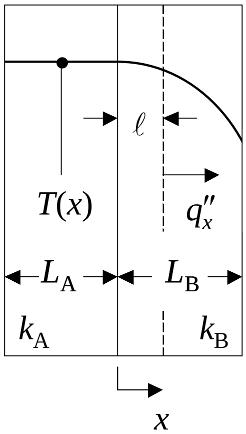
PROBLEM 2.62

KNOWN: Qualitative temperature distribution in a composite wall with one material experiencing uniform volumetric energy generation.

FIND: Which material experiences uniform volumetric generation. The boundary condition at $x = -L_A$. Temperature distribution if the thermal conductivity of Material A is doubled. Temperature distribution if the thermal conductivity of Material B is doubled. Whether a contact resistance exists at the interface between the two materials. Sketch the heat flux distribution $q_x''(x)$ through the composite wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Constant properties.

ANALYSIS: Consider a control volume with the LHS control surface at the interface between the two materials and the RHS control surface located at an arbitrary location within Material B, as shown in the schematic. For this control volume, conservation of energy and Fourier's law may be combined to yield, for uniform volumetric generation in Material B,

$$\dot{q}(\ell) = q_x'' = -k\left.\frac{dT}{dx}\right|_{x=\ell} \quad \text{or} \quad \left.\frac{dT}{dx}\right|_{x=\ell} \propto \ell \qquad (1)$$

The temperature distribution of the problem reflects the preceding proportionality between the temperature gradient and the distance ℓ, and it is appropriate to assume that uniform volumetric generation occurs in Material B but not in Material A. <

The boundary condition at $x = -L_A$ is associated with perfectly insulated conditions,

$$0 = q_x''(x = -L_A) = -k\left.\frac{dT}{dx}\right|_{x=-L_A} \quad \text{or} \quad \left.\frac{dT}{dx}\right|_{x=-L_A} = 0 \qquad <$$

The temperature distribution in Material A corresponds to $q_{x,A}'' = 0$, and is independent of its thermal conductivity. <

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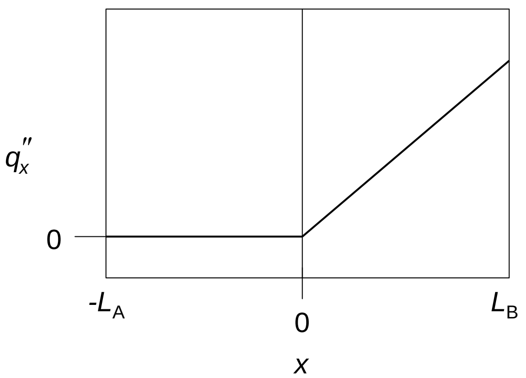
PROBLEM 2.62 (Cont.)

If the volumetric energy generation rate, $\dot{q}$, is unchanged, Equation (1) requires that the temperature gradient everywhere in Material B will be reduced by half if the thermal conductivity of Material B is doubled. Hence, the difference between the minimum and maximum temperatures in the composite wall would be reduced by half. <

Since there is no volumetric energy generation in Material A, and since the surface at $x = -L_A$ is insulated, there can be no conduction of energy into or out of Material A at the LHS of the control volume previously described. Hence, if a contact resistance exists at the interface between Materials A and B, it would not induce any temperature drop across the interface since there is no heat transfer across the interface. Therefore, based on the temperature distribution given in the problem statement, it is impossible to conclude whether a contact resistance exists or not at the interface between Materials A and B. <

Considering Eq. 1, it follows that the heat flux distribution throughout the composite wall is as shown in the sketch below.



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COMMENTS: If you were given information regarding which material experiences internal energy generation, the boundary condition at $x = -L_A$, the thermal conductivities of both materials, and the value of the contact resistance, you should be able to sketch the temperature and heat flux distributions.