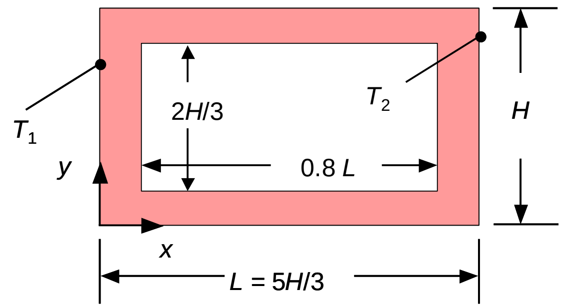
PROBLEM 4.63

KNOWN: Dimensions of a two-dimensional object with isothermal and adiabatic boundaries.

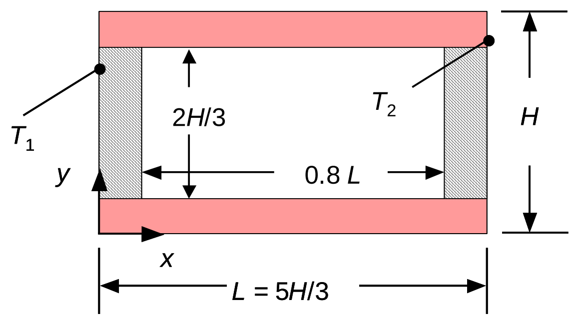
FIND: (a) Estimate of the shape factor using a one-dimensional analysis and (b) estimate of the shape factor using a finite difference method with $\Delta x = \Delta y = 0.05\,L$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) No internal generation, (4) One-dimensional conduction in part (a), (5) Two-dimensional conduction in part (b).

ANALYSIS: (a) As a first approximation, we ignore conduction in the cross-hatched regions.

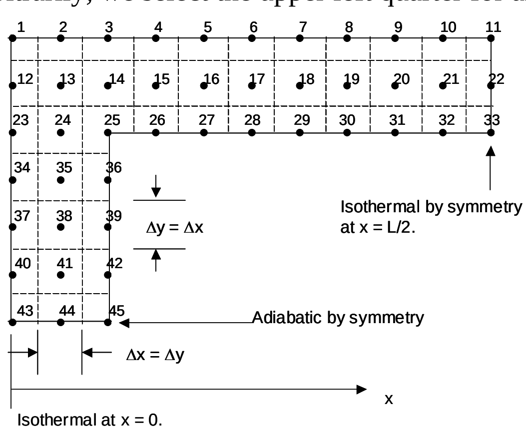


Hence,

$$q' = Sk(T_1 - T_2) \approx (H/3)k(T_1 - T_2)/L = k(T_1 - T_2)/5$$

Therefore, $S \approx 0.20$. <

(b) We begin by taking advantage of the symmetry of the problem. Recognize that the line $y = H/2$ is an adiabat, and the line $x = L/2$ is an isotherm at $T = (T_1 + T_2)/2$. Hence, only one quarter of the domain needs to be modeled. Arbitrarily, we select the upper left quarter for analysis. Note that $\Delta x = \Delta y$.



Continued…

Problem 4.63 (Cont.)

We may use the finite difference equations from the text, or note that for each node, an energy balance can be written for the corresponding control volume. Consider, for example, the control volume about Node 2 for which we may write

$$k \times \frac{\Delta y}{2} \frac{(T_1 - T_2)}{\Delta x} + k \times \frac{\Delta x (T_{13} - T_2)}{\Delta y} + k \times \frac{\Delta y}{2} \frac{(T_3 - T_2)}{\Delta x} = 0$$

or

$$\frac{(T_1 - T_2)}{2} + (T_{13} - T_2) + \frac{(T_3 - T_2)}{2} = 0$$

We let the temperature at $x = 0$ be $T_1 = 1$ and the temperature at $x = L/2$ be $T_2 = 0.5$. The 45 equations are solved simultaneously with the *IHT* code provided in the Comment. The resulting nodal temperatures in the upper left corner are:

1.0000	0.9636	0.9226	0.8737	0.8215	0.7683	0.7147	0.6610	0.6074	0.5537	0.5000
1.0000	0.9659	0.9265	0.8753	0.8220	0.7684	0.7147	0.6611	0.6074	0.5537	0.5000
1.0000	0.9734	0.9423	0.8790	0.8229	0.7686	0.7148	0.6611	0.6074	0.5537	0.5000
1.0000	0.9853	0.9753								
1.0000	0.9923	0.9884								
1.0000	0.9957	0.9938								
1.0000	0.9966	0.9952								

After solving for the temperatures, the heat transfer rate per unit depth may be evaluated at any x location, and for $x = L/2$ it may be expressed as

$$q' = k\left[\frac{\Delta y/2}{\Delta x}(T_{10} - T_{11}) + \frac{\Delta y}{\Delta x}(T_{21} - T_{22}) + \frac{\Delta y/2}{\Delta x}(T_{32} - T_{33})\right] \qquad (1)$$

or, in terms of the shape factor,

$$q' = Sk(T_h - T_c) \qquad (2)$$

Equating Eqs. (1) and (2) yields the shape factor, $S \approx 0.215$. $<$

We note that the shape factor calculated with the finite difference approach is only slightly greater than the shape factor based upon the one-dimensional approximation. The good agreement is expected since the major resistance to heat transfer is posed by the top and bottom slender regions that connect the two isothermal boundaries. We also note that temperatures are nearly isothermal in the region $y \approx H/2$ due to the adiabatic interior.

Continued…

Problem 4.63 (Cont.)

COMMENTS: The *IHT* code is listed below. For each control volume, we note that $\dot{E}_{in} = 0$ and $\Delta y = \Delta x$, yielding the following energy balances for all but the isothermal nodes.

```
Th = 1
Tc =0.5

//Top Row

//Node 1
T1 = Th
//Node 2
(T1 - T2)/2 + (T3 - T2)/2 + (T13 - T2) = 0
//Node 3
(T2 - T3)/2 + (T4 - T3)/2 + (T14 - T3) = 0
//Node 4
(T3 - T4)/2 + (T5 - T4)/2 + (T15 - T4) = 0
//Node 5
(T4 - T5)/2 + (T6 - T5)/2 + (T16 - T5) = 0
//Node 6
(T5 - T6)/2 + (T7 - T6)/2 + (T17 - T6) = 0
//Node 7
(T6 - T7)/2 + (T8 - T7)/2 + (T18 - T7) = 0
//Node 8
(T7 - T8)/2 + (T9 - T8)/2 + (T19 - T8) = 0
//Node 9
(T8 - T9)/2 + (T10 - T9)/2 + (T20 - T9) = 0
//Node 10
(T9 - T10)/2 + (T11 - T10)/2 + (T21 - T10) = 0
//Node 11
T11 = Tc

//Second Row from Top

//Node 12
T12 = Th
//Node 13
(T12 - T13) + (T2 - T13) + (T14 - T13) + (T24 - T13) = 0
//Node 14
(T13 - T14) + (T3 - T14) + (T15 - T14) + (T25 - T14) = 0
//Node 15
(T14 - T15) + (T4 - T15) + (T16 - T15) + (T26 - T15) = 0
//Node 16
(T15 - T16) + (T5 - T16) + (T17 - T16) + (T27 - T16) = 0
//Node 17
(T16 - T17) + (T6 - T17) + (T18 - T17) + (T28 - T17) = 0
//Node 18
(T17 - T18) + (T7 - T18) + (T19 - T18) + (T29 - T18) = 0
//Node 19
(T18 - T19) + (T8 - T19) + (T20 - T19) + (T30 - T19) = 0
//Node 20
(T19 - T20) + (T9 - T20) + (T21 - T20) + (T31 - T20) = 0
//Node 21
(T20 - T21) + (T10 - T21) + (T22 - T21) + (T32 - T21) = 0
//Node 22
T22 = Tc

//Third Row from Top

//Node 23
T23 = Th
//Node 24
(T23 - T24) + (T13 - T24) + (T25 - T24) + (T35 - T24) = 0
//Node 25
(T24 - T25) + (T14 - T25) + (T26 - T25)/2 + (T36 - T25)/2 = 0
//Node 26
```

Continued…

Problem 4.63 (Cont.)

```
(T25 - T26)/2 + (T15 - T26) + (T27 - T26)/2 = 0
//Node 27
(T26 - T27)/2 + (T16 - T27) + (T28 - T27)/2 = 0
//Node 28
(T27 - T28)/2 + (T17 - T28) + (T29 - T28)/2 = 0
//Node 29
(T28 - T29)/2 + (T18 - T29) + (T30 - T29)/2 = 0
//Node 30
(T29 - T30)/2 + (T19 - T30) + (T31 - T30)/2 = 0
//Node 31
(T30 - T31)/2 + (T20 - T31) + (T32 - T31)/2 = 0
//Node 32
(T31 - T32)/2 + (T21 - T32) + (T33 - T32)/2 = 0
//Node 33
T33 = Tc

//Fourth Row from Top

//Node 34
T34 = Th
//Node 35
(T34 - T35) + (T24 - T35) + (T36 - T35) + (T38 - T35) = 0
//Node 36
(T35 - T36) + (T25 - T36)/2 + (T39 - T36)/2 = 0

//Fifth Row from Top

//Node 37
T37 = Th
//Node 38
(T37 - T38) + (T35 - T38) + (T39 - T38) + (T41 - T38) = 0
//Node 39
(T38 - T39) + (T36 - T39)/2 + (T42 - T39)/2 = 0

//Sixth Row from Top
//Node 40
T40 = Th
//Node 41
(T40 - T41) + (T38 - T41) + (T42 - T41) + (T44 - T41) = 0
//Node 42
(T41 - T42) + (T39 - T42)/2 + (T45 - T42)/2 = 0

//Bottom Row
//Node 43
T43 = Th
//Node 44
(T43 - T44)/2 + (T41 - T44) + (T45 - T44)/2 = 0
//Node 45
(T44 - T45)/2 + (T42 - T45)/2 = 0

//Shape Factor

H = 1
L = H*5/3
k = 1
deltay = 0.05*L
deltax = deltay

//Heat Rate at RHS of Computational Domain (W/m)

qprime = ((T10 - T11)*k*deltay/2)/deltax + ((T21 - T22)*k*deltay)/deltax + ((T32 - T33)*k*deltay/2)/deltax
qprime = S*k*(Th - Tc)
```