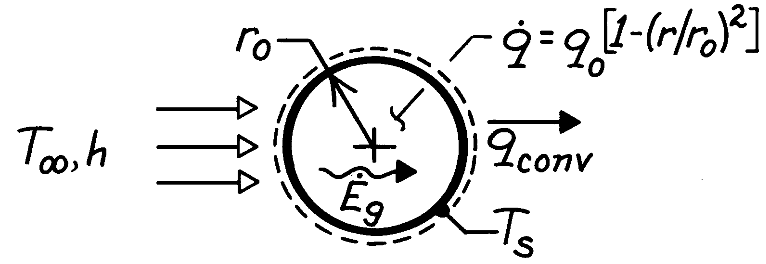
PROBLEM 3.103

KNOWN: Radial distribution of heat dissipation of a spherical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right].$$

Hence $$r^2\frac{dT}{dr} = -\frac{\dot{q}_o}{k}\left(\frac{r^3}{3}-\frac{r^5}{5r_o^2}\right)+C_1$$

$$T = -\frac{\dot{q}_o}{k}\left(\frac{r^2}{6}-\frac{r^4}{20r_o^2}\right)-\frac{C_1}{r}+C_2.$$

From the boundary conditions,

$$dT/dr\,|_{r=0} = 0 \quad \text{and} \quad -k\,dT/dr\,|_{r=r_o} = h\left[T(r_o)-T_\infty\right]$$

it follows that $C_1 = 0$ and

$$\dot{q}_o\left(\frac{r_o}{3}-\frac{r_o}{5}\right) = h\left[-\frac{\dot{q}_o}{k}\left(\frac{r_o^2}{6}-\frac{r_o^2}{20}\right)+C_2-T_\infty\right]$$

$$C_2 = \frac{2r_o\dot{q}_o}{15h}+\frac{7\dot{q}_o r_o^2}{60k}+T_\infty.$$

Hence $$T(r) = T_\infty+\frac{2r_o\dot{q}_o}{15h}+\frac{\dot{q}r_o^2}{k}\left[\frac{7}{60}-\frac{1}{6}\left(\frac{r}{r_o}\right)^2+\frac{1}{20}\left(\frac{r}{r_o}\right)^4\right].$$ <

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + (2r_o\dot{q}_o/15h).$$

The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.