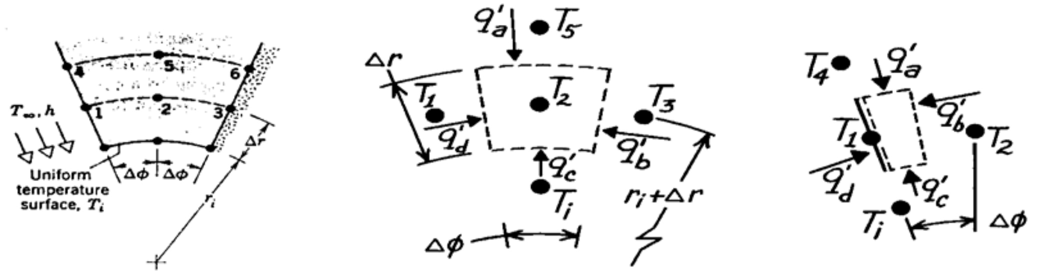
PROBLEM 4.44

KNOWN: Two-dimensional cylindrical configuration with prescribed radial (Δr) and angular ($\Delta\phi$) spacings of nodes.

FIND: Finite-difference equations for nodes 2, 3 and 1.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction in cylindrical coordinates (r,ϕ), (3) Constant properties.

ANALYSIS: The method of solution is to define the appropriate control volume for each node, to identify relevant processes and then to perform an energy balance.

(a) Node 2. This is an *interior* node with control volume as shown above. The energy balance is $\dot{E}_{in} = q'_a + q'_b + q'_c + q'_d = 0$. Using Fourier's law for each process, find

$$k\left[\left[r_i + \frac{3}{2}\Delta r\right]\Delta\phi\right]\frac{(T_5 - T_2)}{\Delta r} + k(\Delta r)\frac{(T_3 - T_2)}{(r_i + \Delta r)\Delta\phi} +$$

$$+ k\left[\left[r_i + \frac{1}{2}\Delta r\right]\Delta\phi\right]\frac{(T_i - T_2)}{\Delta r} + k(\Delta r)\frac{(T_1 - T_2)}{(r_i + \Delta r)\Delta\phi} = 0.$$

Canceling terms and regrouping yields,

$$-2\left[(r_i + \Delta r) + \frac{(\Delta r)^2}{(\Delta\phi)^2}\frac{1}{(r_i + \Delta r)}\right]T_2 + \left[r_i + \frac{3}{2}\Delta r\right]T_5 + \frac{(\Delta r)^2}{(r_i + \Delta r)(\Delta\phi)^2}(T_3 + T_1) + \left[r_i + \frac{1}{2}\Delta r\right]T_i = 0.$$

(b) Node 3. The adiabatic surface behaves as a symmetry surface. We can utilize the result of Part (a) to write the finite-difference equation by inspection as

$$-2\left[(r_i + \Delta r) + \frac{(\Delta r)^2}{(\Delta\phi)^2}\frac{1}{(r_i + \Delta r)}\right]T_3 + \left[r_i + \frac{3}{2}\Delta r\right]T_6 + \frac{2(\Delta r)^2}{(r_i + \Delta r)(\Delta\phi)^2}\cdot T_2 + \left[r_i + \frac{1}{2}\Delta r\right]T_i = 0.$$

(c) Node 1. The energy balance is $q'_a + q'_b + q'_c + q'_d = 0$. Substituting,

$$k\left[\left[r_i + \frac{3}{2}\Delta r\right]\frac{\Delta\phi}{2}\right]\frac{(T_4 - T_1)}{\Delta r} + k(\Delta r)\frac{(T_2 - T_1)}{(r_i + \Delta r)\Delta\phi} +$$

$$+k\left[\left[r_i + \frac{1}{2}\Delta r\right]\frac{\Delta\phi}{2}\right]\frac{(T_i - T_1)}{\Delta r} + h(\Delta r)(T_\infty - T_1) = 0$$ <

This expression could now be rearranged.