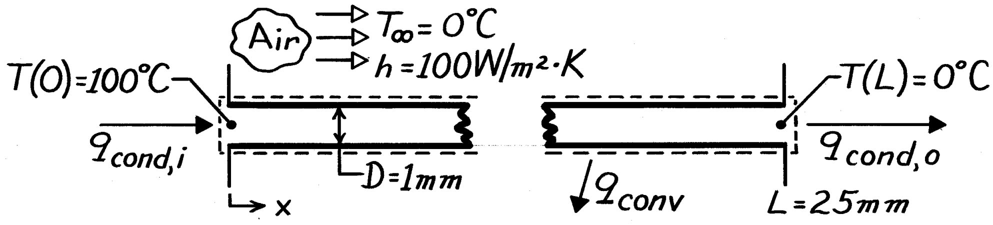
PROBLEM 3.137

KNOWN: Dimensions and end temperatures of pin fins.

FIND: (a) Heat transfer by convection from a single fin and (b) Total heat transfer from a 1 m^2 surface with fins mounted on 4mm centers.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rod, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation.

PROPERTIES: *Table A-1*, Copper, pure (323K): k ≈ 400 W/m·K.

ANALYSIS: (a) By applying conservation of energy to the fin, it follows that

$$q_{conv} = q_{cond,i} - q_{cond,o}$$

where the conduction rates may be evaluated from knowledge of the temperature distribution. The general solution for the temperature distribution is

$$\theta(x) = C_1\, e^{mx} + C_2\, e^{-mx} \qquad \theta \equiv T - T_\infty.$$

The boundary conditions are $\theta(0) \equiv \theta_o = 100°C$ and $\theta(L) = 0$. Hence

$$\theta_o = C_1 + C_2$$

$$0 = C_1\, e^{mL} + C_2\, e^{-mL}$$

Therefore, $$C_2 = C_1\, e^{2mL}$$

$$C_1 = \frac{\theta_o}{1-e^{2mL}}, \qquad C_2 = -\frac{\theta_o\, e^{2mL}}{1-e^{2mL}}$$

and the temperature distribution has the form

$$\theta = \frac{\theta_o}{1-e^{2mL}}\left[e^{mx} - e^{2mL-mx}\right].$$

The conduction heat rate can be evaluated by Fourier's law,

$$q_{cond} = -kA_c\frac{d\theta}{dx} = -\frac{kA_c\theta_o}{1-e^{2mL}}m\left[e^{mx} + e^{2mL-mx}\right]$$

or, with $m = \left(hP/kA_c\right)^{1/2}$,

$$q_{cond} = -\frac{\theta_o\left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left[e^{mx} + e^{2mL-mx}\right].$$

Continued …

Hence at x = 0,

$$q_{cond,i} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(1+e^{2mL}\right)$$

at x = L

$$q_{cond,o} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(2e^{mL}\right)$$

Evaluating the fin parameters:

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4\times100\ W/m^2\cdot K}{400\ W/m\cdot K\times0.001m}\right]^{1/2} = 31.62\ m^{-1}$$

$$\left(hPkA_c\right)^{1/2} = \left[\frac{\pi^2}{4}D^3hk\right]^{1/2} = \left[\frac{\pi^2}{4}\times\left(0.001m\right)^3\times100\frac{W}{m^2\cdot K}\times400\frac{W}{m\cdot K}\right]^{1/2} = 9.93\times10^{-3}\frac{W}{K}$$

$$mL = 31.62\ m^{-1}\times0.025m = 0.791,\qquad e^{mL} = 2.204,\qquad e^{2mL} = 4.865$$

The conduction heat rates are

$$q_{cond,i} = \frac{-100K\left(9.93\times10^{-3}\ W/K\right)}{-3.865}\times5.865 = 1.507\ W$$

$$q_{cond,o} = \frac{-100K\left(9.93\times10^{-3}\ W/K\right)}{-3.865}\times4.408 = 1.133\ W$$

and from the conservation relation,

$$q_{conv} = 1.507\ W - 1.133\ W = 0.374\ W.$$ <

(b) The total heat transfer rate is the heat transfer from N = 250×250 = 62,500 rods and the heat transfer from the remaining (bare) surface ($A = 1m^2 - NA_c$). Hence,

$$q = N\ q_{cond,i} + hA\theta_o = 62,500\ \left(1.507\ W\right) + 100W/m^2\cdot K\left(0.951\ m^2\right)\ 100K$$

$$q = 9.42\times10^4\ W + 0.95\times10^4\ W = 1.037\times10^5\ W.$$

COMMENTS: (1) The fins, which cover only 5% of the surface area, provide for more than 90% of the heat transfer from the surface.

(2) The fin effectiveness, $\varepsilon \equiv q_{cond,i}/hA_c\theta_o$, is ε = 192, and the fin efficiency, $\eta \equiv \left(q_{conv}/h\pi\ DL\theta_o\right)$, is $\eta = 0.48$.

(3) The temperature distribution, $\theta(x)/\theta_o$, and the conduction term, $q_{cond,i}$, could have been obtained directly from Eqs. 3.82 and 3.83, respectively.

(4) Heat transfer by convection from a single fin could also have been obtained from Eq. 3.78.