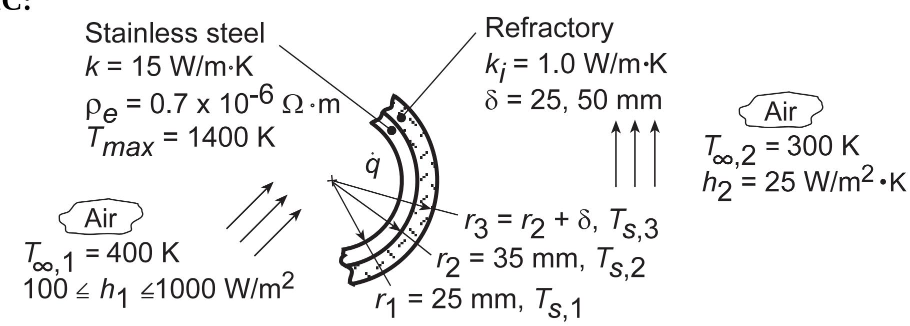
PROBLEM 3.99

KNOWN: Dimensions and properties of tubular heater and external insulation. Internal and external convection conditions. Maximum allowable tube temperature.

FIND: (a) Maximum allowable heater current for adiabatic outer surface, (3) Effect of internal convection coefficient on heater temperature distribution, (c) Extent of heat loss at outer surface.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform heat generation, (4) Negligible radiation at outer surface, (5) Negligible contact resistance.

ANALYSIS: (a) From Eqs. 7 and 10, respectively, of Example 3.8, we know that

$$T_{s,2} - T_{s,1} = \frac{\dot{q}}{2k} r_2^2 \ln\frac{r_2}{r_1} - \frac{\dot{q}}{4k}\left(r_2^2 - r_1^2\right) \tag{1}$$

and

$$T_{s,1} = T_{\infty,1} + \frac{\dot{q}\left(r_2^2 - r_1^2\right)}{2h_1 r_1} \tag{2}$$

Hence, eliminating $T_{s,1}$, we obtain

$$T_{s,2} - T_{\infty,1} = \frac{\dot{q}r_2^2}{2k}\left[\ln\frac{r_2}{r_1} - \frac{1}{2}\left(1 - r_1^2/r_2^2\right) + \frac{k}{h_1 r_1}\left(1 - r_1^2/r_2^2\right)\right]$$

Substituting the prescribed conditions ($h_1 = 100\ \text{W/m}^2\cdot\text{K}$),

$$T_{s,2} - T_{\infty,1} = 1.237\times10^{-4}\left(\text{m}^3\cdot\text{K/W}\right)\dot{q}\left(\text{W/m}^3\right)$$

Hence, with T_{max} corresponding to $T_{s,2}$, the maximum allowable value of $\dot{q}$ is

$$\dot{q}_{max} = \frac{1400-400}{1.237\times10^{-4}} = 8.084\times10^6\ \text{W/m}^3$$

with

$$\dot{q} = \frac{I^2 R_e}{\forall} = \frac{I^2\rho_e L/A_c}{LA_c} = \frac{\rho_e I^2}{\left[\pi\left(r_2^2 - r_1^2\right)\right]^2}$$

$$I_{max} = \pi\left(r_2^2 - r_1^2\right)\left(\frac{\dot{q}}{\rho_e}\right)^{1/2} = \pi\left(0.035^2 - 0.025^2\right)\text{m}^2\left(\frac{8.084\times10^6\ \text{W/m}^3}{0.7\times10^{-6}\ \Omega\cdot\text{m}}\right)^{1/2} = 6406\ \text{A} \quad <$$

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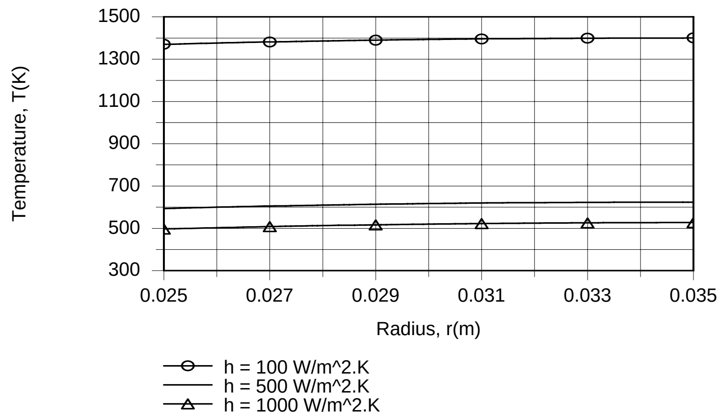
(b) Using the one-dimensional, steady-state conduction model of *IHT* (hollow cylinder; convection at inner surface and adiabatic outer surface), the following temperature distributions were obtained.



The results are consistent with key implications of Eqs. (1) and (2), namely that the value of h_1 has no effect on the temperature drop across the tube ($T_{s,2}$ - $T_{s,1}$ = 30 K, irrespective of h_1), while $T_{s,1}$ decreases with increasing h_1. For h_1 = 100, 500 and 1000 $W/m^2\cdot K$, respectively, the ratio of the temperature drop between the inner surface and the air to the temperature drop across the tube, $(T_{s,1} - T_{\infty,1})/(T_{s,2} - T_{s,1})$, decreases from 970/30 = 32.3 to 194/30 = 6.5 and 97/30 = 3.2. Because the outer surface is insulated, the heat rate to the airflow is fixed by the value of $\dot{q}$ and, irrespective of h_1,

$$q'(r_1) = \pi\left(r_2^2 - r_1^2\right)\dot{q} = -15,240\,W \qquad <$$

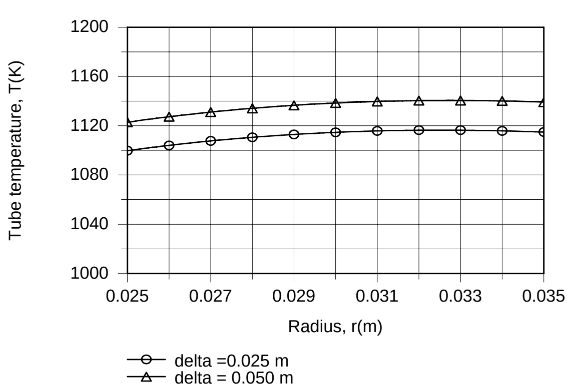
(c) Heat loss from the outer surface of the tube to the surroundings depends on the total thermal resistance

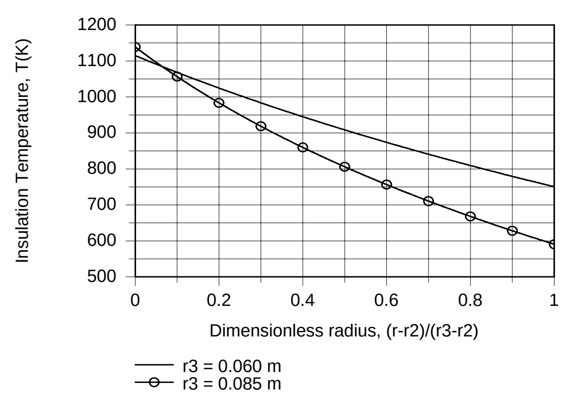
$$R_{tot} = \frac{\ln(r_3/r_2)}{2\pi L k_i} + \frac{1}{2\pi r_3 L h_2}$$

or, for a unit area on surface 2,

$$R''_{tot,2} = (2\pi r_2 L) R_{tot} = \frac{r_2 \ln(r_3/r_2)}{k_i} + \frac{r_2}{r_3 h_2}$$

Again using the capabilities of *IHT* (hollow cylinder; convection at inner surface and heat transfer from outer surface through $R''_{tot,2}$), the following temperature distributions were determined for the tube and insulation.





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PROBLEM 3.99 (Cont.)

Heat losses through the insulation, $q'(r_2)$, are 4250 and 3890 W/m for δ = 25 and 50 mm, respectively, with corresponding values of $q'(r_1)$ equal to -10,990 and -11,350 W/m. Comparing the tube temperature distributions with those predicted for an adiabatic outer surface, it is evident that the losses reduce tube wall temperatures predicted for the adiabatic surface and also shift the maximum temperature from r = 0.035 m to $r \approx 0.033$ m. Although the tube outer and insulation inner surface temperatures, $T_{s,2} = T(r_2)$, increase with increasing insulation thickness, Fig. (c), the insulation outer surface temperature decreases.

COMMENTS: If the intent is to maximize heat transfer to the airflow, heat losses to the ambient should be reduced by selecting an insulation material with a significantly smaller thermal conductivity.