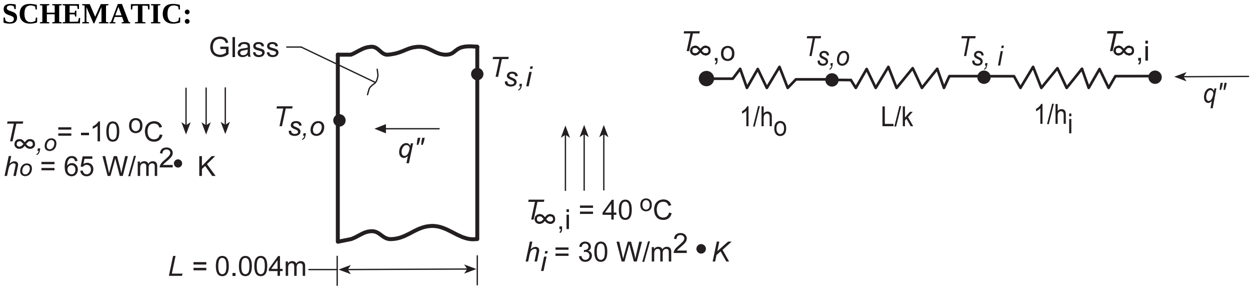
PROBLEM 3.3

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ\mathrm{C} - \left(-10^\circ\mathrm{C}\right)}{\frac{1}{65\,\mathrm{W/m^2\cdot K}} + \frac{0.004\,\mathrm{m}}{1.4\,\mathrm{W/m\cdot K}} + \frac{1}{30\,\mathrm{W/m^2\cdot K}}}$$

$$q'' = \frac{50^\circ\mathrm{C}}{\left(0.0154 + 0.0029 + 0.0333\right)\mathrm{m^2\cdot K/W}} = 969\,\mathrm{W/m^2}\,.$$

Hence, with $q'' = h_i\left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ\mathrm{C} - \frac{969\,\mathrm{W/m^2}}{30\,\mathrm{W/m^2\cdot K}} = 7.7^\circ\mathrm{C} \qquad <$$

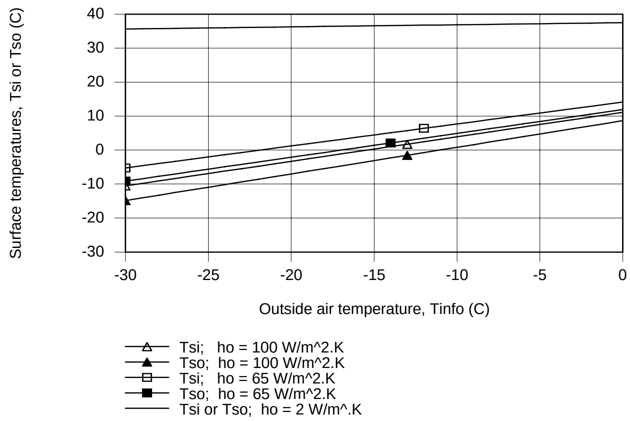
Similarly for the outer surface temperature with $q'' = h_o\left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} + \frac{q''}{h_o} = -10^\circ\mathrm{C} + \frac{969\,\mathrm{W/m^2}}{65\,\mathrm{W/m^2\cdot K}} = 4.9^\circ\mathrm{C} \qquad <$$

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of h_o = 2, 65, and 100 W/m^2·K. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with h_o = 2 W/m^2·K, $T_{s,i}$ - $T_{s,o}$, is too small to show on the plot.

Continued …

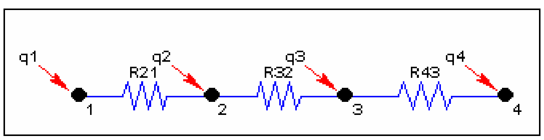
PROBLEM 3.3 (Cont.)



COMMENTS: (1) The largest resistance is that associated with convection at the inner surface. The values of $T_{s,i}$ and $T_{s,o}$ could be increased by increasing the value of h_i.

(2) The *IHT Thermal Resistance Network Model* was used to create a model of the window and generate the above plot. The Workspace is shown below.

```
// Thermal Resistance Network Model:
// The Network:
```




```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 =  Tinfo            // Outside air temperature, C
//q1 =                 // Heat rate, W
T2 =  Tso              // Outer surface temperature, C
q2 =  0                // Heat rate, W; node 2, no external heat source
T3 =  Tsi              // Inner surface temperature, C
q3 =  0                // Heat rate, W; node 2, no external heat source
T4 = Tinfi             // Inside air temperature, C
//q4 =                 // Heat rate, W

// Thermal Resistances:
R21 =  1 / ( ho * As )          // Convection thermal resistance, K/W; outer surface
R32 =  L / ( k * As )           // Conduction thermal resistance, K/W; glass
R43 =  1 / ( hi * As )          // Convection thermal resistance, K/W; inner surface

// Other Assigned Variables:
Tinfo = -10            // Outside air temperature, C
ho = 65                // Convection coefficient, W/m^2.K; outer surface
L = 0.004              // Thickness, m; glass
k = 1.4                // Thermal conductivity, W/m.K; glass
Tinfi = 40             // Inside air temperature, C
hi = 30                // Convection coefficient, W/m^2.K; inner surface
As = 1                 // Cross-sectional area, m^2; unit area
```