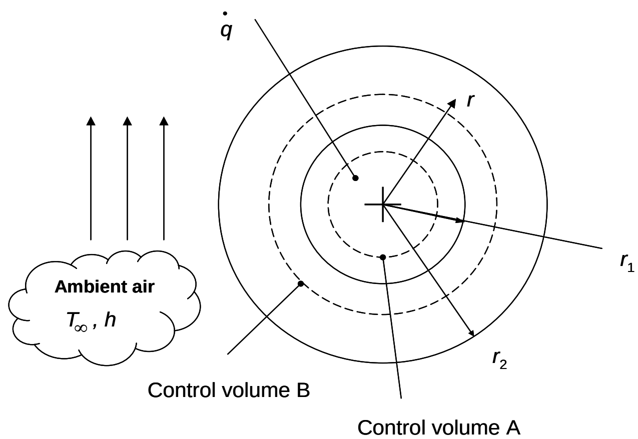
PROBLEM 2.63

KNOWN: Size and thermal conductivities of a spherical particle encased by a spherical shell.

FIND: (a) Relationship between dT/dr and r for $0 \le r \le r_1$, (b) Relationship between dT/dr and r for $r_1 \le r \le r_2$, (c) Sketch of T(r) over the range $0 \le r \le r_2$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer.

ANALYSIS:

(a) The conservation of energy principle, applied to control volume A, results in

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \quad (1)$$

where $\dot{E}_g = \dot{q}\forall = \dot{q}\frac{4}{3}\pi r^3 \quad (2)$

since $\dot{E}_{st} = 0$

$$\dot{E}_{in} - \dot{E}_{out} = q_r'' A = -(-k_1 \frac{dT}{dr})(4\pi r^2) \quad (3)$$

Substituting Eqs. (2) and (3) in Eq. (1) yields

$$\dot{q}\frac{4}{3}\pi r^3 + k_1 \frac{dT}{dr}(4\pi r^2) = 0$$

or

$$\frac{dT}{dr} = -\frac{\dot{q}}{3}\frac{r}{k_1}$$ <

Continued…

PROBLEM 2.63 (Cont.)

(b) For $r > r_1$, the radial heat rate is constant and is

$$\dot{E}_g = q_r = \dot{q}\forall_1 = \dot{q}\frac{4}{3}\pi r_1^3 \tag{4}$$

$$\dot{E}_{in} - \dot{E}_{out} = q_r''A = -(-k_2\frac{dT}{dr})4\pi r^2 \tag{5}$$

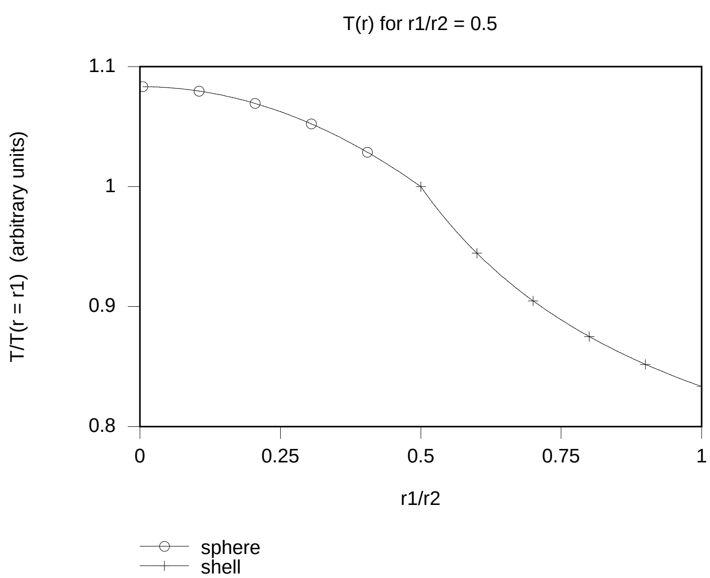
Substituting Eqs. (4) and (5) into Eq. (1) yields

$$k_2\frac{dT}{dr}4\pi r^2 + \dot{q}\frac{4}{3}\pi r_1^3 = 0$$

or

$$\frac{dT}{dr} = -\frac{\dot{q}r_1^3}{3k_2r^2}$$ <

(c) The temperature distribution on T-r coordinates is



COMMENTS: (1) Note the non-linear temperature distributions in both the particle and the shell. (2) The temperature gradient at r = 0 is zero. (3) The discontinuous slope of T(r) at r_1/r_2 = 0.5 is a result of $k_1 = 2k_2$.