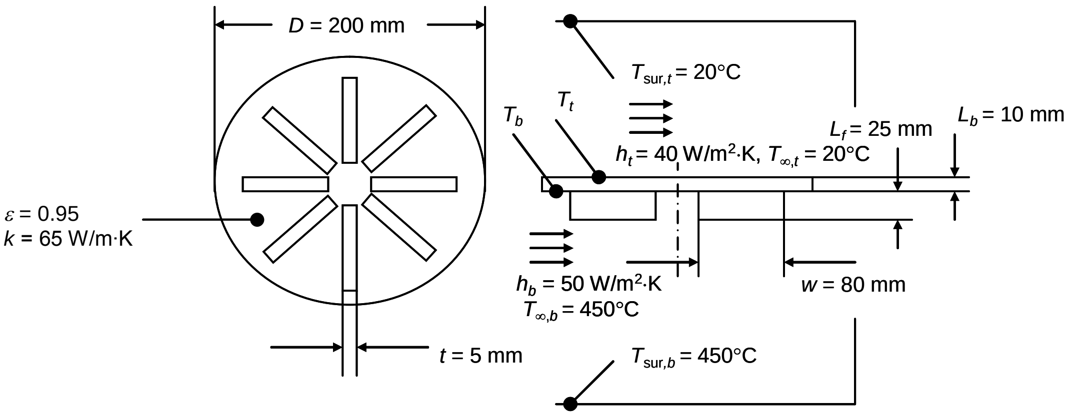
PROBLEM 3.147

KNOWN: Geometry of a cast iron burner with and without fins. Room temperature, combustion temperature, heat transfer coefficient at the top burner surface, heat transfer coefficient at the bottom burner surface, emissivity of burner coating, thermal conductivity of cast iron.

FIND: Temperature of the top burner surface with and without fins.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Constant properties, (3) Convection from fin tip, (4) Large surroundings at top and bottom of burner.

PROPERTIES: *Given*, Cast iron: $k = 65$ W/m·K.

ANALYSIS: Evaluating the radiation heat transfer from the combustion products to the bottom of the burner as $q_{\text{rad},b} = \varepsilon\sigma(\pi D^2/4)(T_{\text{sur},b}^4 - T_b^4)$, the total heat transfer to the bottom of the burner's base is

$$q_b = N\eta_f h_b A_f (T_{\infty,b} - T_b) + h_b A_b (T_{\infty,b} - T_b) + A_t \varepsilon\sigma\left(T_{\text{sur},b}^4 - T_b^4\right) \qquad (1)$$

where $A_b = \pi D^2/4 - Ntw = \pi(0.200\text{m})^2/4 - 8\times 0.005\text{ m} \times 0.080\text{ m} = 0.0282\text{ m}^2$ is the base area without fins and $A_t = \pi D^2/4 = \pi(0.2\text{m})^2/4 = 0.0314\text{ m}^2$. The total heat transfer from the top surface is

$$q_t = h_t A_t (T_t - T_{\infty,t}) + A_t \varepsilon\sigma\left(T_t^4 - T_{\text{sur},t}^4\right) \qquad (2)$$

One-dimensional conduction through the base of the burner is

$$q_{\text{base}} = \left(kA_t / L_b\right)(T_b - T_t) \qquad (3)$$

At steady state, the heat rates must be equal,

$$q_b = q_t; \qquad q_{\text{base}} = q_t \qquad (4, 5)$$

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The fin efficiency may be evaluated using Table 3.5. The corrected fin length is $L_c = L_f + t/2$ = 25 mm + 5 mm/2 = 27.5 mm = 27.5 × 10^{-3} m. The fin area is $A_f = 2wL_c$ = 2 × 0.080 m × 27.5 × 10^{-3} m = 0.0044 m^2. The value of m is $m = \sqrt{2h_b/kt} = \sqrt{2 \times 50\,\text{W/m}^2 \cdot \text{K}/65\,\text{W/m} \cdot \text{K} \times 5 \times 10^{-3}\,\text{m}} = 17.54\,\text{m}^{-1}$ Finally, the fin efficiency is

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \frac{\tanh(17.54\,\text{m}^{-1} \times 27.5 \times 10^{-3}\,\text{m})}{17.54\,\text{m}^{-1} \times 27.5 \times 10^{-3}\,\text{m}} = 0.93$$

Substituting values listed in the schematic, along with values of the various areas, the fin efficiency, and N = 8 into Eqs. (1) through (5) and solving simultaneously yields

T_t = 601.7 K = 328.7°C <

For a burner without fins, Eq. (1) is replaced by

$$q_b = h_b A_t (T_{\infty,b} - T_b) + A_t \varepsilon \sigma \left(T_{\text{sur},b}^4 - T_b^4\right) \tag{6}$$

Substituting values and solving Eqs. (2) through (6) simultaneously yields

T_t = 570.5 K = 297.5°C <

COMMENTS: (1) Adding fins to the bottom of the burner increases the steady-state top temperature by approximately 30 degrees Celsius. (2) The finned burner heat rate is q = 597.2 W, while without fins the heat rate is q = 515.5 W. Hence, the fins increase the heat rate available for cooking. (3) Radiation heat transfer is significant. With fins, radiation accounts for 35% of the heat rate at the top surface and 40% at the bottom surface. Without fins, radiation accounts for 32% at the top surface and 54% at the bottom surface. (4) In general, the treatment of radiation for finned surfaces, such as at the bottom surface of the finned burner is justified, as will be discussed in Chapter 13.