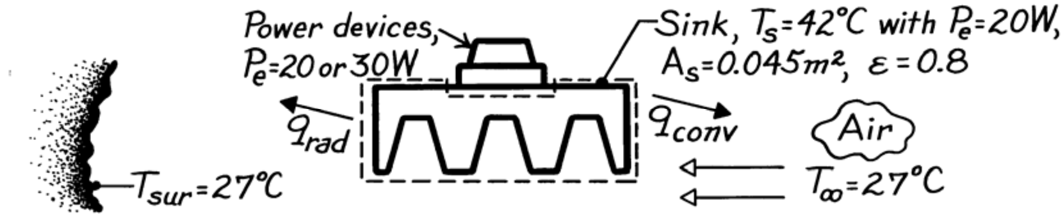
PROBLEM 1.69

KNOWN: Average heat sink temperature when total dissipation is 20 W with prescribed air and surroundings temperature, sink surface area and emissivity.

FIND: Sink temperature when dissipation is 30 W.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) All dissipated power in devices is transferred to the sink, (3) Sink is isothermal, (4) Surroundings and air temperature remain the same for both power levels, (5) Convection coefficient is the same for both power levels, (6) Heat sink is a small surface within a large enclosure, the surroundings.

ANALYSIS: Define a control volume around the heat sink. Power dissipated within the devices is transferred into the sink, while the sink loses heat to the ambient air and surroundings by convection and radiation exchange, respectively.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_e - hA_s\left(T_s - T_\infty\right) - A_s\varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0. \qquad (1)$$

Consider the situation when P_e = 20 W for which T_s = 42°C; find the value of h.

$$h = \left[P_e / A_s - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right] / \left(T_s - T_\infty\right)$$

$$h = \left[20\ \text{W}/0.045\ \text{m}^2 - 0.8\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(315^4 - 300^4\right)\text{K}^4\right] / \left(315-300\right)\text{K}$$

$$h = 24.4\ \text{W/m}^2\cdot\text{K}.$$

For the situation when P_e = 30 W, using this value for h with Eq. (1), obtain

$$30\ \text{W} - 24.4\ \text{W/m}^2\cdot\text{K}\times0.045\ \text{m}^2\left(T_s - 300\right)\text{K}$$

$$-0.045\ \text{m}^2\times0.8\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(T_s^4 - 300^4\right)\text{K}^4 = 0$$

$$30 = 1.098\left(T_s - 300\right) + 2.041\times10^{-9}\left(T_s^4 - 300^4\right).$$

By trial-and-error, find

$$T_s \approx 322\ \text{K} = 49^\circ\text{C}. \qquad <$$

COMMENTS: (1) It is good practice to express all temperatures in kelvin units when using energy balances involving radiation exchange.

(2) Note that we have assumed A_s is the same for the convection and radiation processes. Since not all portions of the fins are completely exposed to the surroundings, $A_{s,rad}$ is less than $A_{s,conv} = A_s$.

(3) Is the assumption that the heat sink is isothermal reasonable?