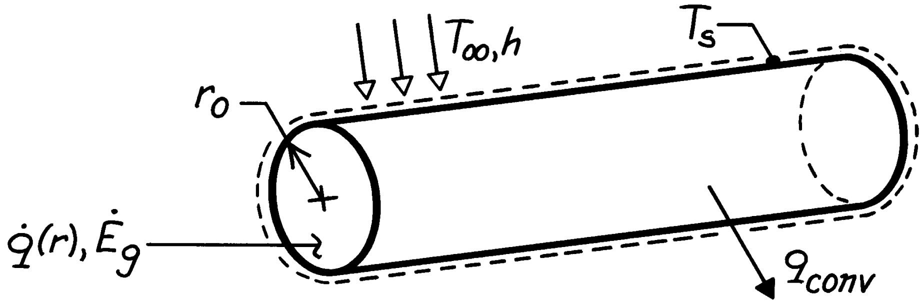
PROBLEM 3.95

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left(1-\frac{r^2}{r_o^2}\right)$$

$$r\frac{dT}{dr} = -\frac{\dot{q}_o r^2}{2k} + \frac{\dot{q}_o r^4}{4kr_o^2} + C_1 \qquad T = -\frac{\dot{q}_o r^2}{4k} + \frac{\dot{q}_o r^4}{16kr_o^2} + C_1 \ln r + C_2.$$

From the boundary conditions,

$$\frac{dT}{dr}\Big|_{r=0} = 0 \rightarrow C_1 = 0 \qquad -k\frac{dT}{dr}\Big|_{r=r_o} = h\left[T(r_o) - T_\infty)\right]$$

$$+\frac{\dot{q}_o r_o}{2} - \frac{\dot{q}_o r_o}{4} = h\left[-\frac{\dot{q}_o r_o^2}{4k} + \frac{\dot{q}_o r_o^2}{16k} + C_2 - T_\infty\right]$$

$$C_2 = \frac{\dot{q}_o r_o}{4h} + \frac{3\dot{q}_o r_o^2}{16k} + T_\infty.$$

Hence

$$T(r) = T_\infty + \frac{\dot{q}_o r_o}{4h} + \frac{\dot{q}_o r_o^2}{k}\left[\frac{3}{16} - \frac{1}{4}\left(\frac{r}{r_o}\right)^2 + \frac{1}{16}\left(\frac{r}{r_o}\right)^4\right]. \qquad <$$

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + (\dot{q}_o r_o)/4h$$

The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.