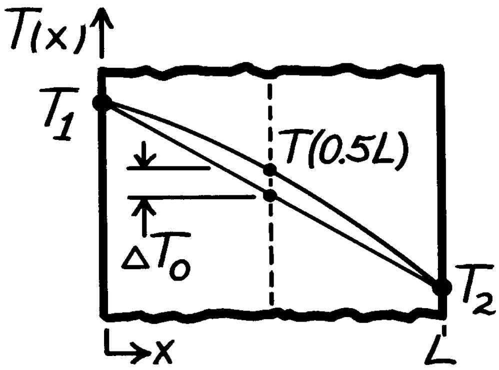
**PROBLEM 3.43**

**KNOWN:** Steady-state temperature distribution of convex shape for material with $k = k_o(1 + \alpha T)$ where $\alpha$ is a constant and the mid-point temperature is $\Delta T_o$ higher than expected for a linear temperature distribution.

**FIND:** Relationship to evaluate $\alpha$ in terms of $\Delta T_o$ and $T_1$, $T_2$ (the temperatures at the boundaries).

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) $\alpha$ is positive and constant.

**ANALYSIS:** At any location in the wall, Fourier's law has the form

$$q''_x = -k_o(1+\alpha T)\frac{dT}{dx}. \tag{1}$$

Since $q''_x$ is a constant, we can separate Eq. (1), identify appropriate integration limits, and integrate to obtain

$$\int_0^L q''_x dx = -\int_{T_1}^{T_2} k_o(1+\alpha T)dT \tag{2}$$

$$q''_x = -\frac{k_o}{L}\left[\left(T_2 + \frac{\alpha T_2^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right]. \tag{3}$$

We could perform the same integration, but with the upper limits at x = L/2, to obtain

$$q''_x = -\frac{2k_o}{L}\left[\left(T_{L/2} + \frac{\alpha T_{L/2}^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right] \tag{4}$$

where

$$T_{L/2} = T(L/2) = \frac{T_1 + T_2}{2} + \Delta T_o. \tag{5}$$

Setting Eq. (3) equal to Eq. (4), substituting from Eq. (5) for $T_{L/2}$, and solving for $\alpha$, it follows that

$$\alpha = \frac{2\Delta T_o}{\left(T_2^2 + T_1^2\right)/2 - \left[\left(T_1 + T_2\right)/2 + \Delta T_o\right]^2}.$$ <