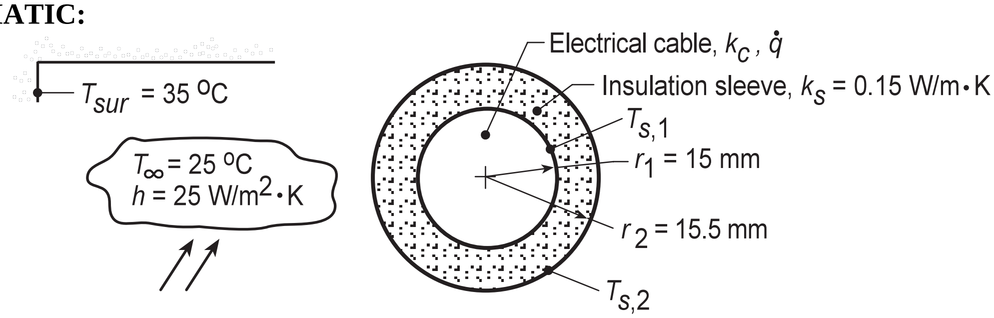
PROBLEM 2.50

KNOWN: An electric cable with an insulating sleeve experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that prescribed temperature distributions for the cable and insulating sleeve satisfy their appropriate heat diffusion equations; sketch temperature distributions labeling key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the sleeve, q'_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the cable to obtain an alternative expression for q'_r in terms of $\dot{q}$ and r_1; (c) Apply surface energy balance around the outer surface of the sleeve to obtain an expression for which $T_{s,2}$ can be evaluated; (d) Determine $T_{s,1}$, $T_{s,2}$, and T_o for the specified geometry and operating conditions; and (e) Plot $T_{s,1}$, $T_{s,2}$, and T_o as a function of the outer radius for the range $15.5 \le r_2 \le 20$ mm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Uniform volumetric heat generation in cable, (3) Negligible thermal contact resistance between the cable and sleeve, (4) Constant properties in cable and sleeve, (5) Surroundings large compared to the sleeve, and (6) Steady-state conditions.

ANALYSIS: (a) The appropriate forms of the heat diffusion equation (HDE) for the insulation and cable are identified. The temperature distributions are valid if they satisfy the relevant HDE.

Insulation: The temperature distribution is given as

$$T(r) = T_{s,2} + \left(T_{s,1} - T_{s,2}\right)\frac{\ln(r/r_2)}{\ln(r_1/r_2)} \quad (1)$$

and the appropriate HDE (radial coordinates, SS, $\dot{q}$ = 0), Eq. 2.26,

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r\left[0 + \left(T_{s,1} - T_{s,2}\right)\frac{1/r}{\ln(r_1/r_2)}\right]\right) = \frac{d}{dr}\left(\frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)}\right) = 0$$

Hence, the temperature distribution satisfies the HDE. <

Continued...

Cable: The temperature distribution is given as

$$T(r) = T_{s,1} + \frac{\dot{q} r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right) \qquad (2)$$

and the appropriate HDE (radial coordinates, SS, $\dot{q}$ uniform), Eq. 2.26,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k_c} = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\left[0 + \frac{\dot{q} r_1^2}{4k_c}\left(0 - \frac{2r}{r_1^2}\right)\right]\right) + \frac{\dot{q}}{k_c} = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(-\frac{\dot{q} r_1^2}{4k_c}\frac{2r^2}{r_1^2}\right) + \frac{\dot{q}}{k_c} = 0$$

$$\frac{1}{r}\left(-\frac{\dot{q} r_1^2}{4k_c}\frac{4r}{r_1^2}\right) + \frac{\dot{q}}{k_c} = 0$$

Hence the temperature distribution satisfies the HDE. <

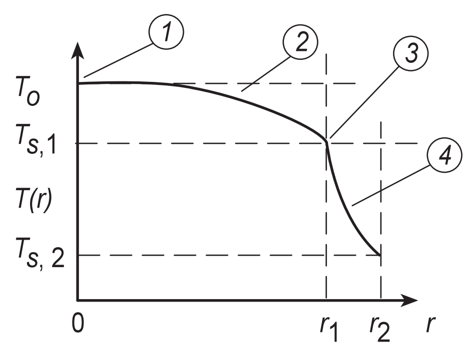
The temperature distributions in the cable, $0 \le r \le r_1$, and sleeve, $r_1 \le r \le r_2$, and their key features are as follows:

(1) Zero gradient, symmetry condition,

(2) Increasing gradient with increasing radius, r, because of $\dot{q}$,

(3) Discontinuous gradient of T(r) across cable-sleeve interface because of different thermal conductivities,

(4) Decreasing gradient with increasing radius, r, since heat rate is constant.



(b) Using Fourier's law for the radial-cylindrical coordinate, the heat rate through the *insulation* (sleeve) per unit length is

$$q'_r = -kA'_r\frac{dT}{dr} = -k2\pi r\frac{dT}{dr}$$ <

and substituting for the temperature distribution, Eq. (1),

$$q'_r = -k_s 2\pi r\left[0 + \left(T_{s,1} - T_{s,2}\right)\frac{1/r}{\ln(r_1/r_2)}\right] = 2\pi k_s\frac{\left(T_{s,1} - T_{s,2}\right)}{\ln(r_2/r_1)} \qquad (3)$$ <

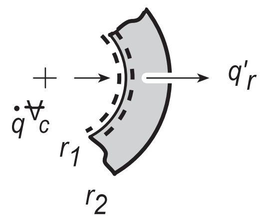
Applying an energy balance to a control surface placed around the cable,

Continued...

PROBLEM 2.50 (Cont.)

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{q}\forall_c - q'_r = 0$$



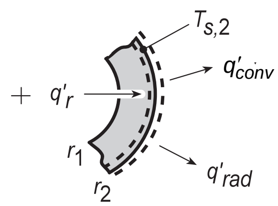
where $\dot{q}\forall_c$ represents the dissipated electrical power in the cable

$$\dot{q}\left(\pi r_1^2\right) - q'_r = 0 \qquad \text{or} \qquad q'_r = \pi \dot{q} r_1^2 \tag{4}$$ <

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q'_r - q'_{conv} - q'_{rad} = 0$$



$$\pi \dot{q} r_1^2 - h\left(2\pi r_2\right)\left(T_{s,2} - T_\infty\right) - \varepsilon\left(2\pi r_2\right)\sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \tag{5}$$ <

This relation can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider a cable-sleeve system with the following prescribed conditions:

r_1 = 15 mm	k_c = 200 W/m·K	h = 25 W/m^2·K	ε = 0.9
r_2 = 15.5 mm	k_s = 0.15 W/m·K	T_∞ = 25°C	T_{sur} = 35°C

For 250 A with R'_e = 0.005 Ω/m, the volumetric heat generation rate is

$$\dot{q} = I^2 R'_e / \forall_c = I^2 R'_e \Big/ \left(\pi r_1^2\right)$$

$$\dot{q} = \left(250\,\text{A}\right)^2 \times 0.005\,\Omega/\text{m} \Big/ \left(\pi \times 0.015^2\,\text{m}^2\right) = 4.42 \times 10^5\,\text{W/m}^3$$

Substituting numerical values in appropriate equations, we can evaluate $T_{s,1}$, $T_{s,2}$ and T_o.

Sleeve outer surface temperature, $T_{s,2}$: Using Eq. (5),

$$\pi \times 4.42 \times 10^5\,\text{W/m}^3 \times \left(0.015\text{m}\right)^2 - 25\,\text{W/m}^2\cdot\text{K} \times \left(2\pi \times 0.0155\text{m}\right)\left(T_{s,2} - 298\text{K}\right)$$
$$-0.9 \times \left(2\pi \times 0.0155\text{m}\right) \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_{s,2}^4 - 308^4\right)\text{K}^4 = 0$$

$$T_{s,2} = 395\,\text{K} = 122^\circ\text{C}$$ <

Sleeve-cable interface temperature, $T_{s,1}$: Using Eqs. (3) and (4), with $T_{s,2}$ = 395 K,

Continued...

PROBLEM 2.50 (Cont.)

$$\pi \dot{q} r_1^2 = 2\pi k_s \frac{\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$\pi \times 4.42 \times 10^5 \ \mathrm{W/m^3} \times \left(0.015\,\mathrm{m}\right)^2 = 2\pi \times 0.15\,\mathrm{W/m \cdot K} \frac{\left(T_{s,1} - 395\,\mathrm{K}\right)}{\ln\left(15.5/15.0\right)}$$

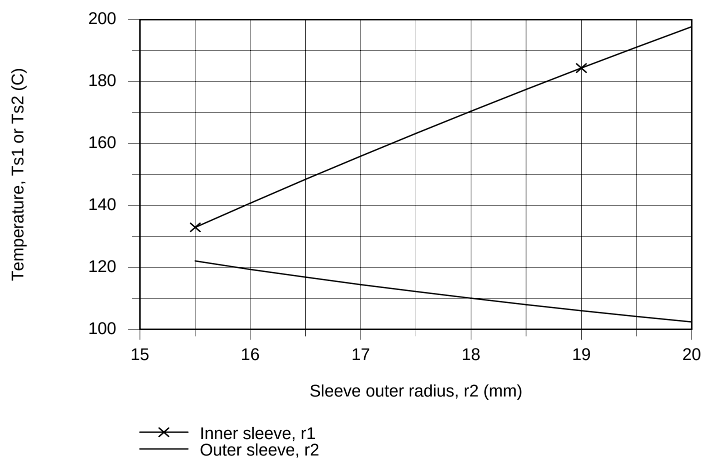
$$T_{s,1} = 406\,\mathrm{K} = 133^\circ\mathrm{C}$$ <

Cable centerline temperature, T_o: Using Eq. (2) with $T_{s,1} = 133°C$,

$$T_o = T(0) = T_{s,1} + \frac{\dot{q} r_1^2}{4k_c}$$

$$T_o = 133^\circ\mathrm{C} + 4.42 \times 10^5 \ \mathrm{W/m^3} \times \left(0.015\,\mathrm{m}\right)^2 \Big/ \left(4 \times 200\,\mathrm{W/m \cdot K}\right) = 133.1^\circ\mathrm{C}$$ <

(e) With all other conditions remaining the same, the relations of part (d) can be used to calculate T_o, $T_{s,1}$ and $T_{s,2}$ as a function of the sleeve outer radius r_2 for the range $15.5 \le r_2 \le 20$ mm.



On the plot above T_o would show the same behavior as $T_{s,1}$ since the temperature rise between cable center and its surface is 0.12°C. With increasing r_2, we expect $T_{s,2}$ to decrease since the heat flux decreases with increasing r_2. We expect $T_{s,1}$ to increase with increasing r_2 since the thermal resistance of the sleeve increases.