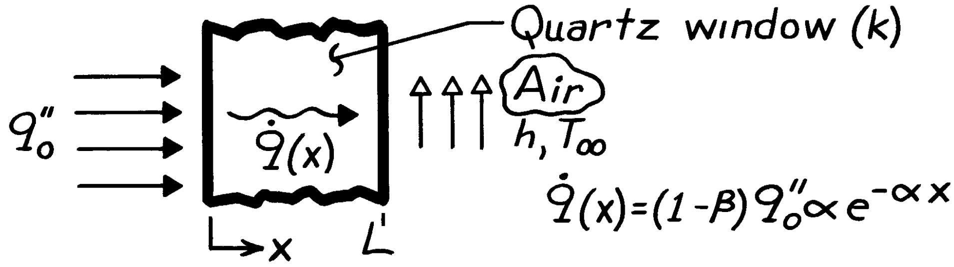
PROBLEM 3.94

KNOWN: Distribution of volumetric heating and surface conditions associated with a quartz window.

FIND: Temperature distribution in the quartz.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation emission and convection at inner surface (x = 0) and negligible emission from outer surface, (4) Constant properties.

ANALYSIS: The appropriate form of the heat equation for the quartz is obtained by substituting the prescribed form of $\dot{q}$ into Eq. 3.44.

$$\frac{d^2T}{dx^2} + \frac{\alpha(1-\beta)q_o''}{k}e^{-\alpha x} = 0$$

Integrating,

$$\frac{dT}{dx} = +\frac{(1-\beta)q_o''}{k}e^{-\alpha x} + C_1 \qquad T = -\frac{(1-\beta)}{k\alpha}q_o''e^{-\alpha x} + C_1x + C_2$$

Boundary Conditions:

$$-k\, dT/dx)_{x=o} = \beta q_o''$$
$$-k\, dT/dx)_{x=L} = h\left[T(L) - T_\infty\right]$$

Hence, at x = 0:

$$-k\left[\frac{(1-\beta)}{k}q_o'' + C_1\right] = \beta q_o''$$
$$C_1 = -q_o''/k$$

At x = L:

$$-k\left[\frac{(1-\beta)}{k}q_o''e^{-\alpha L} + C_1\right] = h\left[-\frac{(1-\beta)}{k\alpha}q_o''e^{-\alpha L} + C_1L + C_2 - T_\infty\right]$$

Substituting for C_1 and solving for C_2,

$$C_2 = \frac{q_o''}{h}\left[1-(1-\beta)e^{-\alpha L}\right] + \frac{q_o''L}{k} + \frac{q_o''(1-\beta)}{k\alpha}e^{-\alpha L} + T_\infty.$$

Hence,
$$T(x) = \frac{(1-\beta)q_o''}{k\alpha}\left[e^{-\alpha L} - e^{-\alpha x}\right] + \frac{q_o''}{k}(L-x) + \frac{q_o''}{h}\left[1-(1-\beta)e^{-\alpha L}\right] + T_\infty. \quad <$$

COMMENTS: The temperature distribution depends strongly on the radiative coefficients, α and β. For $\alpha \to \infty$ or $\beta = 1$, the heating occurs entirely at x = 0 (no volumetric heating).