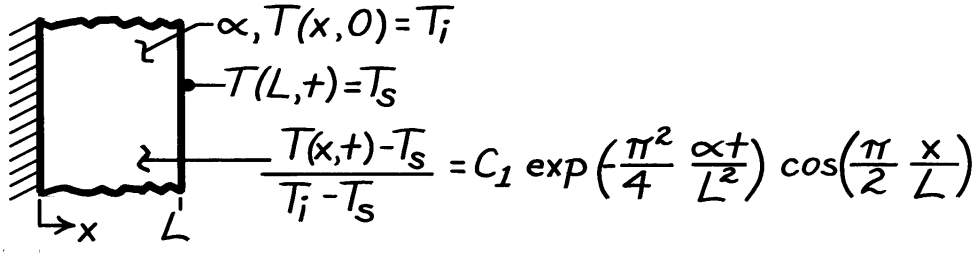
PROBLEM 2.66

KNOWN: Temperature as a function of position and time in a plane wall suddenly subjected to a change in surface temperature, while the other surface is insulated.

FIND: (a) Validate the temperature distribution, (b) Heat fluxes at x = 0 and x = L, (c) Sketch of temperature distribution at selected times and surface heat flux variation with time, (d) Effect of thermal diffusivity on system response.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant properties.

ANALYSIS: (a) To be valid, the temperature distribution must satisfy the appropriate forms of the heat equation and boundary conditions. Substituting the distribution into Equation 2.21, it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$-C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\left(\frac{\pi}{2L}\right)^2 \cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

$$= -\frac{C_1}{\alpha}(T_i - T_s)\left(\frac{\pi^2}{4}\frac{\alpha}{L^2}\right)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right). \qquad <$$

Hence, the heat equation is satisfied. Applying boundary conditions at x = 0 and x = L, it follows that

$$\frac{\partial T}{\partial x}\Big|_{x=0} = -\frac{C_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\Big|_{x=0} = 0 \qquad <$$

and

$$T(L,t) = T_s + C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right)\Big|_{x=L} = T_s. \qquad <$$

Hence, the boundary conditions are also satisfied.

(b) The heat flux has the form

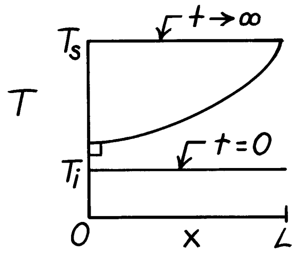
$$q''_x = -k\frac{\partial T}{\partial x} = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right).$$

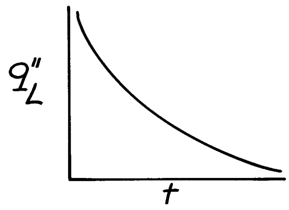
Continued …

Hence, $\quad q''_x(0) = 0,$ <

$$q''_x(L) = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right).$$ <

(c) The temperature distribution and surface heat flux variations are:





(d) For materials A and B of different α,

$$\frac{[T(x,t) - T_s]_A}{[T(x,t) - T_s]_B} = \exp\left[-\frac{\pi^2}{4L^2}(\alpha_A - \alpha_B)t\right]$$

Hence, if $\alpha_A > \alpha_B$, $T(x,t) \to T_s$ more rapidly for Material A. If $\alpha_A < \alpha_B$, $T(x,t) \to T_s$ more rapidly for Material B. <

COMMENTS: Note that the prescribed function for T(x,t) does not reduce to T_i for $t \to 0$. For times at or close to zero, the function is not a valid solution of the problem. At such times, the solution for T(x,t) must include additional terms. The solution is considered in Section 5.5.1 of the text.