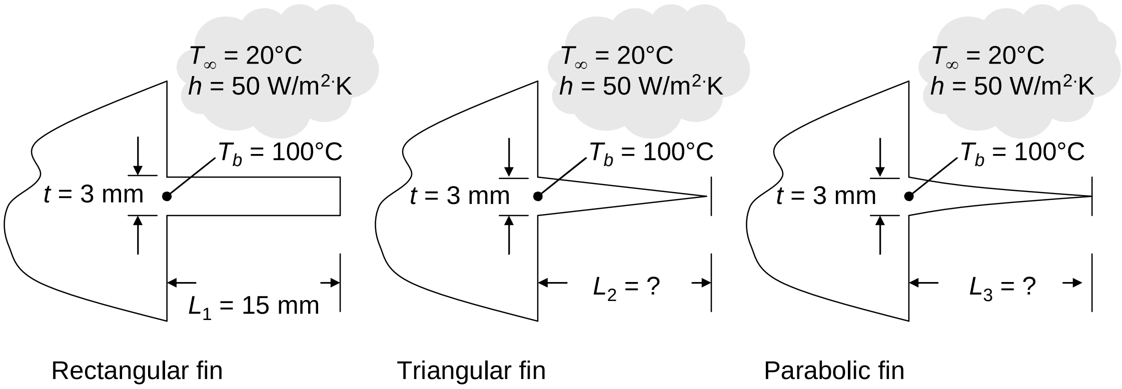
PROBLEM 3.135

KNOWN: Thermal conditions, base thickness, thermal conductivity, and length of a straight rectangular fin.

FIND: (a) Length of triangular straight fin needed to produce the same fin heat rate. Ratio of rectangular straight fin mass to triangular straight fin mass. (b) Repeat part (a) for a parabolic straight fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) Negligible radiation.

PROPERTIES: Aluminum 2024: Given: k = 185 W/m·K, Table A.1: ρ = 2770 kg/m^3.

ANALYSIS: For each of the fins, $m = (2h/kt)^{1/2} = (2 \times 50\ \text{W/m}^2\cdot\text{K}/185\ \text{W/m}\cdot\text{K} \times 0.003\ \text{m})^{1/2}$ = 13.42 m^{-1} and, for fins of unit width, $A_{c,b} = tb$ = 0.003 m × 1 m = 0.003 m^2. Combining the definition of the fin effectiveness, $\varepsilon_f = q_f/(hA_{c,b}\theta_b)$ and the fin efficiency, $\eta_f = q_f/hA_f\theta_b$, yields

$$\varepsilon_f = \eta_f A_f / A_{c,b} \qquad (1)$$

Rectangular Fin: From Table 3.5, $L_c = L + t/2$ = 0.015m + 0.003m/2 = 0.0165m, $\eta_f = \tanh mL_c/(mL_c)$ = tanh 13.42m^{-1}× 0.0165m/(13.42m^{-1}× 0.0165m) = 0.9839, $A_f = 2wL_c$ = 2 × 1m × 0.0165m = 0.033m^2. Hence, ε_{f1} = 0.9839 × 0.033m^2/0.003m^2 = 10.83. The mass of the rectangular fin is $M_1 = \rho tL$ = 2770 kg/m^3 × 0.003 m × 0.015 m = 0.125 kg.

(a) *Triangular Fin*: From Table 3.5,

$$A_f = 2w[L_2^2 + (t/2)^2]^{1/2} = 2 \times 1\text{m} \times [L_2^2 + (0.0015\text{m})^2]^{1/2} \qquad (2a)$$
$$\eta_f = I_1(2mL_2)/[mL_2I_0(2mL_2)] = I_1(2 \times 13.42\text{m}^{-1}L_2)/[\ 13.42\text{m}^{-1}L_2I_0(2\times 13.42\text{m}^{-1}L_2)] \qquad (2b)$$

Equating $\varepsilon_{f2} = \varepsilon_{f1}$ = 10.83, and solving Equations (1) and 2(a, b) simultaneously yields

L_2 = 0.0166m = 16.6mm <

from which $M_2 = \rho(t/2)L_2$ = 2770 kg/m^3 × 0.0015 m × 0.0166 m = 0.069 kg; M_2/M_1 = 0.069kg/0.126kg = 0.55. <

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PROBLEM 3.135 (Cont.)

(b) *Parabolic Fin*: From Table 3.5,

$$C_1 = [1 + (t/L_3)^2]^{1/2} = [1 + (0.003\text{m}/L_3)^2]^{1/2} \qquad (3a)$$
$$A_f = w[C_1L_3 + (L_3^2/t)\ln(t/L_3 + C_1)] = 1\text{m}\times[C_1L_3 + (L_3^2/0.003\text{m})\ln(0.003\text{m}/L_3 + C_1)] \qquad (3b)$$
$$\eta_f = 2/\{[4(mL_3)^2 + 1]^{1/2} + 1\} = 2/\{[4(13.42\text{m}^{-1}L_3)^2 + 1]^{1/2} + 1\} \qquad (3c)$$

Equating $\varepsilon_{f3} = \varepsilon_{f1} = 10.83$, and solving Equations (1) and 3(a - c) simultaneously yields

$$L_3 = 0.0169\text{m} = 16.9\text{mm}$$ <

The mass of the parabolic fin is found from

$$M_3 = \rho A_p w = \rho w t L_3/3 = 2770\text{kg/m}^3 \times 1\text{m} \times 0.003\text{m}/3 = 0.0468\text{kg}$$

and $M_3/M_1 = 0.0468\text{kg}/0.126\text{kg} = 0.37$. <

COMMENTS: (1) The lengths of the three fins are all similar. This is because the fin efficiencies are all near unity ($\eta_R = 0.984$, $\eta_T = 0.976$, $\eta_P = 0.953$) yielding fins of almost constant base temperature. (2) Use of triangular and parabolic fins is appropriate when weight savings is important, such as in aerospace applications. (3) Reduction in cost due to reduction in the amount of raw material used is usually offset by higher manufacturing cost for the triangular and parabolic fins.