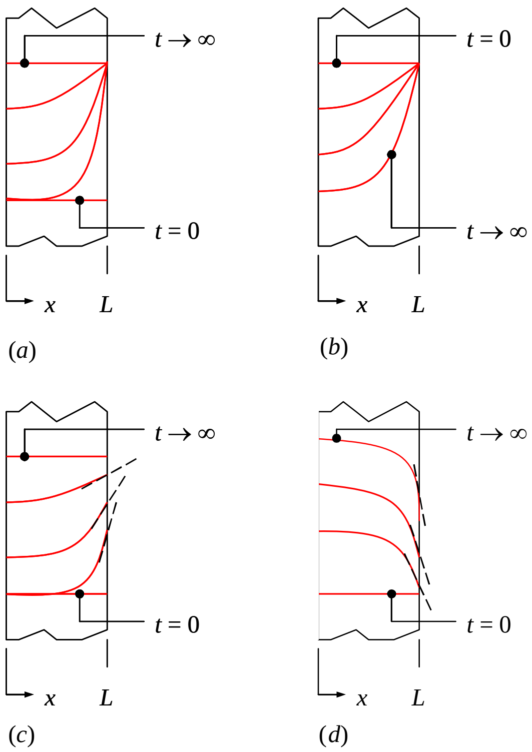
## PROBLEM 2.33

**KNOWN:** Transient temperature distributions in a plane wall.

**FIND:** Appropriate forms of heat equation, initial condition, and boundary conditions.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction, (2) Constant properties, (3) Negligible radiation.

**ANALYSIS:** The general form of the heat equation in Cartesian coordinates for constant *k* is Equation 2.21. For one-dimensional conduction it reduces to

$$\frac{\partial^2 T}{\partial x^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

At steady state this becomes

$$\frac{d^2 T}{dx^2}+\frac{\dot{q}}{k}=0$$

If there is no thermal energy generation the steady-state temperature distribution is linear (or could be constant). If there is uniform thermal energy generation the steady-state temperature distribution must be parabolic.

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## PROBLEM 2.33 (Cont.)

In case (*a*), the steady-state temperature distribution is constant, therefore there must not be any thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

The initial temperature is uniform throughout the solid, thus the initial condition is

$$T(x,0) = T_i$$ <

At $x = 0$, the slope of the temperature distribution is zero at all times, therefore the heat flux is zero (insulated condition). The boundary condition is

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$ <

At $x = L$, the temperature is the same for all $t > 0$. Therefore the surface temperature is constant:

$$T(L,t) = T_s$$ <

For case (*b*), the steady-state temperature distribution is not linear and appears to be parabolic, therefore there is thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

The initial temperature is uniform, the temperature gradient at $x = 0$ is zero, and the temperature at $x = L$ is equal to the initial temperature for all $t > 0$, therefore the initial and boundary conditions are

$$T(x,0) = T_i, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \quad T(L,t) = T_i$$ <

With the left side insulated and the right side maintained at the initial temperature, the cause of the decreasing temperature must be a negative value of thermal energy generation.

In case (*c*), the steady-state temperature distribution is constant, therefore there is no thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

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**PROBLEM 2.33 (Cont.)**

The initial temperature is uniform throughout the solid. At $x = 0$, the slope of the temperature distribution is zero at all times. Therefore the initial condition and boundary condition at $x = 0$ are

$$T(x,0) = T_i, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \qquad <$$

At $x = L$, neither the temperature nor the temperature gradient are constant for all time. Instead, the temperature gradient is decreasing with time as the temperature approaches the steady-state temperature. This corresponds to a convection heat transfer boundary condition. As the surface temperature approaches the fluid temperature, the heat flux at the surface decreases. The boundary condition is:

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right] \qquad <$$

The fluid temperature, $T_\infty$, must be higher than the initial solid temperature to cause the solid temperature to increase.

For case (*d*), the steady-state temperature distribution is not linear and appears to be parabolic, therefore there is thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad <$$

Since the temperature is increasing with time and it is *not* due to heat conduction due to a high surface temperature, the energy generation must be positive.

The initial temperature is uniform and the temperature gradient at $x = 0$ is zero. The boundary condition at $x = L$ is convection. The temperature gradient and heat flux at the surface are *increasing* with time as the thermal energy generation causes the temperature to rise further and further above the fluid temperature. The initial and boundary conditions are:

$$T(x,0) = T_i, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \quad -k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right] \qquad <$$

**COMMENTS:** 1. You will learn to solve for the temperature distribution in transient conduction in Chapter 5. 2. Case (b) might correspond to a situation involving a spatially-uniform endothermic chemical reaction. Such situations, although they can occur, are not common.