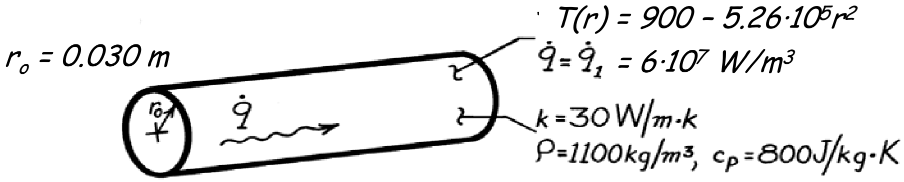
PROBLEM 2.28

KNOWN: Steady-state temperature distribution in a cylindrical rod having uniform heat generation of $\dot{q}_1 = 6 \times 10^7 \text{ W/m}^3$.

FIND: (a) Steady-state centerline and surface heat transfer rates per unit length, q'_r. (b) Initial time rate of change of the centerline and surface temperatures in response to a change in the generation rate from $\dot{q}_1$ to $\dot{q}_2 = 10^8 \text{ W/m}^3$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the r direction, (2) Uniform generation, and (3) Steady-state for $\dot{q}_1 = 6 \times 10^7 \text{ W/m}^3$.

ANALYSIS: (a) From the rate equations for cylindrical coordinates,

$$q''_r = -k\frac{\partial T}{\partial r} \qquad q = -kA_r\frac{\partial T}{\partial r}.$$

Hence,

$$q_r = -k(2\pi rL)\frac{\partial T}{\partial r}$$

or

$$q'_r = -2\pi kr\frac{\partial T}{\partial r} \tag{1}$$

where $\partial T/\partial r$ may be evaluated from the prescribed temperature distribution, T(r).

At r = 0, the gradient is $(\partial T/\partial r) = 0$. Hence, from Equation (1) the heat rate is

$$q'_r(0) = 0.$$ <

At $r = r_o$, the temperature gradient is

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -2\left[5.26\times10^5 \frac{K}{m^2}\right](r_o) = -2\left(5.26\times10^5\right)(0.030\text{m})$$

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -31.6\times10^3 \text{ K/m}.$$

Continued ...

Hence, the heat rate at the outer surface ($r = r_o$) per unit length is

$$q'_r(r_o) = -2\pi\left[30\ \mathrm{W/m\cdot K}\right](0.030\mathrm{m})\left[-31.6\times10^{3}\ \mathrm{K/m}\right]$$

$$q'_r(r_o) = 1.785\times10^{5}\ \mathrm{W/m}. \qquad <$$

(b) Transient (time-dependent) conditions will exist when the generation is changed, and for the prescribed assumptions, the temperature is determined by the following form of the heat equation, Equation 2.26

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q}_2 = \rho c_p \frac{\partial T}{\partial t}$$

Hence

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q}_2\right].$$

However, initially (at t = 0), the temperature distribution is given by the prescribed form, $T(r) = 800 - 5.26\times10^{5} r^{2}$, and

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] = \frac{k}{r}\frac{\partial}{\partial r}\left[r\left(-10.52\times10^{5}\cdot r\right)\right]$$

$$= \frac{k}{r}\left(-21.04\times10^{5}\cdot r\right)$$

$$= 30\ \mathrm{W/m\cdot K}\left[-21.04\times10^{5}\ \mathrm{K/m^2}\right]$$

$$= -6.31\times10^{7}\ \mathrm{W/m^3}\ \left(\text{the original } \dot{q}=\dot{q}_1\right).$$

Hence, everywhere in the wall,

$$\frac{\partial T}{\partial t} = \frac{1}{1100\ \mathrm{kg/m^3}\times800\ \mathrm{J/kg\cdot K}}\left[-6.31\times10^{7} + 10^{8}\right]\ \mathrm{W/m^3}$$

or

$$\frac{\partial T}{\partial t} = 41.91\ \mathrm{K/s} \qquad <$$

COMMENTS: (1) The value of ($\partial T/\partial t$) will decrease with increasing time, until a new steady-state condition is reached and once again ($\partial T/\partial t$) = 0. (2) By applying the energy conservation requirement, Equation 1.12c, to a unit length of the rod for the steady-state condition, $\dot{E}'_{in} - E'_{out} + \dot{E}'_{gen} = 0.$

Hence $q'_r(0) - q'_r(r_o) = -\dot{q}_1\left(\pi r_o^2\right).$