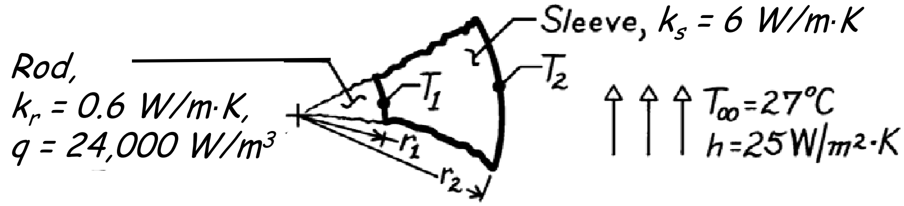
PROBLEM 3.101

KNOWN: Long rod experiencing uniform volumetric generation encapsulated by a circular sleeve exposed to convection.

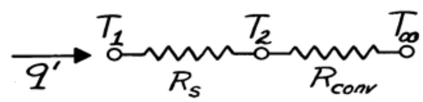
FIND: (a) Temperature at the interface between rod and sleeve and on the outer surface, (b) Temperature at center of rod.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction in rod and sleeve, (2) Steady-state conditions, (3) Uniform volumetric generation in rod, (4) Negligible contact resistance between rod and sleeve.

ANALYSIS: (a) Construct a thermal circuit for the sleeve,



where

$$q' = \dot{E}'_{gen} = \dot{q}\pi\, D_1^2/4 = 24{,}000\ \mathrm{W/m^3} \times \pi \times (0.24\ \mathrm{m})^2/4 = 1086\ \mathrm{W/m}$$

$$R'_s = \frac{\ln(r_2/r_1)}{2\pi\, k_s} = \frac{\ln(440/240)}{2\pi \times 6\ \mathrm{W/m\cdot K}} = 1.608\times10^{-2}\,\mathrm{m\cdot\ K/W}$$

$$R_{conv} = \frac{1}{h\pi\, D_2} = \frac{1}{25\ \mathrm{W/m^2\cdot K}\times\pi\times0.440\ \mathrm{m}} = 2.894\times10^{-2}\,\mathrm{m\cdot\ K/W}$$

The rate equation can be written as

$$q' = \frac{T_1 - T_\infty}{R'_s + R'_{conv}} = \frac{T_2 - T_\infty}{R'_{conv}}$$

$$T_1 = T_\infty + q'\left(R'_s + R'_{conv}\right) = 27^\circ\mathrm{C} + 1086\ \mathrm{W/m}\left(1.608\times10^{-2} + 2.894\times10^{-2}\right)\mathrm{K/W\cdot m} = 75.8^\circ\mathrm{C} \qquad <$$

$$T_2 = T_\infty + q'R'_{conv} = 27^\circ\mathrm{C} + 1086\ \mathrm{W/m}\times2.894\times10^{-2}\mathrm{m\cdot\ K/W} = 58.4^\circ\mathrm{C}. \qquad <$$

(b) The temperature at the center of the rod is

$$T(0) = T_o = \frac{\dot{q}r_1^2}{4k_r} + T_1 = \frac{24{,}000\ \mathrm{W/m^3}(0.120\ \mathrm{m})^2}{4\times0.6\ \mathrm{W/m\cdot K}} + 75.9^\circ\mathrm{C} = 220^\circ\mathrm{C}. \qquad <$$

COMMENTS: The thermal resistances due to conduction in the sleeve and convection are comparable. Will increasing the sleeve outer diameter cause the surface temperature T_2 to increase or decrease?