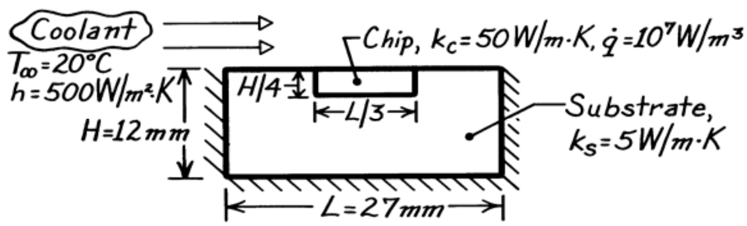
PROBLEM 4.80

KNOWN: Silicon chip mounted in a dielectric substrate. One surface of system is convectively cooled while the remaining surfaces are well insulated.

FIND: Whether maximum temperature in chip will exceed 85°C.

SCHEMATIC:



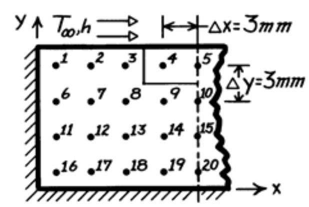
ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Negligible contact resistance between chip and substrate, (4) Upper surface experiences uniform convection coefficient, (5) Other surfaces are perfectly insulated.

ANALYSIS: Performing an energy balance on the chip assuming it is *perfectly insulated from* the substrate, the maximum temperature occurring at the interface with the dielectric substrate will be, according to Eqs. 3.48 and 3.51,

$$T_{max} = \frac{\dot{q}(H/4)^2}{2k_c} + \frac{\dot{q}(H/4)}{h} + T_\infty = \frac{10^7\ \text{W/m}^3 (0.003\ \text{m})^2}{2\times 50\ \text{W/m}\cdot\text{K}} + \frac{10^7\ \text{W/m}^3 (0.003\ \text{m})}{500\ \text{W/m}^2\cdot\text{K}} + 20^\circ\text{C} = 80.9^\circ\text{C}.$$

Since $T_{max} < 85°C$ for the assumed situation, for the actual two-dimensional situation with the conducting dielectric substrate, the maximum temperature should be less than 80°C.

Using the suggested grid spacing of 3 mm, construct the nodal network and write the finite-difference equation for each of the nodes taking advantage of symmetry of the system. Note that we have chosen to *not* locate nodes on the system surfaces for two reasons: (1) fewer total number of nodes, 20 vs. 25, and (2) Node 5 corresponds to center of chip which is likely the point of maximum temperature. Using these numerical values,



$$\frac{h\Delta x}{k_s} = \frac{500\ \text{W/m}^2\cdot\text{K}\times 0.003\ \text{m}}{5\ \text{W/m}\cdot\text{K}} = 0.30 \qquad \alpha = \frac{2}{(k_s/k_c)+1} = \frac{2}{5/50+1} = 1.818$$

$$\frac{h\Delta x}{k_c} = \frac{500\ \text{W/m}^2\cdot\text{K}\times 0.003\ \text{m}}{5\ \text{W/m}\cdot\text{K}} = 0.030 \qquad \beta = \frac{2}{(k_c/k_s)+1} = \frac{2}{50/5+1} = 0.182$$

$$\frac{\dot{q}\Delta x\Delta y}{k_c} = 1.800 \qquad \gamma = \frac{1}{k_c/k_s+1} = 0.0910$$

find the nodal equations:

Node 1 $$k_s\Delta x\frac{T_6-T_1}{\Delta y} + k_s\Delta y\frac{T_2-T_1}{\Delta x} + h\Delta x(T_\infty - T_1) = 0$$

Continued …

$$-\left(2+\frac{h\Delta x}{k_s}\right)T_1 + T_2 + T_6 = -\frac{h\Delta x}{k_s}T_\infty \qquad -2.30T_1 + T_2 + T_6 = -6.00 \tag{1}$$

Node 2 $$T_1 - 3.3T_2 + T_3 + T_7 = -6.00 \tag{2}$$

Node 3

$$k_s\Delta y\frac{T_2 - T_3}{\Delta x} + \frac{T_4 - T_3}{(\Delta x/2)/k_c\Delta y + (\Delta x/2)/k_s\Delta y} + k_s\Delta x\frac{T_8 - T_3}{\Delta y} + h\Delta x\left(T_\infty - T_3\right) = 0$$

$$T_2 - \left(2 + \alpha + \left(h\Delta x/k_s\right)T_3\right) + \alpha T_4 + T_8 = -\left(h\Delta x/k\right)T_\infty$$

$$T_2 - 4.12T_3 + 1.82T_4 + T_8 = -6.00 \tag{3}$$

Node 4

$$\frac{T_3 - T_4}{(\Delta x/2)/k_s\Delta y + (\Delta x/2)/k_c\Delta y} + k_c\Delta y\frac{T_5 - T_4}{\Delta x} + \frac{T_9 - T_4}{(\Delta y/2)/k_s\Delta x + (\Delta y/2)k_c\Delta x}$$

$$+\dot{q}\left(\Delta x\Delta y\right) + h\Delta x\left(T_\infty - T_4\right) = 0$$

$$\beta T_3 - \left(1 + 2\beta + \left[h\Delta x/k_c\right]\right)T_4 + T_5 + \beta T_9 = -\left(h\Delta x/k_c\right)T_\infty - \dot{q}\Delta x\Delta y/k_c$$

$$0.182T_3 - 1.39T_4 + T_5 + 0.182T_9 = -2.40 \tag{4}$$

Node 5

$$k_c\Delta y\frac{T_4 - T_5}{\Delta x} + \frac{T_{10} - T_5}{(\Delta y/2)/k_s\left(\Delta x/2\right) + (\Delta y/2)/k_c\left(\Delta x/2\right)} + h\left(\Delta x/2\right)\left(T_\infty - T_5\right) + \dot{q}\Delta y\left(\Delta x/2\right) = 0$$

$$2T_4 - 2.21T_5 + 0.182T_{10} = -2.40 \tag{5}$$

Nodes 6 and 11

$$k_s\Delta x\left(T_1 - T_6\right)/\Delta y + k_s\Delta y\left(T_7 - T_6\right)/\Delta x + k_s\Delta x\left(T_{11} - T_6\right)/\Delta y = 0$$

$$T_1 - 3T_6 + T_7 + T_{11} = 0 \qquad T_6 - 3T_{11} + T_{12} + T_{16} = 0 \tag{6,11}$$

Nodes 7, 8, 12, 13, 14 Treat as interior points,

$$T_2 + T_6 - 4T_7 + T_8 + T_{12} = 0 \qquad T_3 + T_7 - 4T_8 + T_9 + T_{13} = 0 \tag{7,8}$$

$$T_7 + T_{11} - 4T_{12} + T_{13} + T_{17} = 0 \qquad T_8 + T_{12} - 4T_{13} + T_{14} + T_{18} = 0 \tag{12,13}$$

$$T_9 + T_{13} - 4T_{14} + T_{15} + T_{19} = 0 \tag{14}$$

Node 9

$$k_s\Delta y\frac{T_8 - T_9}{\Delta x} + \frac{T_4 - T_9}{(\Delta y/2)/k_c\Delta x + (\Delta y/2)/k_s\Delta x} + k_s\Delta y\frac{T_{10} - T_9}{\Delta x} + k_s\Delta x\frac{T_{14} - T_9}{\Delta y} = 0$$

$$1.82T_4 + T_8 - 4.82T_9 + T_{10} + T_{14} = 0 \tag{9}$$

Node 10 Using the result of Node 9 and considering symmetry,

$$1.82T_5 + 2T_9 - 4.82T_{10} + T_{15} = 0 \tag{10}$$

Node 15 Interior point considering symmetry $$T_{10} + 2T_{14} - 4T_{15} + T_{20} = 0 \tag{15}$$

Node 16 By inspection, $$T_{11} - 2T_{16} + T_{17} = 0 \tag{16}$$

Continued …

PROBLEM 4.80 (Cont.)

Nodes 17, 18, 19, 20

$$T_{12} + T_{16} - 3T_{17} + T_{18} = 0 \qquad T_{13} + T_{17} - 3T_{18} + T_{19} = 0 \qquad (17,18)$$

$$T_{14} + T_{18} - 3T_{19} + T_{20} = 0 \qquad T_{15} + 2T_{19} - 3T_{20} = 0 \qquad (19,20)$$

Using the matrix inversion method, the above system of finite-difference equations is written in matrix notation, Eq. 4.48, [A][T] = [C] where

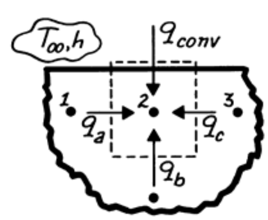
$$[A] = \begin{array}{cccccccccccccccccccc}
-2.3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -3.3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -4.12 & 1.82 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .182 & -1.39 & 1 & 0 & 0 & 0 & .182 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & -2.21 & 0 & 0 & 0 & 0 & .182 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.82 & 0 & 0 & 0 & 1 & -4.82 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.82 & 0 & 0 & 0 & 2 & -4.82 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & -4 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & -3
\end{array} \qquad [C] = \begin{array}{c} -6 \\ -6 \\ -6 \\ -2.4 \\ -2.4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

and the temperature distribution (°C), in geometrical representation, is

34.46	36.13	40.41	45.88	46.23
37.13	38.37	40.85	43.80	44.51
38.56	39.38	40.81	42.72	42.78
39.16	39.77	40.76	41.70	42.06

The maximum temperature is T_5 = 46.23°C which is indeed less than 85°C. $<$

COMMENTS: (1) The convection process for the energy balances of Nodes 1 through 5 were simplified by assuming the node temperature is also that of the surface. Considering Node 2, the energy balance processes for q_a, q_b and q_c are identical (see Eq. (2)); however,



$$q_{conv} = \frac{T_\infty - T_2}{1/h + \Delta y/2k} \approx h\left(T_\infty - T_2\right)$$

where $h\Delta y/2k = 5\ \mathrm{W/m^2 \cdot K} \times 0.003\ \mathrm{m}/2 \times 50\ \mathrm{W/m \cdot K} = 1.5\times10^{-4} \ll 1$. Hence, for this situation, the simplification is justified.