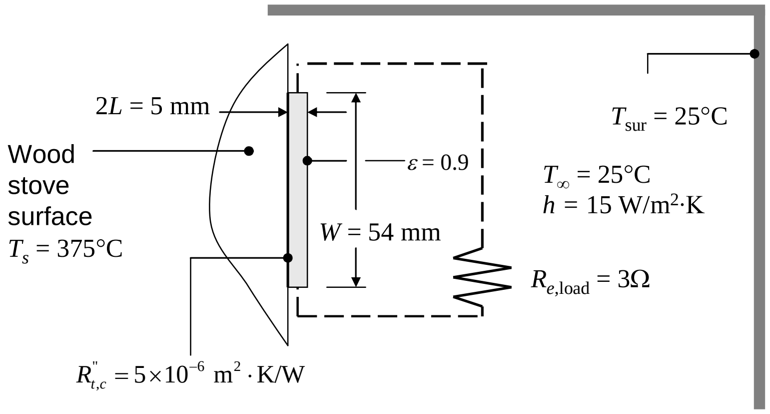
PROBLEM 3.170

KNOWN: Dimensions of thermoelectric module. Convection conditions, thermoelectric module performance parameters, load electrical resistance, contact resistance between thermoelectric module and stove surface, emissivity of the exposed surface of the thermoelectric module, temperature of surroundings.

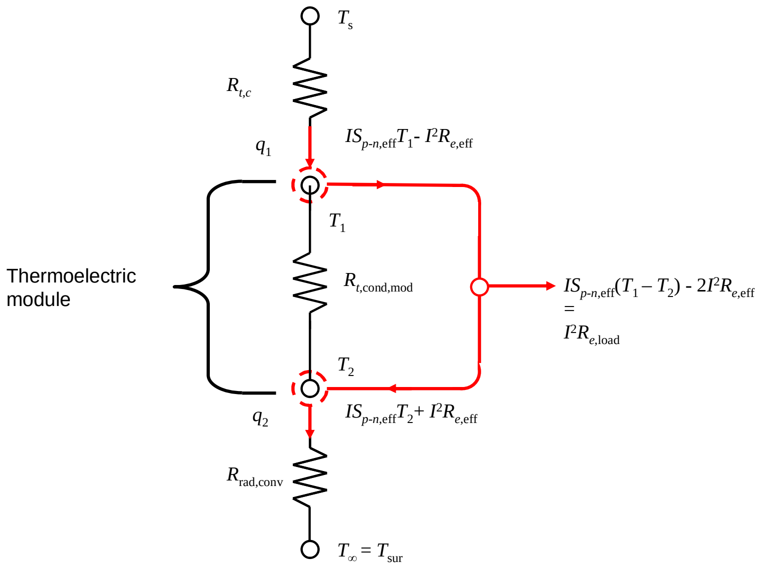
FIND: Sketch of the equivalent thermal circuit and electrical power generated by the module.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) Large surroundings.

ANALYSIS: The portion of the equivalent thermal circuit that describes the thermoelectric module is the same as shown in Figure 3.24*b*. However, the external thermal resistances are different. The high temperature side of the TEM is exposed to the stove surface through a contact resistance, $R_{t,c}$. The low temperature side exchanges heat with the surroundings through radiation and convection. Since T_∞ = T_{sur}, the radiation and convection thermal resistances can be combined into a single resistance, $R_{rad,conv}$, as shown below. Also, $q_{conv,1}$ and $q_{conv,2}$ have been replaced with the more general terms q_1 and q_2.



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The two external resistances can be calculated as follows:

$$R_{t,c} = R''_{t,c} / W^2 = 5 \times 10^{-6} \text{ m}^2 \cdot \text{K/W} / (0.054 \text{ m})^2 = 1.71 \times 10^{-3} \text{ K/W}$$

$$R_{\text{rad,conv}} = 1/[(h_{\text{conv}} + h_r)W^2]$$

$$h_r = \varepsilon\sigma(T_2 + T_{\text{sur}})(T_2^2 + T_{\text{sur}}^2) \quad (1)$$

The radiation heat transfer coefficient, h_r, depends on the unknown TEM surface temperature, T_2. This can be left as an unknown in solving the simultaneous equations.

The analysis proceeds as in Example 3.13. The conduction resistance of one module is the same as in the example, namely

$$R_{t,\text{cond,mod}} = \frac{L}{NA_{c,s}k_s} = \frac{2.5 \times 10^{-3} \text{ m}}{100 \times 1.2 \times 10^{-5} \text{ m}^2 \times 1.2 \text{ W/m} \cdot \text{K}} = 1.736 \text{ K/W}$$

From Equations 3.125 and 3.126,

$$q_1 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_1 - I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736 \text{ K/W}} + I \times 0.1435 \text{ V/K} \times T_1 - I^2 \times 4 \ \Omega \quad (2)$$

$$q_2 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_2 + I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736 \text{ K/W}} + I \times 0.1435 \text{ V/K} \times T_2 + I^2 \times 4 \ \Omega \quad (3)$$

Additional relationships can be written by considering heat transfer through the external resistances.

$$q_1 = (T_s - T_1)/R_{t,c} = \left[(375 + 273) \text{ K - } T_1\right]/1.71 \times 10^{-3} \text{ K/W} \quad (4)$$

$$\begin{aligned} q_2 &= (T_2 - T_\infty)/R_{\text{rad,conv}} = (T_2 - T_\infty)(h + h_r)W^2 \\ &= \left[T_2 - (25 + 273) \text{ K}\right] \times (15 \text{ W/m}^2 \cdot \text{K} + h_r) \times (0.054 \text{ m})^2 \end{aligned} \quad (5)$$

The electric power produced by the single module, P_N, is equal to the electric power dissipated in the load resistance. Equating the expression for P_N from Equation 3.127 to the electric power dissipated in the load gives

$$P_N = IS_{p\text{-}n,\text{eff}}(T_1 - T_2) - 2I^2R_{e,\text{eff}} = I^2R_{e,\text{load}}$$

$$I \times 0.1435 \text{ V/K} \times (T_1 - T_2) - 2I^2 \times 4 \ \Omega = I^2 \times 3 \ \Omega \quad (6)$$

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Equations 1 through 6 may be solved simultaneously, for example using IHT, to yield I = 0.27 A, and

$$P_N = I^2 R_{e,\text{load}} = (0.27\text{ A})^2 \times 3\ \Omega = 0.22\text{ W} \qquad <$$

COMMENTS: (1) Radiation is significant. If radiation heat transfer were neglected, the electrical power output would be decreased to 0.036 W. (2) The electrical power output is quite low. The power output could be raised by increasing the temperature difference across the module. For example, the electrical power could be used to rotate a small fan to increase the value of the heat transfer coefficient. If h were to increase to 30 $W/m^2 \cdot K$, for example, the electrical power would increase to 0.40 W. A tradeoff exists between the extra power provided by the fan and the power needed to operate the fan.