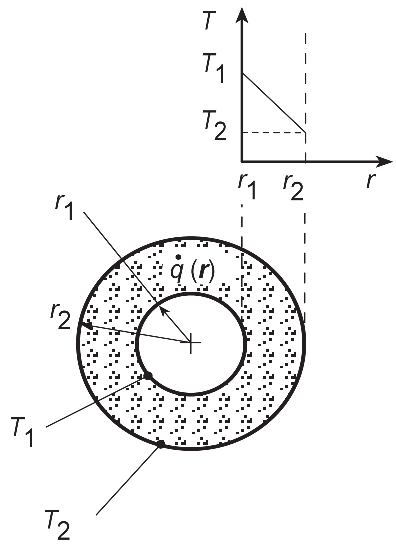
PROBLEM 2.47

KNOWN: Inner and outer radii and surface temperatures of a long circular tube with internal energy generation.

FIND: Conditions for which a linear radial temperature distribution may be maintained.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties.

ANALYSIS: For the assumed conditions, Eq. 2.26 reduces to

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\dot{q}=0$$

If $\dot{q}$ = 0 or $\dot{q}$ = constant, it is clearly impossible to have a linear radial temperature distribution. However, we may use the heat equation to infer a special form of $\dot{q}$ (r) for which dT/dr is a constant (call it C_1). It follows that

$$\frac{k}{r}\frac{d}{dr}\left(r\,C_1\right)+\dot{q}=0$$

$$\dot{q}=-\frac{C_1 k}{r} \qquad <$$

where $C_1 = (T_2 - T_1)/(r_2 - r_1)$. Hence, if the generation rate varies inversely with radial location, the radial temperature distribution is linear.

COMMENTS: Conditions for which $\dot{q} \propto (1/r)$ would be unusual.