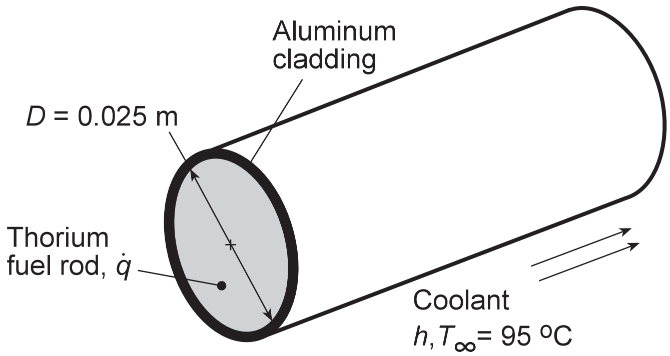
PROBLEM 3.97

KNOWN: Energy generation in an aluminum-clad, thorium fuel rod under specified operating conditions.

FIND: (a) Whether prescribed operating conditions are acceptable, (b) Effect of $\dot{q}$ and h on acceptable operating conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r-direction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible temperature gradients in aluminum and contact resistance between aluminum and thorium.

PROPERTIES: *Table A-1*, Aluminum, pure: M.P. = 933 K; *Table A-1*, Thorium: M.P. = 2023 K, k ≈ 60 W/m·K.

ANALYSIS: (a) System failure would occur if the melting point of either the thorium or the aluminum were exceeded. From Eq. 3.58, the maximum thorium temperature, which exists at r = 0, is

$$T(0) = \frac{\dot{q}r_o^2}{4k} + T_s = T_{Th,max}$$

where, from the energy balance equation, Eq. 3.60, the surface temperature, which is also the aluminum temperature, is

$$T_s = T_\infty + \frac{\dot{q}r_o}{2h} = T_{Al}$$

Hence,

$$T_{Al} = T_s = 95^\circ C + \frac{7\times10^8\ W/m^3 \times 0.0125\,m}{14,000\ W/m^2\cdot K} = 720^\circ C = 993\,K$$

$$T_{Th,max} = \frac{7\times10^8\ W/m^3\,(0.0125m)^2}{4\times60\ W/m\cdot K} + 993\,K = 1449\,K \qquad <$$

Although $T_{Th,max}$ < M.P.$_{Th}$ and the thorium would not melt, T_{al} > M.P.$_{Al}$ and the cladding would melt under the proposed operating conditions. The problem could be eliminated by *decreasing* $\dot{q}$ or r_o, *increasing* h or using a cladding material with a higher melting point.

(b) Using the one-dimensional, steady-state conduction model (solid cylinder) of the IHT software, the following radial temperature distributions were obtained for parametric variations in $\dot{q}$ and h.

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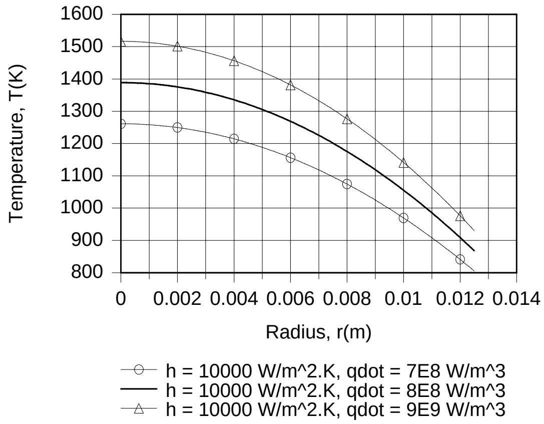
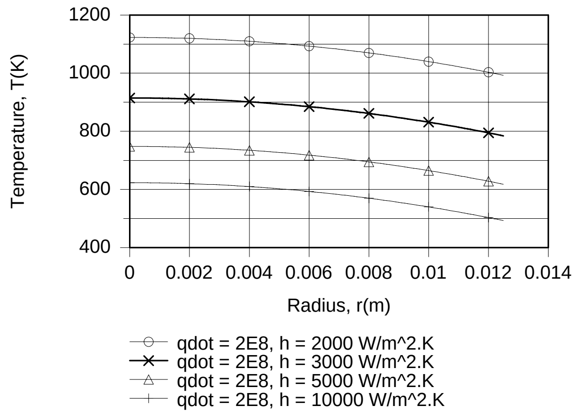
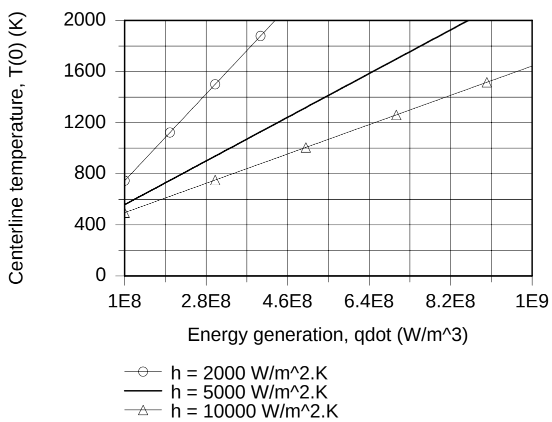
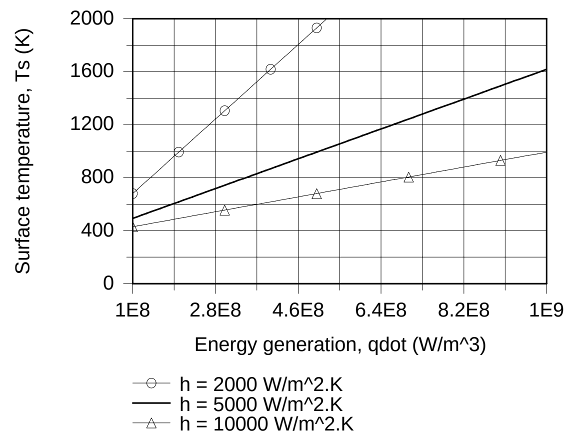




For h = 10,000 W/m^2·K, which represents a reasonable upper limit with water cooling, the temperature of the aluminum would be well below its melting point for $\dot{q} = 7 \times 10^8$ W/m^3, but would be close to the melting point for $\dot{q} = 8 \times 10^8$ W/m^3 and would exceed it for $\dot{q} = 9 \times 10^8$ W/m^3. Hence, under the best of conditions, $\dot{q} \approx 7 \times 10^8$ W/m^3 corresponds to the maximum allowable energy generation. However, if coolant flow conditions are constrained to provide values of h < 10,000 W/m^2·K, volumetric heating would have to be reduced. Even for $\dot{q}$ as low as 2×10^8 W/m^3, operation could not be sustained for h = 2000 W/m^2·K.

The effects of $\dot{q}$ and h on the centerline and surface temperatures are shown below.





For h = 2000 and 5000 W/m^2·K, the melting point of thorium would be approached for $\dot{q} \approx 4.4 \times 10^8$ and 8.5×10^8 W/m^3, respectively. For h = 2000, 5000 and 10,000 W/m^2·K, the melting point of aluminum would be approached for $\dot{q} \approx 1.6 \times 10^8$, 4.3×10^8 and 8.7×10^8 W/m^3. Hence, the envelope of acceptable operating conditions must call for a reduction in $\dot{q}$ with decreasing h, from a maximum of $\dot{q} \approx 7 \times 10^8$ W/m^3 for h = 10,000 W/m^2·K.

COMMENTS: Note the problem which would arise in the event of a *loss of coolant*, for which case h would *decrease* drastically.