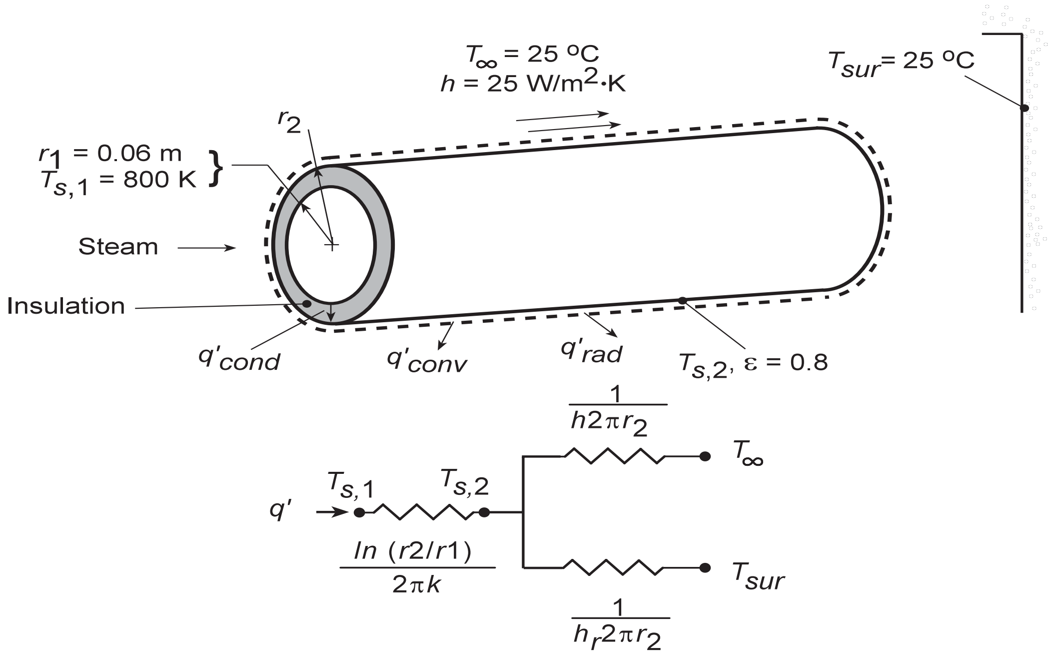
PROBLEM 3.45

KNOWN: Thickness and inner surface temperature of calcium silicate insulation on a steam pipe. Convection and radiation conditions at outer surface.

FIND: (a) Heat loss per unit pipe length for prescribed insulation thickness and outer surface temperature. (b) Heat loss and radial temperature distribution as a function of insulation thickness.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-3*, Calcium Silicate (T = 645 K): k = 0.089 W/m·K.

ANALYSIS: (a) From Eq. 3.32 with $T_{s,2}$ = 490 K, the heat rate per unit length is

$$q' = q_r/L = \frac{2\pi k\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$q' = \frac{2\pi\left(0.089\,\mathrm{W/m\cdot K}\right)\left(800 - 490\right)\mathrm{K}}{\ln\left(0.08\,\mathrm{m}/0.06\,\mathrm{m}\right)}$$

$$q' = 603\,\mathrm{W/m}\,.$$ <

(b) Performing an energy for a control surface around the outer surface of the insulation, it follows that

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,1} - T_{s,2}}{\ln\left(r_2/r_1\right)/2\pi k} = \frac{T_{s,2} - T_\infty}{1/\left(2\pi r_2 h\right)} + \frac{T_{s,2} - T_{sur}}{1/\left(2\pi r_2 h_r\right)}$$

where $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. Solving this equation for $T_{s,2}$, the heat rate may be determined from

$$q' = 2\pi r_2\left[h\left(T_{s,2} - T_\infty\right) + h_r\left(T_{s,2} - T_{sur}\right)\right]$$

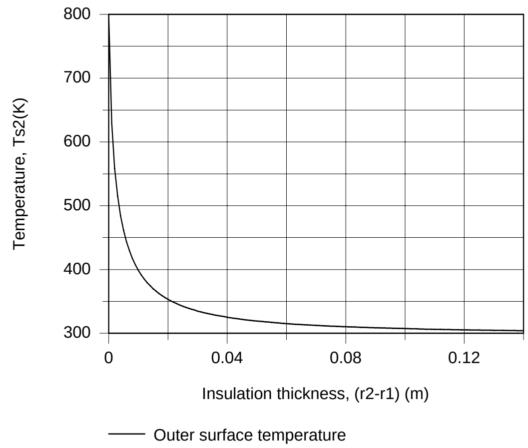
Continued...

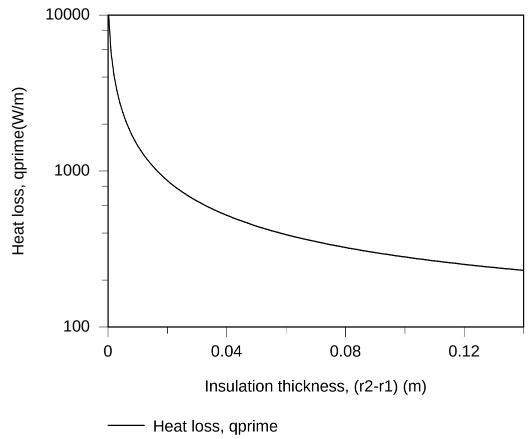
PROBLEM 3.45 (Cont.)

and from Eq. 3.31 the temperature distribution is

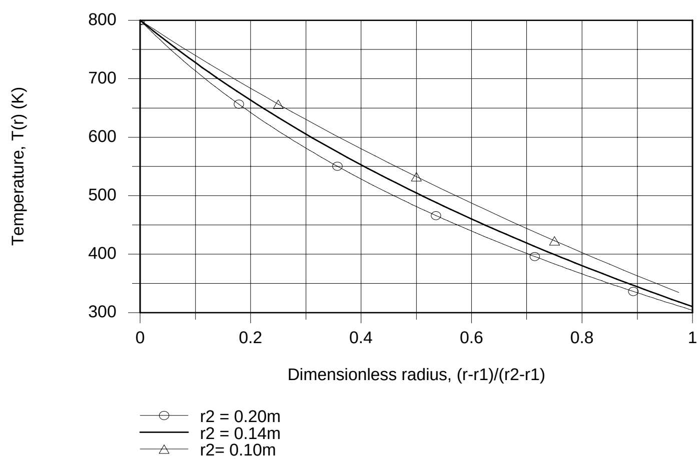
$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln\left(r_1/r_2\right)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

As shown below, the outer surface temperature of the insulation $T_{s,2}$ and the heat loss q' decay precipitously with increasing insulation thickness from values of $T_{s,2} = T_{s,1} = 800$ K and $q' = 11{,}600$ W/m, respectively, at $r_2 = r_1$ (no insulation).





When plotted as a function of a dimensionless radius, $(r - r_1)/(r_2 - r_1)$, the temperature decay becomes more pronounced with increasing r_2.



Note that $T(r_2) = T_{s,2}$ increases with decreasing r_2 and a linear temperature distribution is approached as r_2 approaches r_1.

COMMENTS: An insulation layer thickness of 20 mm is sufficient to maintain the outer surface temperature and heat rate below 350 K and 1000 W/m, respectively.