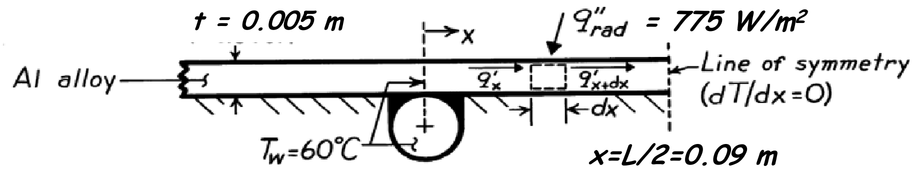
PROBLEM 3.108

KNOWN: Net radiative flux to absorber plate.

FIND: (a) Maximum absorber plate temperature, (b) Rate of energy collected per tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (x) conduction along absorber plate, (3) Uniform radiation absorption at plate surface, (4) Negligible losses by conduction through insulation, (5) Negligible losses by convection at absorber plate surface, (6) Temperature of absorber plate at x = 0 is approximately that of the water.

PROPERTIES: *Table A-1*, Aluminum alloy (2024-T6): k ≈ 180 W/m·K.

ANALYSIS: The absorber plate acts as an extended surface (a conduction-radiation system), and a differential equation which governs its temperature distribution may be obtained by applying Eq.1.11b to a differential control volume. For a unit length of tube

$$q'_x + q''_{rad}(dx) - q'_{x+dx} = 0.$$

With $$q'_{x+dx} = q'_x + \frac{dq'_x}{dx}dx$$

and $$q'_x = -kt\frac{dT}{dx}$$

it follows that,

$$q''_{rad} - \frac{d}{dx}\left[-kt\frac{dT}{dx}\right] = 0$$

$$\frac{d^2T}{dx^2} + \frac{q''_{rad}}{kt} = 0$$

Integrating twice it follows that, the general solution for the temperature distribution has the form,

$$T(x) = -\frac{q''_{rad}}{2kt}x^2 + C_1x + C_2.$$

Continued …

The boundary conditions are:

$$T(0) = T_w \qquad C_2 = T_w$$

$$\left.\frac{dT}{dx}\right]_{x=L/2} = 0 \qquad C_1 = \frac{q''_{rad}L}{2kt}$$

Hence,

$$T(x) = \frac{q''_{rad}}{2kt}\, x(L-x) + T_w.$$

The maximum absorber plate temperature, which is at x = L/2, is therefore

$$T_{max} = T(L/2) = \frac{q''_{rad}L^2}{8kt} + T_w.$$

The rate of energy collection per tube may be obtained by applying Fourier's law at x = 0. That is, energy is transferred to the tubes via conduction through the absorber plate. Hence,

$$q' = 2\left[-k\,t\left.\frac{dT}{dx}\right]_{x=0}\right]$$

where the factor of two arises due to heat transfer from both sides of the tube. Hence,

$$q' = -Lq''_{rad}.$$

Hence $$T_{max} = \frac{775\frac{W}{m^2}(0.18m)^2}{8\left[180\frac{W}{m\cdot K}\right](0.005m)} + 60^\circ C$$

or $$T_{max} = 63.6^\circ C$$ <

and $$q' = -0.18m \times 775\ W/m^2$$

or $$q' = -140\ W/m.$$ <

COMMENTS: Convection losses in the typical flat plate collector, which is not evacuated, would reduce the value of q'.