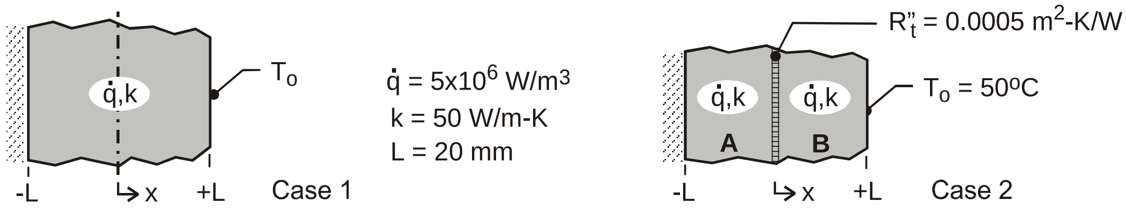
PROBLEM 3.89

KNOWN: Plane wall of thickness 2L, thermal conductivity k with uniform energy generation $\dot{q}$. For case 1, boundary at x = -L is perfectly insulated, while boundary at x = +L is maintained at T_o = 50°C. For case 2, the boundary conditions are the same, but a thin dielectric strip with thermal resistance $R''_t = 0.0005\ m^2 \cdot K/W$ is inserted at the mid-plane.

FIND: (a) Sketch the temperature distribution for case 1 on T-x coordinates and describe key features; identify and calculate the maximum temperature in the wall, (b) Sketch the temperature distribution for case 2 on the same T-x coordinates and describe the key features; (c) What is the temperature difference between the two walls at x = 0 for case 2? And (d) What is the location of the maximum temperature of the composite wall in case 2; calculate this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the plane and composite walls, and (3) Constant properties.

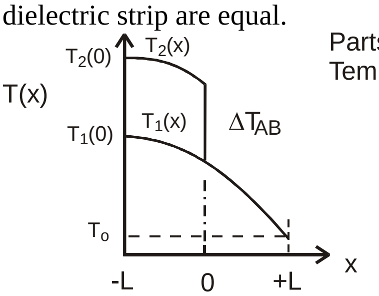
ANALYSIS: (a) For case 1, the temperature distribution, $T_1(x)$ vs. x, is parabolic as shown in the schematic below and the gradient is zero at the insulated boundary, x = -L. From Eq. 3.48 (see discussion after Eq. 3.49),

$$T_1(-L) - T_1(+L) = \frac{\dot{q}(2L)^2}{2k} = \frac{5 \times 10^6 \, W/m^3 (2 \times 0.020\ m)^2}{2 \times 50\ W/m \cdot K} = 80°C$$

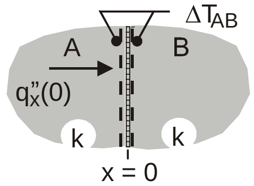
and since $T_1(+L) = T_o$ = 50°C, the maximum temperature occurs at x = -L,

$$T_1(-L) = T_1(+L) + 80°C = 130°C$$

(b) For case 2, the temperature distribution, $T_2(x)$ vs. x, is piece-wise parabolic, with zero gradient at x = -L and a drop across the dielectric strip, ΔT_{AB}. The temperature gradients at either side of the dielectric strip are equal.



Parts (a,b)
Temperature distributions



Part (d) Surface energy balance

(c) For case 2, the temperature drop across the thin dielectric strip follows from the surface energy balance shown above.

$$q''_x(0) = \Delta T_{AB} / R''_t \qquad\qquad q''_x(0) = \dot{q}L$$

$$\Delta T_{AB} = R''_t \, \dot{q}L = 0.0005\ m^2 \cdot K/W \times 5 \times 10^6 \, W/m^3 \times 0.020\ m = 50°C.$$

(d) For case 2, the maximum temperature in the composite wall occurs at x = -L, with the value,

$$T_2(-L) = T_1(-L) + \Delta T_{AB} = 130°C + 50°C = 180°C$$ <