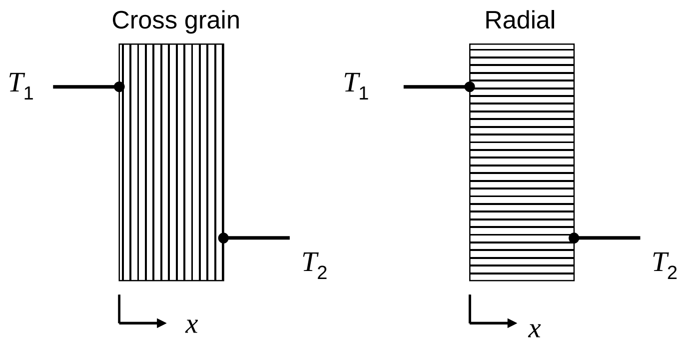
PROBLEM 3.34

KNOWN: Oak wood with a grain structure. Grains are highly porous and the wood is dry.

FIND: Fraction of oak cross-section that appears as being grained.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) Thermal conductivity of highly porous grain is that of air.

PROPERTIES: *Table A.3*, Oak, cross grain (300 K): k_{cross} = 0.17 W/m·K; Oak, radial (300 K): k_{rad} = 0.19 W/m·K. *Table A.4*, Air (300 K): k_{air} = 0.0263 W/m·K.

ANALYSIS: The cross grain condition is characterized by the lowest effective thermal conductivity. Therefore,

$$k_{cross} = k_{min} = 0.17 \text{W/m} \cdot \text{K} = \frac{1}{(1-\varepsilon)/k_s + \varepsilon/k_f} = \frac{1}{(1-\varepsilon)/k_s + \varepsilon/0.0263 \text{ W/m} \cdot \text{K}} \quad (1)$$

Likewise, the radial condition exhibits the highest effective thermal conductivity. Hence,

$$k_{rad} = k_{max} = 0.19 \text{W/m} \cdot \text{K} = \varepsilon k_f + (1-\varepsilon)k_s = \varepsilon \times 0.0263 \text{ W/m} \cdot \text{K} + (1-\varepsilon)k_s \quad (2)$$

The preceding two equations may be solved simultaneously to determine the two unknowns, k_s and ε, yielding

$$\varepsilon = 0.022$$ <

COMMENTS: (1) The predicted value of the solid thermal conductivity is k_s = 0.1937 W/m·K. (2) The predicted value of ε is rather low. In reality, the thermal conductivity of the grain is greater than that of air. Doubling the value of k_f to 0.0526 W/m·K yields ε = 0.061, which is consistent with estimates of ε for oak.