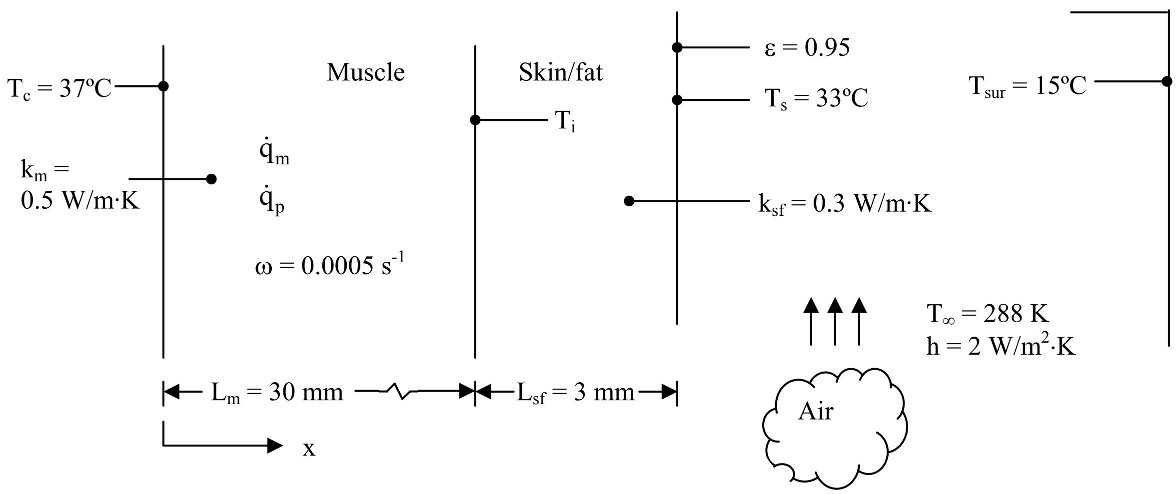
PROBLEM 3.166

KNOWN: Dimensions and thermal conductivities of a muscle layer and a skin/fat layer. Skin emissivity and surface area. Skin temperature. Perfusion rate within the muscle layer. Core body and arterial temperatures. Blood density and specific heat. Ambient conditions.

FIND: Metabolic heat generation rate to maintain skin temperature at 33ºC.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer through the muscle and skin/fat layers, (3) Metabolic heat generation rate, perfusion rate, arterial temperature, blood properties, and thermal conductivities are all uniform, (4) Solar radiation is negligible, (5) Conditions are the same everywhere on the torso, limbs, etc.

ANALYSIS: Since we know the skin temperature and environment temperature, we can find the heat loss rate from the skin surface to the environment:

$$q = hA\,(T_s - T_\infty) + \varepsilon\sigma A(T_s^4 - T_{sur}^4)$$
$$= 2\ \text{W/m}^2\cdot\text{K} \times 1.8\ \text{m}^2 (33 - 15)°\text{C} + 0.95 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 \times 1.8\ \text{m}^2 \left(306^4 - 288^4\right)\text{K}^4$$
$$= 248\ \text{W}$$

We can then find T_i, the interface temperature between the skin/fat layer and the muscle layer, by analyzing heat transfer through the skin/fat layer:

$$T_i = T_s + \frac{qL_{sf}}{k_{sf}A} = 33°\text{C} + \frac{248\ \text{W} \times 0.003\ \text{m}}{0.3\ \text{W/m}\cdot\text{K} \times 1.8\ \text{m}^2} = 34.4°\text{C}$$

Heat transfer in the muscle layer is governed by Equation 3.114. In Example 3.12, this equation was solved subject to specified surface temperature boundary conditions, and the rate at which heat leaves the muscle and enters the skin/fat layer was found to be

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PROBLEM 3.166 (Cont.)

$$q = -k_m A\tilde{m}\frac{\theta_i \cosh \tilde{m}L_m - \theta_c}{\sinh \tilde{m}L_m}$$

This must equal the rate at which heat is transferred across the skin/fat layer, as calculated above. Inserting the definitions of θ_i and θ_c, we can solve for the metabolic heat generation rate:

$$\dot{q}_m = \omega\rho_b c_b \frac{\dfrac{q}{k_m A\tilde{m}}\sinh \tilde{m}L_m + (T_i - T_a)\cosh \tilde{m}L_m + (T_c - T_a)}{\cosh \tilde{m}L_m + 1} \qquad (1)$$

where

$$\tilde{m} = \sqrt{\omega\rho_b c_b/k_m} = \left[0.0005\ \text{s}^{-1} \times 1000\ \text{kg/m}^3 \times 3600\ \text{J/kg}\cdot\text{K}\Big/0.5\ \text{W/m}\cdot\text{K}\right]^{1/2} = 60\ \text{m}^{-1}$$

$$\sinh(\tilde{m}L_m) = \sinh(60\ \text{m}^{-1} \times 0.03\ \text{m}) = 2.94\ ; \quad \cosh(\tilde{m}L_m) = \cosh(60\ \text{m}^{-1} \times 0.03\ \text{m}) = 3.11$$

With $T_c = T_a$, Equation (1) yields

$$\dot{q}_m = 0.0005\ \text{s}^{-1} \times 1000\ \text{kg/m}^3 \times 3600\ \text{J/kg}\cdot\text{K}$$

$$\times \left[\frac{\dfrac{248\ \text{W}}{0.5\ \text{W/m}\cdot\text{K} \times 1.8\ \text{m}^2 \times 60\ \text{m}^{-1}} \times 2.94 + (34.4 - 37)°\text{C} \times 3.11}{3.11 + 1}\right]$$

$$= 2341\ \text{W/m}^3$$ $<$

COMMENT: (1) Shivering can increase the metabolic heat generation rate by up to five to six times the resting metabolic rate. The value found here is approximately three times the metabolic heat generation rate given in Example 3.12, so it is well within what can be produced by shivering. (2) In the water environment, even with the original 24ºC water temperature, shivering would be insufficient to maintain a comfortable skin temperature.