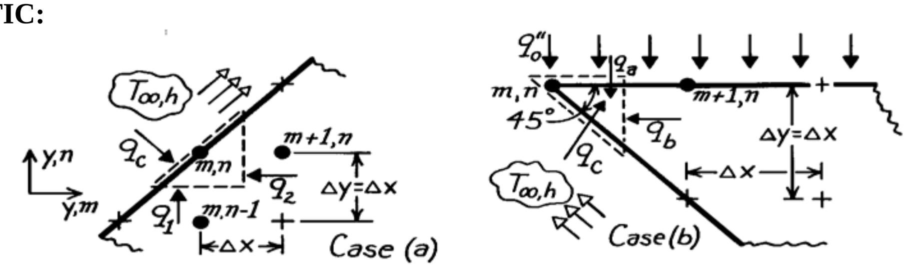
PROBLEM 4.46

KNOWN: Nodal point configurations corresponding to a diagonal surface boundary subjected to a convection process and to the tip of a machine tool subjected to constant heat flux and convection cooling.

FIND: Finite-difference equations for the node m,n in the two situations shown.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: (a) The control volume about node m,n has triangular shape with sides Δx and Δy while the diagonal (surface) length is $\sqrt{2}\ \Delta x$. The heat rates associated with the control volume are due to conduction, q_1 and q_2, and to convection, q_c. Performing an energy balance, find

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_c = 0$$

$$k(\Delta x \cdot 1)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k(\Delta y \cdot 1)\frac{T_{m+1,n} - T_{m,n}}{\Delta x} + h\left(\sqrt{2}\ \Delta x \cdot 1\right)\left(T_\infty - T_{m,n}\right) = 0.$$

Note that we have considered the solid to have unit depth normal to the page. Recognizing that $\Delta x = \Delta y$, dividing each term by k and regrouping, find

$$T_{m,n-1} + T_{m+1,n} + \sqrt{2}\ \cdot\frac{h\Delta x}{k}T_\infty - \left[2 + \sqrt{2}\ \cdot\frac{h\Delta x}{k}\right]T_{m,n} = 0. \qquad <$$

(b) The control volume about node m,n has triangular shape with sides $\Delta x/2$ and $\Delta y/2$ while the lower diagonal surface length is $\sqrt{2}(\Delta x/2)$. The heat rates associated with the control volume are due to the constant heat flux, q_a, to conduction, q_b, and to the convection process, q_c. Perform an energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_a + q_b + q_c = 0$$

$$q''_o \cdot\left[\frac{\Delta x}{2}\cdot 1\right] + k\cdot\left[\frac{\Delta y}{2}\cdot 1\right]\frac{T_{m+1,n} - T_{m,n}}{\Delta x} + h\cdot\left[\sqrt{2}\cdot\frac{\Delta x}{2}\right]\left(T_\infty - T_{m,n}\right) = 0.$$

Recognizing that $\Delta x = \Delta y$, dividing each term by k/2 and regrouping, find

$$T_{m+1,n} + \sqrt{2}\cdot\frac{h\Delta x}{k}\cdot T_\infty + q''_o\cdot\frac{\Delta x}{k} - \left(1 + \sqrt{2}\cdot\frac{h\Delta x}{k}\right)T_{m,n} = 0. \qquad <$$

COMMENTS: Note the appearance of the term $h\Delta x/k$ in both results, which is a dimensionless parameter (the *Biot number*) characterizing the relative effects of convection and conduction.