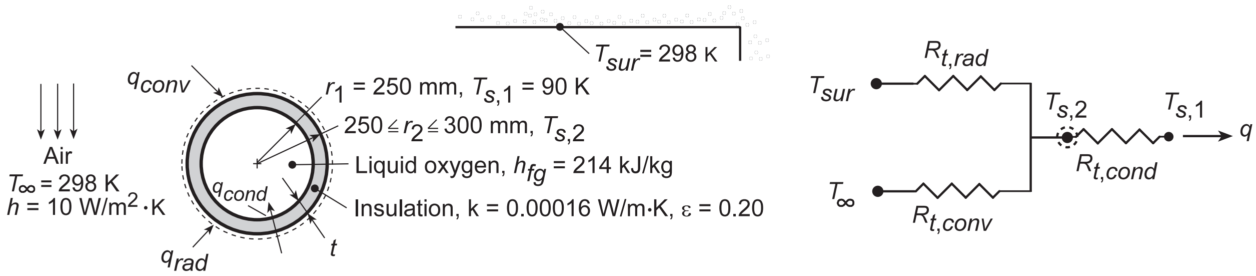
PROBLEM 3.64

KNOWN: Diameter of a spherical container used to store liquid oxygen and properties of insulating material. Environmental conditions.

FIND: (a) Reduction in evaporative oxygen loss associated with a prescribed insulation thickness, (b) Effect of insulation thickness on evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Negligible conduction resistance of container wall and contact resistance between wall and insulation, (3) Container wall at boiling point of liquid oxygen.

ANALYSIS: (a) Applying an energy balance to a control surface about the insulation, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that $q_{conv} + q_{rad} = q_{cond} = q$. Hence,

$$\frac{T_\infty - T_{s,2}}{R_{t,conv}} + \frac{T_{sur} - T_{s,2}}{R_{t,rad}} = \frac{T_{s,2} - T_{s,1}}{R_{t,cond}} = q \qquad (1)$$

where $R_{t,conv} = \left(4\pi r_2^2 h\right)^{-1}$, $R_{t,rad} = \left(4\pi r_2^2 h_r\right)^{-1}$, $R_{t,cond} = (1/4\pi k)\left[(1/r_1) - (1/r_2)\right]$, and, from Eq. 1.9, the radiation coefficient is $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. With t = 10 mm ($r_2$ = 260 mm), ε = 0.2 and $T_\infty = T_{sur}$ = 298 K, an iterative solution of the energy balance equation yields $T_{s,2} \approx 297.7$ K, where $R_{t,conv}$ = 0.118 K/W, $R_{t,rad}$ = 0.982 K/W and $R_{t,cond}$ = 76.5 K/W. With the insulation, it follows that the heat gain is

$$q_w \approx 2.72 \text{ W}$$

Without the insulation, the heat gain is

$$q_{wo} = \frac{T_\infty - T_{s,1}}{R_{t,conv}} + \frac{T_{sur} - T_{s,1}}{R_{t,rad}}$$

where, with $r_2 = r_1$, $T_{s,1}$ = 90 K, $R_{t,conv}$ = 0.127 K/W and $R_{t,rad}$ = 3.14 K/W. Hence,

$$q_{wo} = 1702 \text{ W}$$

With the oxygen mass evaporation rate given by $\dot{m} = q/h_{fg}$, the percent reduction in evaporated oxygen is

$$\% \text{ Reduction} = \frac{\dot{m}_{wo} - \dot{m}_w}{\dot{m}_{wo}} \times 100\% = \frac{q_{wo} - q_w}{q_{wo}} \times 100\%$$

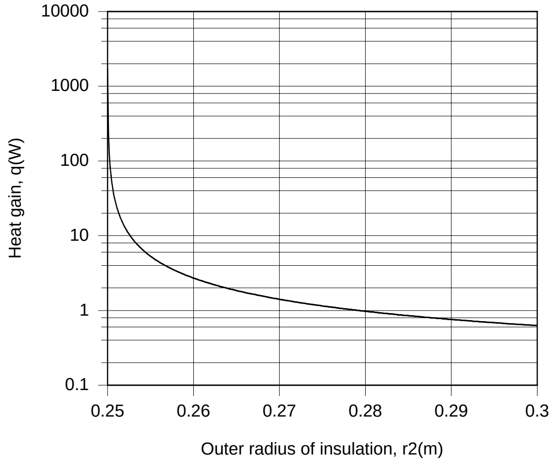
Hence,

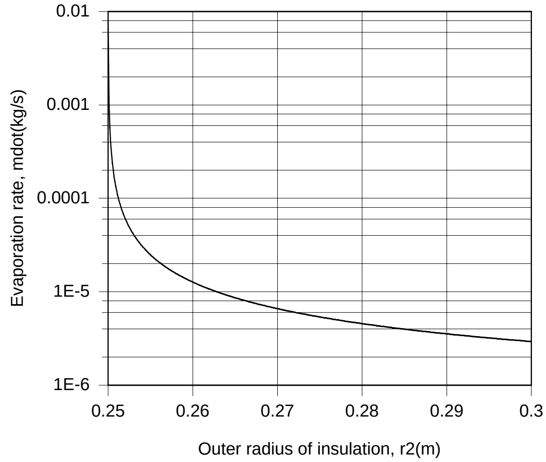
$$\% \text{ Reduction} = \frac{(1702 - 2.7)\text{ W}}{1702 \text{ W}} \times 100\% = 99.8\% \qquad <$$

Continued...

PROBLEM 3.64 (Cont.)

(b) Using Equation (1) to compute $T_{s,2}$ and q as a function of r_2, the corresponding evaporation rate, $\dot{m} = q/h_{fg}$, may be determined. Variations of q and $\dot{m}$ with r_2 are plotted as follows.





Because of its extremely low thermal conductivity, significant benefits are associated with using even a thin layer of insulation. Nearly three-order magnitude reductions in q and $\dot{m}$ are achieved with $r_2 = 0.26$ m. With increasing r_2, q and $\dot{m}$ decrease from values of 1702 W and 8×10^{-3} kg/s at $r_2 = 0.25$ m to 0.627 W and 2.9×10^{-6} kg/s at $r_2 = 0.30$ m.

COMMENTS: Laminated metallic-foil/glass-mat insulations are extremely effective and corresponding conduction resistances are typically much larger than those normally associated with surface convection and radiation.