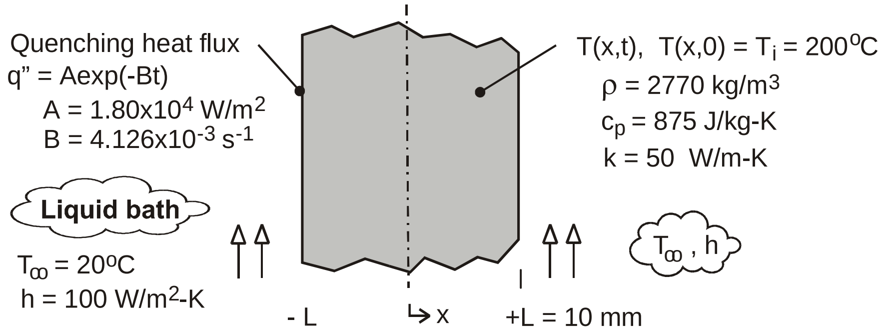
PROBLEM 2.56

KNOWN: Plate of thickness 2L, initially at a uniform temperature of T_i = 200°C, is suddenly quenched in a liquid bath of T_∞ = 20°C with a convection coefficient of 100 W/m^2·K.

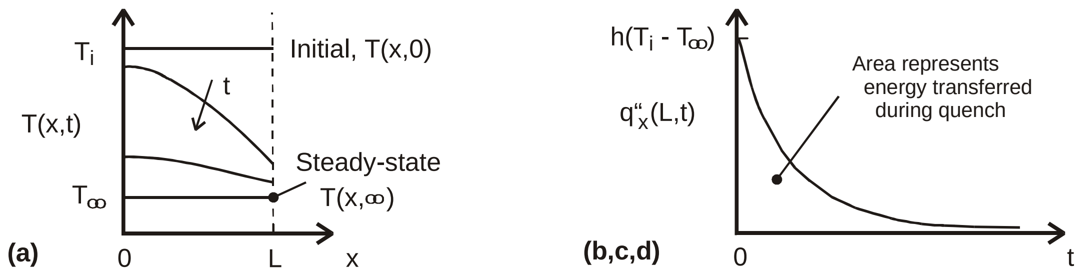
FIND: (a) On T-x coordinates, sketch the temperature distributions for the initial condition (t ≤ 0), the steady-state condition (t → ∞), and two intermediate times; (b) On $q''_x - t$ coordinates, sketch the variation with time of the heat flux at x = L, (c) Determine the heat flux at x = L and for t = 0; what is the temperature gradient for this condition; (d) By performing an energy balance on the plate, determine the amount of energy per unit surface area of the plate (J/m^2) that is transferred to the bath over the time required to reach steady-state conditions; and (e) Determine the energy transferred to the bath during the quenching process using the exponential-decay relation for the surface heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) No internal heat generation.

ANALYSIS: (a) The temperature distributions are shown in the sketch below.



(b) The heat flux at the surface x = L, $q''_x(L,t)$, is initially a maximum value, and decreases with increasing time as shown in the sketch above.

(c) The heat flux at the surface x = L at time t = 0, $q''_x(L,0)$, is equal to the convection heat flux with the surface temperature as T(L,0) = T_i.

$$q''_x(L,0) = q''_{conv}(t=0) = h(T_i - T_\infty) = 100\,W/m^2\cdot K\,(200-20)°C = 18.0\,kW/m^2 \quad <$$

From a surface energy balance as shown in the sketch considering the conduction and convection fluxes at the surface, the temperature gradient can be calculated.

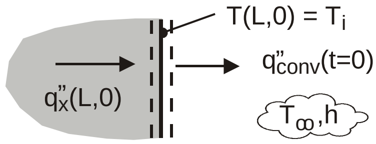
Continued …

PROBLEM 2.56 (Cont.)

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(L,0) - q''_{conv}(t=0) = 0 \qquad \text{with} \qquad q''_x(L,0) = -k\left.\frac{\partial T}{\partial x}\right)_{x=L}$$

$$\left.\frac{\partial T}{\partial x}\right)_{L,0} = -q''_{conv}(t=0)/k = -18\times10^3\,W/m^2/50\,W/m\cdot K = -360\,K/m$$ <



(d) The energy transferred from the plate to the bath over the time required to reach steady-state conditions can be determined from an energy balance on a time interval basis, Eq. 1.12b. For the initial state, the plate has a uniform temperature T_i; for the final state, the plate is at the temperature of the bath, T_∞.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0,$$

$$-E''_{out} = \rho c_p(2L)[T_\infty - T_i]$$

$$E''_{out} = -2770\,kg/m^3\times875\,J/kg\cdot K(2\times0.010\ m)[20-200]K = +8.73\times10^6\,J/m^2$$ <

(e) The energy transfer from the plate to the bath during the quenching process can be evaluated from knowledge of the surface heat flux as a function of time. The area under the curve in the $q''_x(L,t)$ vs. time plot (see schematic above) represents the energy transferred during the quench process.

$$E''_{out} = 2\int_{t=0}^{\infty} q''_x(L,t)\,dt = 2\int_{t=0}^{\infty} Ae^{-Bt}dt$$

$$E''_{out} = 2A\left[-\frac{1}{B}e^{-Bt}\right]_0^{\infty} = 2A\left[-\frac{1}{B}(0-1)\right] = 2A/B$$

$$E''_{out} = 2\times1.80\times10^4\,W/m^2/4.126\times10^{-3}s^{-1} = 8.73\times10^6\,J/m^2$$ <

COMMENTS: (1) Can you identify and explain the important features in the temperature distributions of part (a)?

(2) The maximum heat flux from the plate occurs at the instant the quench process begins and is equal to the convection heat flux. At this instant, the gradient in the plate at the surface is a maximum. If the gradient is too large, excessive thermal stresses could be induced and cracking could occur.

(3) In this thermodynamic analysis, we were able to determine the energy transferred during the quenching process. We cannot determine the rate at which cooling of the plate occurs without solving the heat diffusion equation.