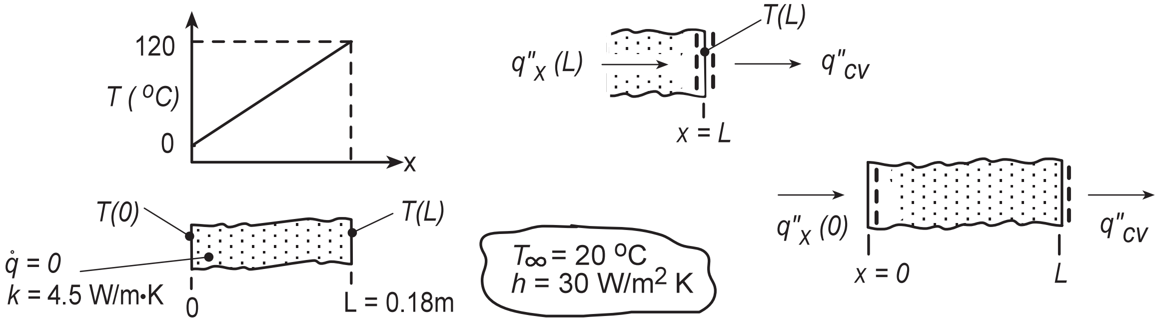
PROBLEM 2.41

KNOWN: Plane wall with no internal energy generation.

FIND: Determine whether the prescribed temperature distribution is possible; explain your reasoning. With the temperatures T(0) = 0°C and T_∞ = 20°C fixed, compute and plot the temperature T(L) as a function of the convection coefficient for the range $10 \le h \le 100$ W/m^2·K.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal energy generation, (3) Constant properties, (4) No radiation exchange at the surface x = L, and (5) Steady-state conditions.

ANALYSIS: (a) Is the prescribed temperature distribution possible? If so, the energy balance at the surface x = L as shown above in the Schematic, must be satisfied.

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q''_x(L) - q''_{cv} = 0 \qquad (1,2)$$

where the conduction and convection heat fluxes are, respectively,

$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = -k\frac{T(L)-T(0)}{L} = -4.5\,W/m\cdot K \times (120-0)^\circ C/0.18\,m = -3000\,W/m^2$$

$$q''_{cv} = h\left[T(L) - T_\infty\right] = 30\,W/m^2\cdot K\times(120-20)^\circ C = 3000\,W/m^2$$

Substituting the heat flux values into Eq. (2), find (-3000) - (3000) ≠ 0 and therefore, the temperature distribution is not possible.

(b) With T(0) = 0°C and T_∞ = 20°C, the temperature at the surface x = L, T(L), can be determined from an overall energy balance on the wall as shown above in the schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q''_x(0) - q''_{cv} = 0 \qquad -k\frac{T(L)-T(0)}{L} - h\left[T(L)-T_\infty\right] = 0$$

$$-4.5\,W/m\cdot K\left[T(L) - 0^\circ C\right]\Big/0.18\,m - 30\,W/m^2\cdot K\left[T(L) - 20^\circ C\right] = 0$$

$$T(L) = 10.9^\circ C$$

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Using this same analysis, T(L) as a function of the convection coefficient can be determined and plotted. We don't expect T(L) to be linearly dependent upon h. Note that as h increases to larger values, T(L) approaches T_∞. To what value will T(L) approach as h decreases?