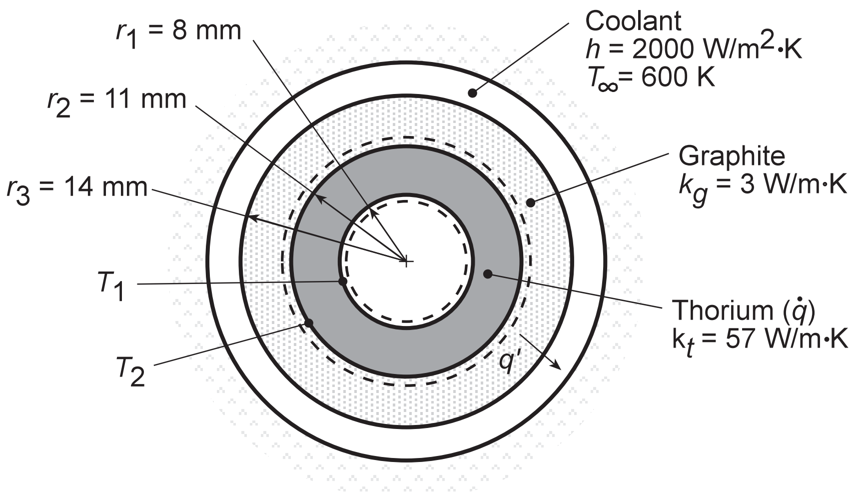
# PROBLEM 3.100

**KNOWN:** Materials, dimensions, properties and operating conditions of a gas-cooled nuclear reactor.

**FIND:** (a) Inner and outer surface temperatures of fuel element, (b) Temperature distributions for different heat generation rates and maximum allowable generation rate.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

**PROPERTIES:** *Table A.1*, Thorium: $T_{mp} \approx 2000$ K; *Table A.2*, Graphite: $T_{mp} \approx 2300$ K.

**ANALYSIS:** (a) The outer surface temperature of the fuel, $T_2$, may be determined from the rate equation

$$q' = \frac{T_2 - T_\infty}{R'_{tot}}$$

where

$$R'_{tot} = \frac{\ln(r_3/r_2)}{2\pi k_g} + \frac{1}{2\pi r_3 h} = \frac{\ln(14/11)}{2\pi(3\,\text{W/m}\cdot\text{K})} + \frac{1}{2\pi(0.014\,\text{m})\left(2000\,\text{W/m}^2\cdot\text{K}\right)} = 0.0185\,\text{m}\cdot\text{K/W}$$

and the heat rate per unit length may be determined by applying an energy balance to a control surface about the fuel element. Since the interior surface of the element is essentially adiabatic, it follows that

$$q' = \dot{q}\pi\left(r_2^2 - r_1^2\right) = 10^8\,\text{W/m}^3 \times \pi\left(0.011^2 - 0.008^2\right)\text{m}^2 = 17,907\,\text{W/m}$$

Hence,

$$T_2 = q'R'_{tot} + T_\infty = 17,907\,\text{W/m}\left(0.0185\,\text{m}\cdot\text{K/W}\right) + 600\,\text{K} = 931\,\text{K}$$ <

With zero heat flux at the inner surface of the fuel element, Eq. C.14 yields

$$T_1 = T_2 + \frac{\dot{q}r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) - \frac{\dot{q}r_1^2}{2k_t}\ln\left(\frac{r_2}{r_1}\right)$$

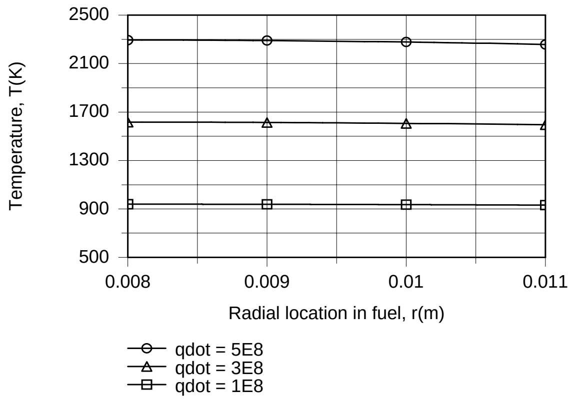
$$T_1 = 931\,\text{K} + \frac{10^8\,\text{W/m}^3(0.011\,\text{m})^2}{4\times 57\,\text{W/m}\cdot\text{K}}\left[1 - \left(\frac{0.008}{0.011}\right)^2\right] - \frac{10^8\,\text{W/m}^3(0.008\,\text{m})^2}{2\times 57\,\text{W/m}\cdot\text{K}}\ln\left(\frac{0.011}{0.008}\right)$$

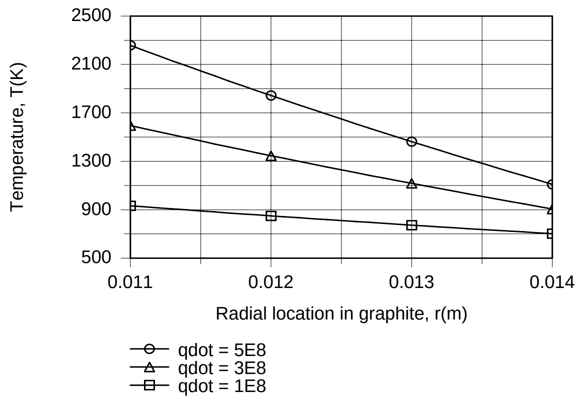
Continued...

## PROBLEM 3.100 (Cont.)

$$T_1 = 931\,\text{K} + 25\,\text{K} - 18\,\text{K} = 938\,\text{K}$$ <

(b) The temperature distributions may be obtained by using the IHT model for one-dimensional, steady-state conduction in a hollow tube. For the fuel element ($\dot{q} > 0$), an adiabatic surface condition is prescribed at $r_1$, while heat transfer from the outer surface at $r_2$ to the coolant is governed by the thermal resistance $R''_{tot,2} = 2\pi r_2 R'_{tot} = 2\pi(0.011\text{ m})0.0185\text{ m}\cdot\text{K/W} = 0.00128\text{ m}^2\cdot\text{K/W}$. For the graphite ($\dot{q} = 0$), the value of $T_2$ obtained from the foregoing solution is prescribed as an inner boundary condition at $r_2$, while a convection condition is prescribed at the outer surface ($r_3$). For $1 \times 10^8 \le \dot{q} \le 5 \times 10^8\text{ W/m}^3$, the following distributions are obtained.




The comparatively large value of $k_t$ yields small temperature variations across the fuel element, while the small value of $k_g$ results in large temperature variations across the graphite. Operation at $\dot{q} = 5 \times 10^8\text{ W/m}^3$ is clearly unacceptable, since the melting points of thorium and graphite are exceeded and approached, respectively. To prevent softening of the materials, which would occur below their melting points, the reactor should not be operated much above $\dot{q} = 3 \times 10^8\text{ W/m}^3$.

**COMMENTS:** A contact resistance at the thorium/graphite interface would increase temperatures in the fuel element, thereby reducing the maximum allowable value of $\dot{q}$.