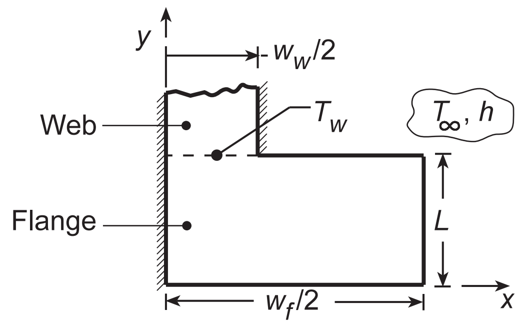
PROBLEM 4.83

KNOWN: Bottom half of an I-beam exposed to hot furnace gases.

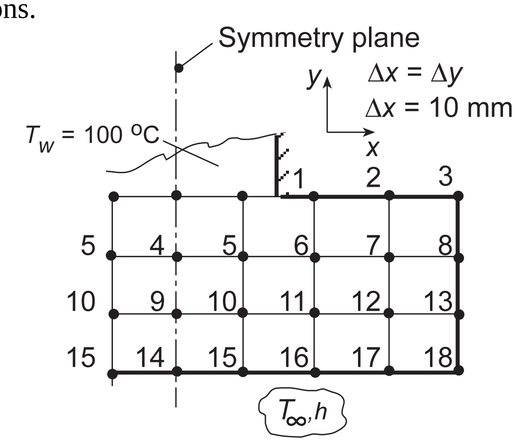
FIND: (a) The heat transfer rate per unit length into the beam using a coarse nodal network (5 × 4) considering the temperature distribution across the web is uniform and (b) Assess the reasonableness of the uniform web-flange interface temperature assumption.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, and (2) Constant properties.

ANALYSIS: (a) The symmetrical section of the I-beam is shown in the Schematic above indicating the web-flange interface temperature is uniform, T_w = 100°C. The nodal arrangement to represent this system is shown below. The nodes on the line of symmetry have been shown for convenience in deriving the nodal finite-difference equations.



Using the *IHT Finite-Difference Equations Tool*, the set of nodal equations can be readily formulated. The temperature distribution (°C) is tabulated in the same arrangement as the nodal network.

100.00	100.00	215.8	262.9	284.8
166.6	177.1	222.4	255.0	272.0
211.7	219.5	241.9	262.7	274.4
241.4	247.2	262.9	279.3	292.9

The heat rate to the beam can be determined from energy balances about the web-flange interface nodes as shown in the sketch below.

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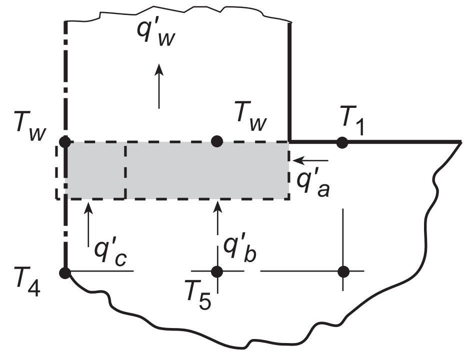

$$q'_w = q'_a + q'_b + q'_c$$

$$q'_w = k(\Delta y/2)\frac{T_1 - T_w}{\Delta x} + k(\Delta x)\frac{T_5 - T_w}{\Delta y} + k(\Delta x/2)\frac{T_4 - T_w}{\Delta y}$$

$$q'_w = 10\,\text{W/m}\cdot\text{K}\left[(215.8-100)/2 + (177.1-100) + (166.6-100)/2\right]\text{K} = 1683\,\text{W/m}$$ <

(b) The schematic below poses the question concerning the reasonableness of the uniform temperature assumption at the web-flange interface. From the analysis above, note that T_1 = 215.8°C vs. T_w = 100°C indicating that this assumption is a poor one. This L-shaped section has strong two-dimensional behavior. To illustrate the effect, we performed an analysis with T_w = 100°C located nearly 2 × times further up the web than it is wide. For this situation, the temperature difference at the web-flange interface across the width of the web was nearly 40°C. The steel beam with its low thermal conductivity has substantial internal thermal resistance and given the L-shape, the uniform temperature assumption (T_w) across the web-flange interface is inappropriate.