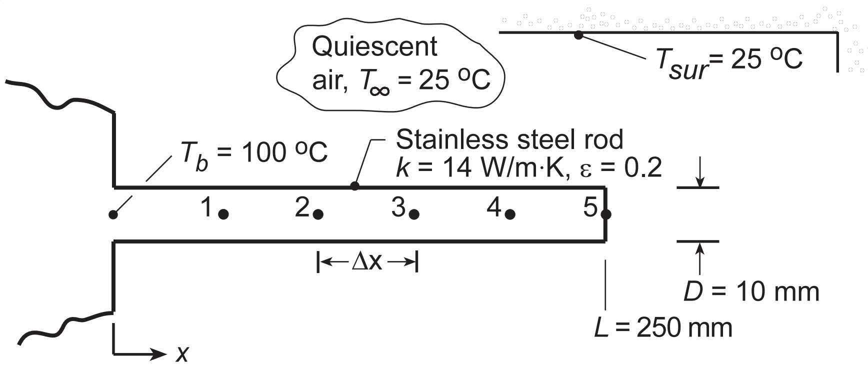
PROBLEM 4.79

KNOWN: Pin fin of 10 mm diameter and length 250 mm with base temperature of 100°C experiencing radiation exchange with the surroundings and free convection with ambient air.

FIND: Temperature distribution using finite-difference method with five nodes. Fin heat rate and relative contributions by convection and radiation.

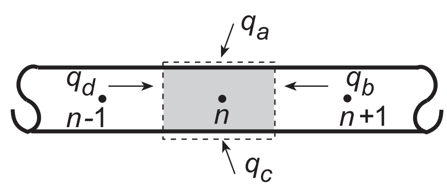
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fin, (3) Constant properties, (4) Fin approximates small object in large enclosure, (5) Fin tip experiences convection and radiation, (6) $h_{fc} = 2.89[0.6 + 0.624(T - T_\infty)^{1/6}]^2$.

ANALYSIS: To apply the finite-difference method, define the 5-node system shown above where Δx = L/5. Perform energy balances on the nodes to obtain the finite-difference equations for the nodal temperatures.

Interior node, n = 1, 2, 3 or 4



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_c + q_d = 0 \qquad (1)$$

$$h_{r,n}P\Delta x\left(T_{sur} - T_n\right) + kA_c\frac{T_{n+1} - T_n}{\Delta x} + h_{fc,n}P\Delta x\left(T_\infty - T_n\right) + kA_c\frac{T_{n-1} - T_n}{\Delta x} = 0 \qquad (2)$$

where the free convection coefficient is

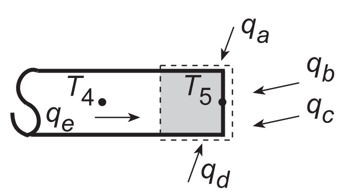
$$h_{fc,n} = 2.89\left[0.6 + 0.624\left(T_n - T_\infty\right)^{1/6}\right]^2 \qquad (3)$$

and the linearized radiation coefficient is

$$h_{r,n} = \varepsilon\sigma\left(T_n + T_{sur}\right)\left(T_n^2 + T_{sur}^2\right) \qquad (4)$$

with P = πD and $A_c = \pi D^2/4$. (5,6)

Tip node, n = 5



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_c + q_d + q_e = 0$$

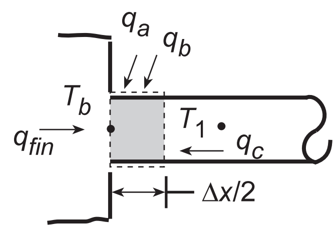
$$\begin{aligned} &h_{r,5}\left(P\Delta x/2\right)\left(T_{sur} - T_5\right) + h_{r,5}A_c\left(T_{sur} - T_5\right) + h_{fc,5}A_c\left(T_\infty - T_5\right) \\ &\quad + h_{fc,5}\left(P\Delta x/2\right)\left(T_\infty - T_5\right) + kA_c\frac{T_4 - T_5}{\Delta x} = 0 \end{aligned} \qquad (7)$$

Continued...

PROBLEM 4.79 (Cont.)

Knowing the nodal temperatures, the heat rates are evaluated as:

Fin Heat Rate: Perform an energy balance around Node b.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_c + q_{fin} = 0$$

$$h_{r,b}\left(P\Delta x/2\right)\left(T_{sur} - T_b\right) + h_{fc,b}\left(P\Delta x/2\right)\left(T_\infty - T_b\right) + kA_c\frac{\left(T_1 - T_b\right)}{\Delta x} + q_{fin} = 0 \qquad (8)$$

where $h_{r,b}$ and $h_{fc,b}$ are evaluated at T_b.

Convection Heat Rate: To determine the portion of the heat rate by convection from the fin surface, we need to sum contributions from each node. Using the convection heat rate terms from the foregoing energy balances, for, respectively, node b, nodes 1, 2, 3, 4 and node 5.

$$q_{cv} = -\left.q_b\right)_b - \sum \left.q_c\right)_{1-4} - \left.\left(q_c + q_d\right)\right)_5 \qquad (9)$$

Radiation Heat Rate: In the same manner,

$$q_{rad} = -\left.q_a\right)_b - \sum \left.q_b\right)_{1-4} - \left.\left(q_a + q_b\right)\right)_5$$

The above equations were entered into the IHT workspace and the set of equations solved for the nodal temperatures and the heat rates. Summary of key results including the temperature distribution and heat rates is shown below.

Node	b	1	2	3	4	5	Fin
T_j (°C)	100	58.5	40.9	33.1	29.8	28.8	-
q_{cv} (W)	0.603	0.451	0.183	0.081	0.043	0.015	1.375
q_{fin} (W)	-	-	-	-	-	-	1.604
q_{rad} (W)	-	-	-	-	-	-	0.229
h_{cv} (W/m^2·K)	10.1	8.6	7.3	6.4	5.7	5.5	-
h_{rad} (W/m^2·K)	1.5	1.4	1.3	1.3	1.2	1.2	-

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COMMENTS: From the tabulated results, it is evident that free convection is the dominant node. Note that the free convection coefficient varies almost by a factor of two over the length of the fin.