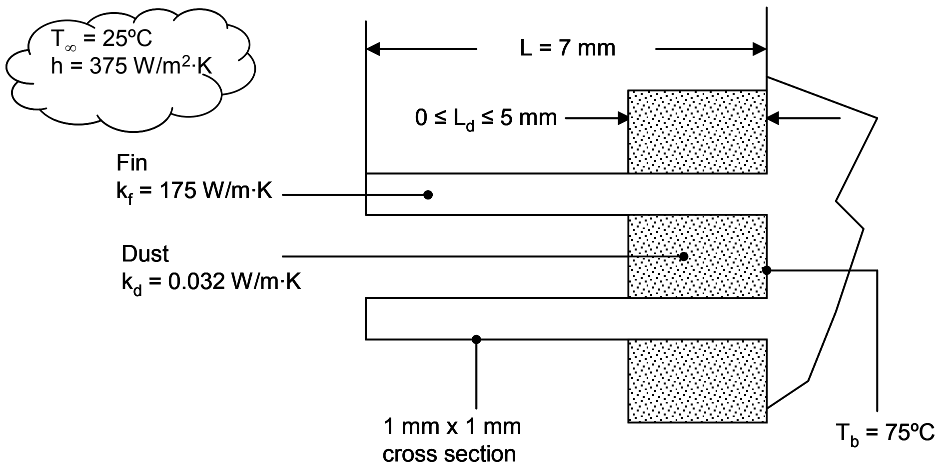
PROBLEM 3.152

KNOWN: Dimensions of a fin array and dust layer. Aluminum and dust thermal conductivities. Base temperature. Air temperature and heat transfer coefficient.

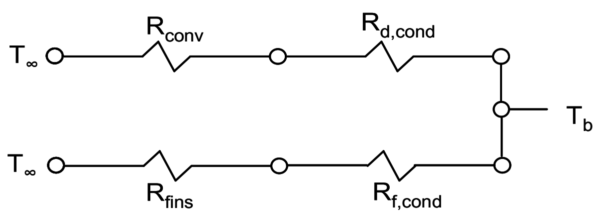
FIND: Allowable heat rate for dust layer thickness in the range of $0 \le L_d \le 5$ mm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible temperature variation across fin thickness, (3) Constant properties, (4) Uniform heat transfer coefficient, including over fin tips.

ANALYSIS: There are two heat transfer paths, one through the dust and into the air, and the other through the fin. The thermal circuit is



The thermal resistances are given by

$$R_{d,cond} = \frac{L_d}{k_d A_d} = \frac{L_d}{k_d (A_p - NA_c)}$$

where $A_p = 53 \times 10^{-3}\ \text{m} \times 57 \times 10^{-3}\ \text{m} = 3.02 \times 10^{-3}\ \text{m}^2$, $N = 14 \times 17 = 238$ and $A_c = w^2 = 10^{-6}\ \text{m}^2$.

$$R_{conv} = \frac{1}{hA_d}, \quad R_{f,cond} = \frac{L_d}{k_f N A_c} \quad \text{and} \quad R_{fins} = \frac{R_{t,f}}{N}$$

where from Equation 3.88

$$R_{t,f} = \frac{\theta_b}{q_f}$$

where q_f is given by Equation 3.77,

$$R_{t,f} = \frac{\cosh(mL_f) + (h/mk_f)\sinh(mL_f)}{\sqrt{4hw^3k}\ (\sinh(mL_f) + (h/mk_f)\cosh(mL_f))}$$

Continued…

Here, $m = (4h / k_f w)^{1/2} = (4 \times 375\ W/m^2{\cdot}K / 175\ W/m{\cdot}K \times 10^{-3}\ m)^{1/2} = 92.6\ m^{-1}$

and $L_f = L - L_d$.

Finally,

$$q = q_{dust} + q_{fin}$$

$$= \frac{T_b - T_\infty}{R_{d,cond} + R_{conv}} + \frac{T_b - T_\infty}{R_{f,cond} + R_{fins}}$$

Performing the calculation for a dust layer thickness of $L_d = 5$ mm yields

$R_{d,cond} = 56.1$ K/W

$R_{conv} = 0.958$ K/W

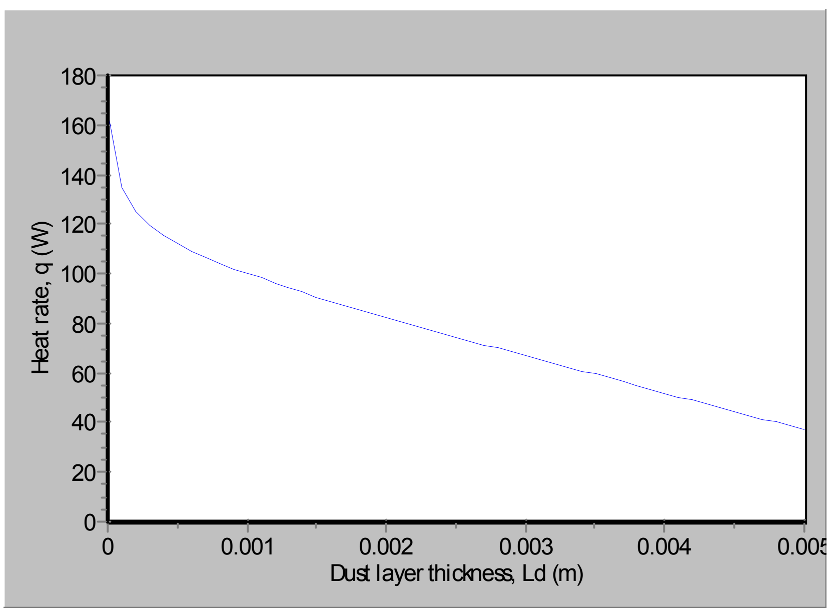
$R_{f,cond} = 0.120$ K/W

$R_{t,f} = 301$ K/W, $R_{fins} = 1.26$ K/W

$$q = \frac{75°C - 25°C}{(56.1 + 0.958)\ K/W} + \frac{75°C - 25°C}{0.120 + 1.26} = 0.876\ W + 36.1\ W = 37.0\ W$$

<

The figure shows the variation of the allowable heat rate as the dust layer thickness varies.



COMMENTS: The figure below shows the two contributions to the heat rate, q_{dust} and q_{fin}. The heat transfer through the dust layer decreases rapidly as the dust layer thickness increases and insulates the surface. The fin heat transfer also decreases with increasing dust layer thickness as more of the fin surface is insulated by the dust.

Continued…

PROBLEM 3.152 (Cont.)