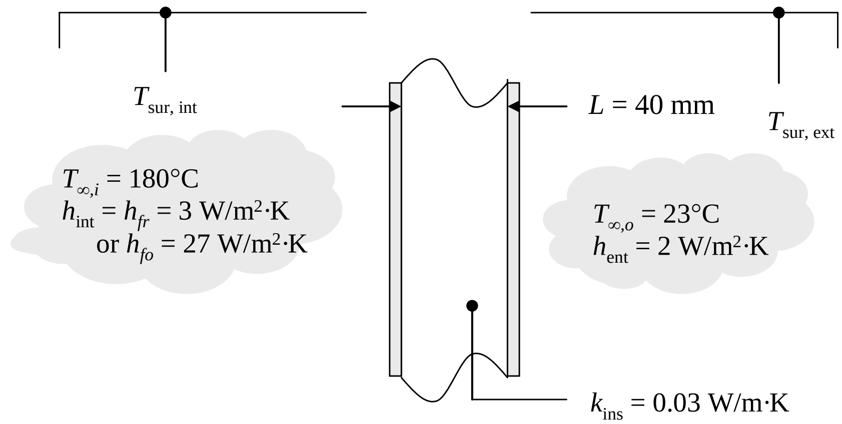
PROBLEM 3.17

KNOWN: Thickness and thermal conductivity of oven wall insulation. Exterior air temperature and convection heat transfer coefficient. Interior air temperature and convection heat transfer coefficients under free and forced convection conditions.

FIND: Heat flux through oven walls under free and forced convection conditions. Impact of forced convection on heat loss. Effect of radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through walls, (3) Thermal resistance of sheet metal layers is negligible, (4) Interior and exterior radiation is to large surroundings at the air temperatures, (5) Emissivity is approximately 1.0, (6) Negligible contact resistances.

ANALYSIS: Neglecting radiation, the heat flux through the oven wall can be calculated from Equations 3.11 and 3.12 on a per unit area basis.

$$q_x'' = (T_{\infty,\text{int}} - T_{\infty,\text{ext}})/R_{\text{tot}}''$$

where

$$R_{\text{tot}}'' = \frac{1}{h_{\text{int}}} + \frac{L}{k_{\text{ins}}} + \frac{1}{h_{\text{ext}}}$$

When forced convection is disabled inside the oven, we have $h_{\text{int}} = h_{fr}$ and

$$R_{\text{tot}}'' = \frac{1}{h_{\text{int}}} + \frac{L}{k_{\text{ins}}} + \frac{1}{h_{\text{ext}}} = \frac{1}{3\ \text{W/m}^2\cdot\text{K}} + \frac{0.04\ \text{m}}{0.03\ \text{W/m}\cdot\text{K}} + \frac{1}{2\ \text{W/m}^2\cdot\text{K}}$$
$$= 0.333\ \text{m}^2\cdot\text{K/W} + 1.333\ \text{m}^2\cdot\text{K/W} + 0.5\ \text{m}^2\cdot\text{K/W} = 2.17\ \text{m}^2\cdot\text{K/W}$$

Therefore

$$q_x'' = (T_{\infty,\text{int}} - T_{\infty,\text{ext}})/R_{\text{tot}}'' = (180°\text{C} - 23°\text{C})/2.17\ \text{m}^2\cdot\text{K/W} = 72.5\ \text{W/m}^2 \qquad <$$

When forced convection is activated inside the oven, we have $h_{\text{int}} = h_{fo}$ and

Continued…

PROBLEM 3.17 (Cont.)

$$R''_{\text{tot}} = \frac{1}{h_{\text{int}}} + \frac{L}{k_{\text{ins}}} + \frac{1}{h_{\text{ext}}} = \frac{1}{27\ \text{W/m}^2\cdot\text{K}} + \frac{0.04\ \text{m}}{0.03\ \text{W/m}\cdot\text{K}} + \frac{1}{2\ \text{W/m}^2\cdot\text{K}}$$
$$= 0.037\ \text{m}^2\cdot\text{K/W} + 1.333\ \text{m}^2\cdot\text{K/W} + 0.5\ \text{m}^2\cdot\text{K/W} = 1.87\ \text{m}^2\cdot\text{K/W}$$

and

$$q''_x = (T_{\infty,\text{int}} - T_{\infty,\text{ext}})/R''_{\text{tot}} = (180°\text{C} - 23°\text{C})/1.87\ \text{m}^2\cdot\text{K/W} = 83.9\ \text{W/m}^2 \qquad <$$

Operation in forced convection mode increases oven heat loss by around 16%. Although h_{int} increases by a factor of 9, other thermal resistances tend to dominate the total thermal resistance. <

Radiation at both the inner and outer oven surfaces can be accounted for by combining convection and radiation heat transfer in parallel. This results in a total heat transfer coefficient at each surface that is the sum of the convection and radiation heat transfer coefficients, see Example 3.1. Therefore, the total thermal resistance can be expressed as

$$R''_{\text{tot}} = \frac{1}{h_{\text{int}} + h_{r,\text{int}}} + \frac{L}{k_{\text{ins}}} + \frac{1}{h_{\text{ext}} + h_{r,\text{ext}}}$$

The radiation heat transfer coefficient is given by Equation 1.9 and can be approximated as

$$h_r \approx 4\varepsilon\sigma\overline{T}^3$$

where $\overline{T} = (T_s + T_{\text{sur}})/2$. We do not know the oven surface temperatures and will approximate both surface temperatures as the average of the interior and exterior air temperatures, $T_s = (T_{\infty,\text{int}} + T_{\infty,\text{ext}})/2$ = 101.5°C ≈ 375 K. Thus, $\overline{T}_{\text{int}} = 414$ K and $\overline{T}_{\text{ext}} = 335.5$ K. Assuming the emissivity of both surfaces is $\varepsilon \approx 1$, we find $h_{r,\text{int}} \approx 16\ \text{W/m}^2\cdot\text{K}$ and $h_{r,\text{ext}} \approx 8.6\ \text{W/m}^2\cdot\text{K}$. Thus, under free convection conditions,

$$R''_{\text{tot}} = \frac{1}{(3+16)\ \text{W/m}^2\cdot\text{K}} + 1.333\ \text{m}^2\cdot\text{K/W} + \frac{1}{(2+8.6)\ \text{W/m}^2\cdot\text{K}}$$
$$= (0.053 + 1 + 0.094)\text{m}^2\cdot\text{K/W} = 1.48\ \text{m}^2\cdot\text{K/W}$$

and under forced convection conditions,

$$R''_{\text{tot}} = \frac{1}{(27+16)\ \text{W/m}^2\cdot\text{K}} + 1.333\ \text{m}^2\cdot\text{K/W} + 0.094\ \text{m}^2\cdot\text{K/W}$$
$$= (0.023 + 1 + 0.094)\text{m}^2\cdot\text{K/W} = 1.45\ \text{m}^2\cdot\text{K/W}$$

When radiation is accounted for, the internal and external heat transfer resistances become small compared to the thermal resistance of the insulation, and there is little difference between the total thermal resistance with or without forced convection inside the oven. <

COMMENTS: The radiation heat flux can be calculated more accurately by solving for the oven interior and exterior surface temperatures, however without knowledge of the surface emissivity there is no reason to be so precise.