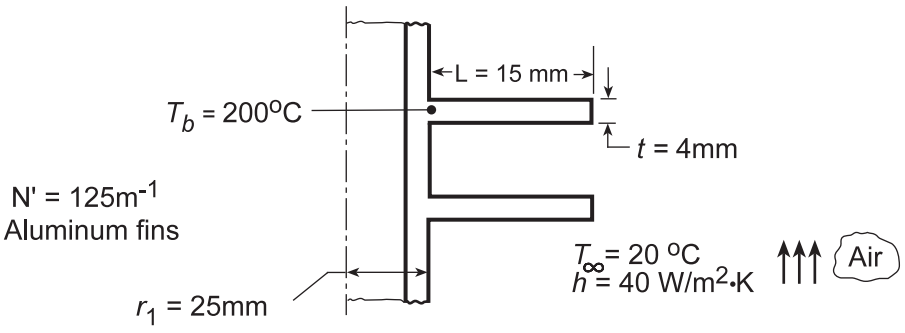
PROBLEM 3.159

KNOWN: Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

FIND: (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.20 are

$$r_{2c} = r_2 + t/2 = 40\,\text{mm} + 2\,\text{mm} = 0.042\,\text{m} \qquad L_c = L + t/2 = 15\,\text{mm} + 2\,\text{mm} = 0.017\,\text{m}$$

$$r_{2c}/r_1 = 0.042\,\text{m}/0.025\,\text{m} = 1.68 \qquad A_p = L_c t = 0.017\,\text{m} \times 0.004\,\text{m} = 6.8\times10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.017\,\text{m})^{3/2}\left[40\,\text{W/m}^2\cdot\text{K}/240\,\text{W/m}\cdot\text{K}\times6.8\times10^{-5}\,\text{m}^2\right]^{1/2} = 0.11$$

The fin efficiency is $\eta_f \approx 0.97$. From Eq. 3.91,

$$q_f = \eta_f q_{max} = \eta_f h A_{f(ann)}\theta_b = 2\pi\eta_f h\left[r_{2c}^2 - r_1^2\right]\theta_b$$

$$q_f = 2\pi\times0.97\times40\,\text{W/m}^2\cdot\text{K}\left[(0.042)^2 - (0.025)^2\right]\text{m}^2\times180^\circ\text{C} = 50\,\text{W} \quad <$$

From Eq. 3.86, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{50\,\text{W}}{40\,\text{W/m}^2\cdot\text{K}\;2\pi(0.025\,\text{m})(0.004\,\text{m})180^\circ\text{C}} = 11.05 \quad <$$

(b) The rate of heat transfer per unit length is

$$q' = N'q_f + h(1 - N't)(2\pi r_1)\theta_b$$

$$q' = 125\times50\,\text{W/m} + 40\,\text{W/m}^2\cdot\text{K}\,(1 - 125\times0.004)(2\pi\times0.025\,\text{m})\times180^\circ\text{C}$$

$$q' = (6250 + 565)\,\text{W/m} = 6.82\,\text{kW/m} \quad <$$

COMMENTS: Note the dominant contribution made by the fins to the total heat transfer.