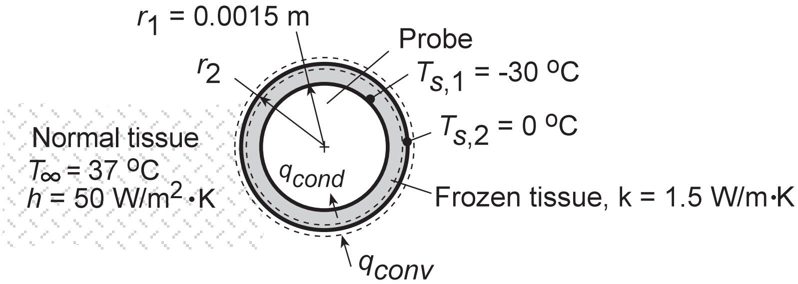
PROBLEM 3.69

KNOWN: Diameter and surface temperature of a spherical cryoprobe. Temperature of surrounding tissue and effective convection coefficient at interface between frozen and normal tissue.

FIND: Thickness of frozen tissue layer.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible contact resistance between probe and frozen tissue, (3) Constant properties, (4) Negligible perfusion effects.

ANALYSIS: Performing an energy balance for a control surface about the phase front, it follows that

$$q_{conv} - q_{cond} = 0$$

Hence,

$$h\left(4\pi r_2^2\right)\left(T_\infty - T_{s,2}\right) = \frac{T_{s,2} - T_{s,1}}{\left[\left(1/r_1\right) - \left(1/r_2\right)\right]/4\pi k}$$

$$r_2^2\left[\left(1/r_1\right) - \left(1/r_2\right)\right] = \frac{k}{h}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)}$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = \frac{k}{hr_1}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)} = \frac{1.5\,\mathrm{W/m\cdot K}}{\left(50\,\mathrm{W/m^2\cdot K}\right)\left(0.0015\,\mathrm{m}\right)}\left(\frac{30}{37}\right)$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = 16.2$$

$$\left(r_2/r_1\right) = 4.56$$

It follows that r_2 = 6.84 mm and the thickness of the frozen tissue is

$$\delta = r_2 - r_1 = 5.34\,\mathrm{mm}$$ <