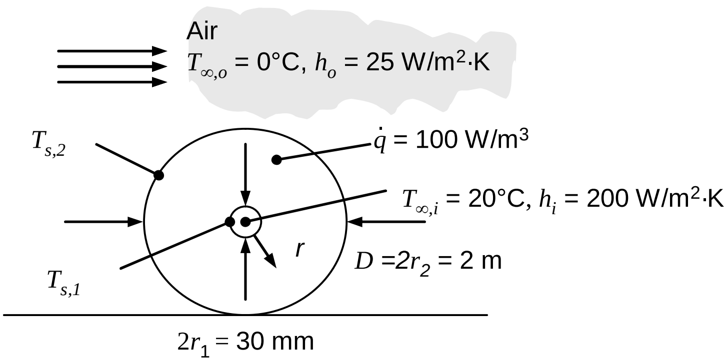
PROBLEM 3.83

KNOWN: Diameter, thermal conductivity and microbial energy generation rate in cylindrical hay bales. Thin-walled tube diameter and insertion location. Temperature of flowing water and convective heat transfer coefficient inside the tube. Ambient conditions.

FIND: (a) Steady-state heat transfer to the water per unit length of tube, (b) Plot of the radial temperature distribution, $T(r)$, in the hay (c) Plot of the heat transfer to the water per unit length of tube for bale diameters of 0.2 m $\le D \le$ 2 m for $\dot{q}$ = 100 W/m^3.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer (4) Uniform volumetric generation, (5) Negligible radiation, (6) Negligible conduction to or from the ground.

PROPERTIES: k = 0.04 W/m·K (given).

ANALYSIS: (a) The temperature distribution is found by utilizing the general solution given by Eq. 3.56 with mixed boundary conditions applied at r_1 and r_2. Specifically,

$$\text{at } r_1\text{:} \quad -k\left.\frac{dT}{dr}\right|_{r=r_1} = h_i\left[T_{\infty,i} - T_{s,1}\right] \qquad \text{at } r_2\text{:} -k\left.\frac{dT}{dr}\right|_{r=r_2} = h_o\left[T_{s,2} - T_{\infty,o}\right]$$

The solutions are given by Eqs. C.16, C.17, and C.2.

From Eq. C.16,

$$h_{\infty,i}(T_{\infty,i} - T_{s,1}) = 200\ \text{W/m}^2\cdot\text{K} \times (20°\text{C} - \text{T}_{s,1})$$

$$= \frac{\dot{q}r_1}{2} - \frac{k\left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{r_1 \ln(r_2/r_1)}$$

$$= \frac{100\text{W/m}^3 \times 15\times10^{-3}\text{m}}{2} - \frac{0.04\text{W/m}\cdot\text{K}\left[\frac{100\text{W/m}^3(1\text{m})^2}{4\times0.04\text{W/m}\cdot\text{K}}\left(1 - \frac{(15\times10^{-3}\text{m})^2}{(1\text{m})^2}\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{15\times10^{-3}\text{m}\times\ln(1000/15)} \quad (1)$$

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From Eq. C.17,

$$h_{\infty,o}(T_{s,2} - T_{\infty,o}) = 25\ \mathrm{W/m^2 \cdot K} \times (T_{s,2} - 0°C)$$

$$= \frac{\dot{q} r_2}{2} - \frac{k\left[\frac{\dot{q} r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{r\,2\ln(r_2/r_1)}$$

$$= \frac{100\mathrm{W/m^3} \times 1\mathrm{m}}{2} - \frac{0.04\mathrm{W/m \cdot K}\left[\frac{100\mathrm{W/m^3}(1\mathrm{m})^2}{4 \times 0.04\mathrm{W/m \cdot K}}\left(1 - \frac{(15\times10^{-3}\mathrm{m})^2}{(1\mathrm{m})^2}\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{1\mathrm{m} \times \ln(1000/15)} \quad (2)$$

Equations (1) and (2) may be solved simultaneously to yield $T_{s,1}$ = 21.54°C, $T_{s,2}$ = 1.75°C. The heat transfer to the cold fluid per unit length is

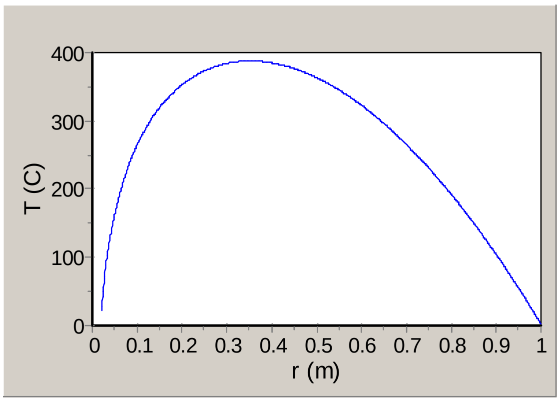
$$q' = h_i(2\pi r_i)(T_{s,i} - T_{\infty,i}) = 200\mathrm{W/m^2 \cdot K} \times 2 \times \pi \times 15\times10^{-3}\mathrm{m} \times (21.54 - 20)°C = 38.7\mathrm{W/m} \quad <$$

(b) The radial temperature distribution is evaluated from Eq. C.2 and is shown below.

$$T(r) = T_{s,2} + \frac{\dot{q} r_2^2}{4k}\left(1 - \frac{r^2}{r_2^2}\right) - \left[\frac{\dot{q} r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + \left(T_{s,2} - T_{s,1}\right)\right]\frac{\ln(r_2/r)}{\ln(r_2/r_1)}$$

$$= 1.75°\mathrm{C} + \frac{100\mathrm{W/m^3} \times (1\mathrm{m})^2}{4 \times 0.04\mathrm{W/m \cdot K}}\left(1 - \frac{r^2}{(1\mathrm{m})^2}\right) - \left[\frac{100\mathrm{W/m^3} \times (1\mathrm{m})^2}{4 \times 0.04\mathrm{W/m \cdot K}}\left(1 - \frac{(15\times10^{-3}\mathrm{m})^2}{(1\mathrm{m})^2}\right) + \left(1.75°\mathrm{C} - 21.54°\mathrm{C}\right)\right]$$

$$\times \frac{\ln(1\mathrm{m}/r)}{\ln(1\mathrm{m}/15\times10^{-3}\mathrm{m})}$$



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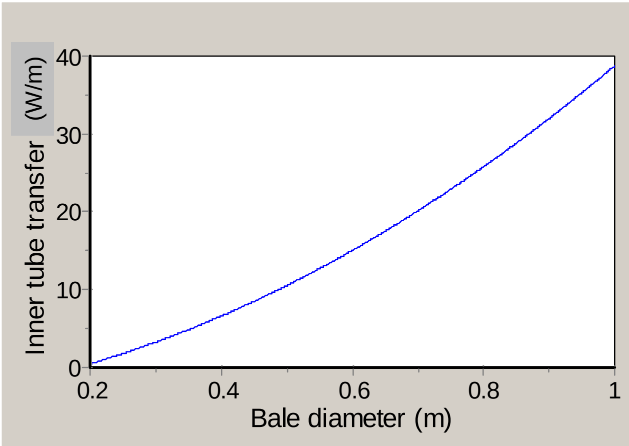
PROBLEM 3.83 (Cont.)

Note that the maximum temperature occurs at $r \approx 0.35$ m.

(c) The rate of heat transfer to the cool fluid, per unit length, is shown versus the bale diameter in the plot below.



Note that at very small bale diameters, the heat transfer to the inner tube will become negative. That is, the energy generation in the bale is not sufficient to offset conduction losses from the relatively warm tube liquid to the relatively cold outside air.

COMMENTS: (1) The energy generated in the bale per unit length is $\dot{E}_g' = \dot{q} \times \pi \times (r_2^2 - r_1^2) = 100\text{W/m}^3 \times \pi \times (1\text{m}^2 - (0.015\text{m})^2) = 314$ W/m. Hence, the heat transfer to the inner tube represents (38.7/314)×100 = 12.3% of the total generated. The remaining 87.6% is lost to the ambient air. (2) The performance could be improved by inserting more tubes, or by stacking the bales in adjacent rows so that heat losses from the exterior surface would be minimized. (3) Evaluation of the two constants appearing in the analytical solution (Eq. 3.56) using the two mixed boundary conditions is very tedious, resulting in a cumbersome expression. Utilization of the results of Appendix C saves considerable time.