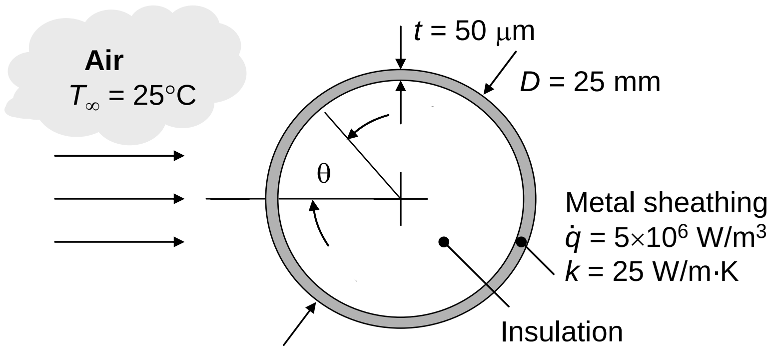
PROBLEM 4.69

KNOWN: Dimensions of long cylinder, thickness of metal sheathing, volumetric generation rate within the sheathing, thermal conductivity of sheathing and convection heat transfer coefficient dependence upon angle θ.

FIND: (a) Temperature distribution within the thin sheathing neglecting θ-direction conduction heat transfer, (b) temperature distribution in the sheathing accounting for θ-direction conduction heat transfer in the metal.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform internal generation, (4) Metal sheathing is very thin relative to cylinder diameter, (5) One-dimensional conduction, (6) Negligible radiation.

ANALYSIS: (a) Neglecting conduction in the θ-direction in the sheathing, an energy balance at any θ location yields

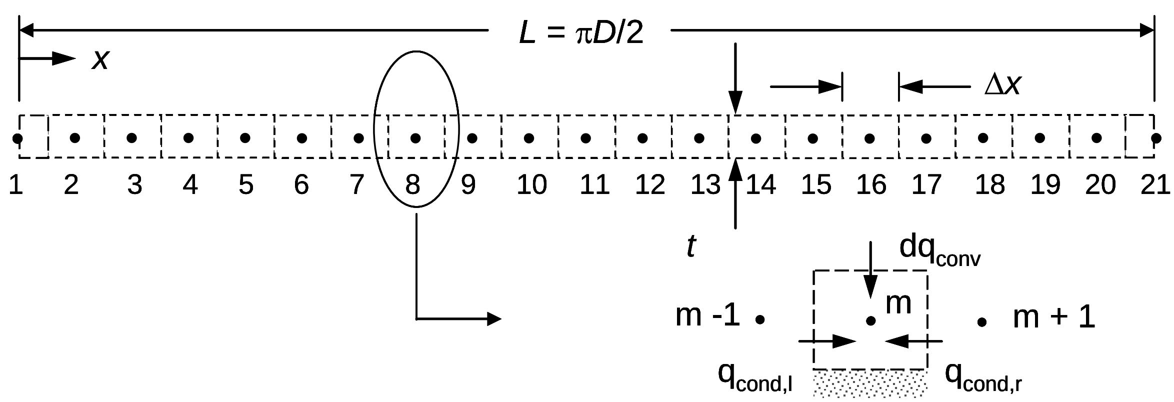
$$\dot{q}t = h(\theta)(T_s - T_\infty) \quad \text{or} \quad T_s = T_\infty + \dot{q}t/h(\theta) \ \text{and}$$

$$h(\theta) = 26 + 0.637\theta - 8.92\theta^2 \text{ for } 0 \le \theta < \pi/2; \ h(\theta) = 5 \text{ for } \pi/2 \le \theta \le \pi$$

These equations may be solved to yield the temperature distribution that is plotted on the next page where $x = \theta D/2$.

(b) Since the sheathing is thin relative to the cylinder diameter, we may evaluate one-dimensional conduction in the x-direction using the Cartesian coordinate system. The finite difference equations are derived by combining expressions for heat fluxes based upon Fourier's law and Newton's law of cooling, along with conservation of energy for each control volume within the discretized domain. Application of conservation of energy for each control volume yields the expression $\dot{E}_{\text{in}} + \dot{E}_g = 0$.

The discretized domain is shown below.



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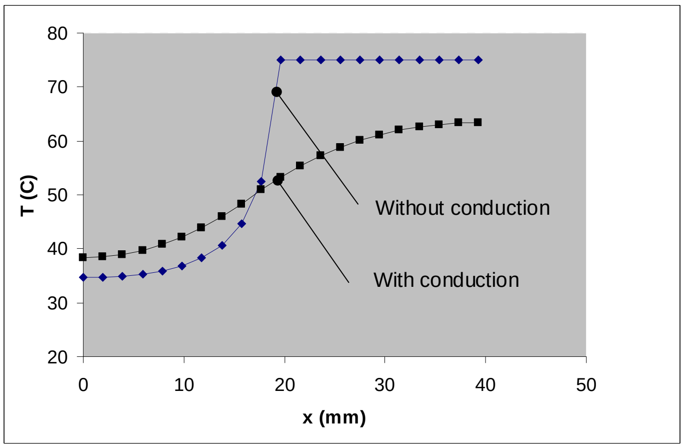
The finite difference equations are as follows.

Node 1:
$$k\frac{(T_2 - T_1)}{\Delta x}t + h(\Delta x/2)(T_\infty - T_1) + \dot{q}(\Delta x/2)t = 0$$

Nodes 2 - 20:
$$k\frac{(T_{m-1} - T_m)}{\Delta x}t + k\frac{(T_{m+1} - T_m)}{\Delta x}t + h(\Delta x)(T_\infty - T_m) + \dot{q}(\Delta x)t = 0$$

Node 21:
$$k\frac{(T_{20} - T_{21})}{\Delta x}t + h(\Delta x/2)(T_\infty - T_{21}) + \dot{q}(\Delta x/2)t = 0$$

The temperature distribution is plotted below.



COMMENTS: (1) Conduction in the θ-direction within the sheathing smears the temperature distribution, increasing the low temperatures on the upstream half of the cylinder and lowering the temperatures on the downstream half of the cylinder. (2) The *IHT* code is listed below.

```
qdot = 5*10^6       //W/m^3
t = 50*10^(-6)      //m
Tinf = 25 + 273     //K
k = 25              //W/m·K
D = 25/1000         //m
L = pi*D/2          //m
dx = L/20           //m

h1 = 26             //W/m^2·K
h2 = 25.88
h3 = 25.32
h4 = 24.32
h5 = 22.88
h6 = 21.00
h7 = 18.68
h8 = 15.92
h9 = 12.71
h10 = 9.07
h11 = 5
h12 = 5
h13 = 5
h14 = 5
h15 = 5
h16 = 5
h17 = 5
h18 = 5
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Continued…

PROBLEM 4.69 (Cont.)

```
h19 = 5
h20 = 5
h21 = 5

//Node 1
k*(T2 - T1)*t/dx + h1*(dx/2)*(Tinf - T1) + qdot*dx*t/2 = 0
//Node 2
k*(T1 - T2)*t/dx + k*(T3 - T2)*t/dx + h2*dx*(Tinf - T2) + qdot*dx*t = 0
//Node 3
k*(T2 - T3)*t/dx + k*(T4 - T3)*t/dx + h3*dx*(Tinf - T3) + qdot*dx*t = 0
//Node 4
k*(T3 - T4)*t/dx + k*(T5 - T4)*t/dx + h4*dx*(Tinf - T4) + qdot*dx*t = 0
//Node 5
k*(T4 - T5)*t/dx + k*(T6 - T5)*t/dx + h5*dx*(Tinf - T5) + qdot*dx*t = 0
//Node 6
k*(T5 - T6)*t/dx + k*(T7 - T6)*t/dx + h6*dx*(Tinf - T6) + qdot*dx*t = 0
//Node 7
k*(T6 - T7)*t/dx + k*(T8 - T7)*t/dx + h7*dx*(Tinf - T7) + qdot*dx*t = 0
//Node 8
k*(T7 - T8)*t/dx + k*(T9 - T8)*t/dx + h8*dx*(Tinf - T8) + qdot*dx*t = 0
//Node 9
k*(T8 - T9)*t/dx + k*(T10 - T9)*t/dx + h9*dx*(Tinf - T9) + qdot*dx*t  = 0
//Node 10
k*(T9 - T10)*t/dx + k*(T11 - T10)*t/dx + h10*dx*(Tinf - T10) + qdot*dx*t = 0
//Node 11
k*(T10 - T11)*t/dx + k*(T12 - T11)*t/dx + h11*dx*(Tinf - T11) + qdot*dx*t = 0
//Node 12
k*(T11 - T12)*t/dx + k*(T13 - T12)*t/dx + h12*dx*(Tinf - T12) + qdot*dx*t = 0
//Node 13
k*(T12 - T13)*t/dx + k*(T14 - T13)*t/dx + h13*dx*(Tinf - T13) + qdot*dx*t = 0
//Node 14
k*(T13 - T14)*t/dx + k*(T15 - T14)*t/dx + h14*dx*(Tinf - T14) + qdot*dx*t = 0
//Node 15
k*(T14 - T15)*t/dx + k*(T16 - T15)*t/dx + h15*dx*(Tinf - T15) + qdot*dx*t = 0
//Node 16
k*(T15 - T16)*t/dx + k*(T17 - T16)*t/dx + h16*dx*(Tinf - T16) + qdot*dx*t  = 0
//Node 17
k*(T16 - T17)*t/dx + k*(T18 - T17)*t/dx + h17*dx*(Tinf - T17) + qdot*dx *t= 0
//Node 18
k*(T17 - T18)*t/dx + k*(T19 - T18)*t/dx + h18*dx*(Tinf - T18) + qdot*dx*t  = 0
//Node 19
k*(T18 - T19)*t/dx + k*(T20 - T19)*t/dx + h19*dx*(Tinf - T19) + qdot*dx*t = 0
//Node 20
k*(T19 - T20)*t/dx + k*(T21 - T20)*t/dx + h20*dx*(Tinf - T20) + qdot*dx*t = 0
//Node 21
k*(T20 - T21)*t/dx +h21*(dx/2)*(Tinf - T21) + + qdot*dx*t/2 = 0
```