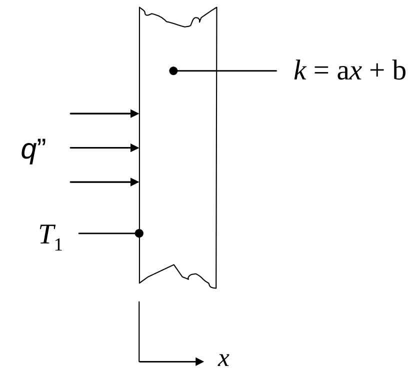
PROBLEM 1.14

KNOWN: Expression for variable thermal conductivity of a wall. Constant heat flux. Temperature at x = 0.

FIND: Expression for temperature gradient and temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction.

ANALYSIS: The heat flux is given by Fourier's law, and is known to be constant, therefore

$$q_x'' = -k\frac{dT}{dx} = \text{constant}$$

Solving for the temperature gradient and substituting the expression for k yields

$$\frac{dT}{dx} = -\frac{q_x''}{k} = -\frac{q_x''}{ax+b}$$ <

This expression can be integrated to find the temperature distribution, as follows:

$$\int \frac{dT}{dx}dx = -\int \frac{q_x''}{ax+b}dx$$

Since $q_x'' = \text{constant}$, we can integrate the right hand side to find

$$T = -\frac{q_x''}{a}\ln(ax+b) + c$$

where c is a constant of integration. Applying the known condition that $T = T_1$ at x = 0, we can solve for c.

Continued…

PROBLEM 1.14 (Cont.)

$$T(x = 0) = T_1$$

$$-\frac{q''_x}{a}\ln b + c = T_1$$

$$c = T_1 + \frac{q''_x}{a}\ln b$$

Therefore, the temperature distribution is given by

$$T = -\frac{q''_x}{a}\ln\left(ax + b\right) + T_1 + \frac{q''_x}{a}\ln b \qquad <$$

$$= T_1 + \frac{q''_x}{a}\ln\frac{b}{ax + b} \qquad <$$

COMMENTS: Temperature distributions are not linear in many situations, such as when the thermal conductivity varies spatially or is a function of temperature. Non-linear temperature distributions may also evolve if internal energy generation occurs or non-steady conditions exist.