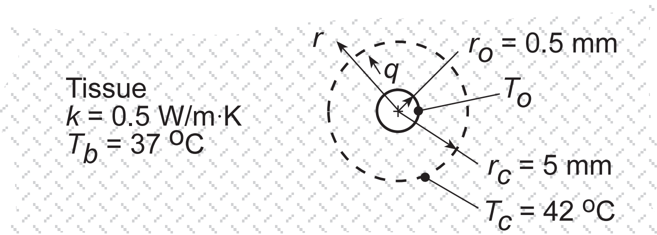
PROBLEM 3.78

KNOWN: Critical and normal tissue temperatures. Radius of spherical heat source and radius of tissue to be maintained above the critical temperature. Tissue thermal conductivity.

FIND: General expression for radial temperature distribution in tissue. Heat rate required to maintain prescribed thermal conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant k, (3) Negligible contact resistance.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

Integrating twice,

$$\frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

Since $T \to T_b$ as $r \to \infty$, $C_2 = T_b$. At $r = r_o$, $q = -k\left(4\pi r_o^2\right)dT/dr|_{r_o} = -4\pi k r_o^2 C_1/r_o^2 = -4\pi k C_1$.

Hence, $C_1 = -q/4\pi k$ and the temperature distribution is

$$T(r)=\frac{q}{4\pi kr}+T_b$$ <

It follows that

$$q=4\pi kr\left[T(r)-T_b\right]$$

Applying this result at $r = r_c$,

$$q=4\pi(0.5\,\text{W/m}\cdot\text{K})(0.005\,\text{m})(42-37)^\circ\text{C}=0.157\,\text{W}$$ <

COMMENTS: At $r_o = 0.0005$ m, $T(r_o) = \left[q/(4\pi k r_o)\right]+T_b = 87°C$. Proximity of this temperature to the boiling point of water suggests the need to operate at a lower power dissipation level.