yields a steady-state temperature of T_{22} = 1004°C, significantly higher than when conduction is accounted for.(4) Although conduction losses from the irradiated areas are significant in the stainless steel, relatively high temperatures can be induced because radiation losses are reduced relative to plywood because of the lower emissivity. (5) Stainless steel will have a smaller absorptivity relative to plywood. Many more students would be needed to focus their individual mirrors to induce the absorbed irradiation value given in the problem statement. (6) The preceding analysis is based upon the assumption of two-dimensional heat transfer, implying the edge effects are negligible. However, for Node 22 (location A) the edge surface area is significant and constitutes approximately 30% of the entire exposed surface area. Including edge losses at location A will decrease the temperature of the wood mockup at that location to approximately T_{22} = 527°C while inclusion of edge losses will not affect the predicted temperature at location B. For the stainless steel case, T_{22} = 700°C and the temperature at location B is not affected. Hence, the recommended irradiation location is highly-dependent upon whether edge losses are accounted for.