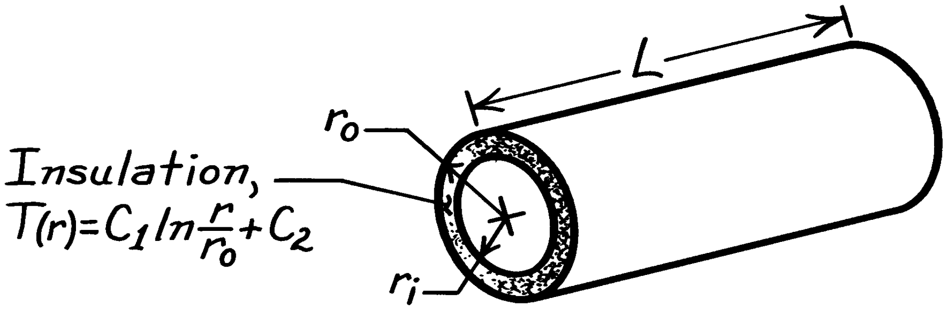
PROBLEM 2.46

KNOWN: Temperature distribution in steam pipe insulation.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.26, the heat equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{C_1}{r}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.23, the radial component of the heat flux is

$$q_r''=-k\frac{\partial T}{\partial r}=-k\frac{C_1}{r}.$$

Hence, q_r'' decreases with increasing $r\left(q_r''\alpha 1/r\right)$. <

At any radial location, the heat rate is

$$q_r=2\pi r L q_r''=-2\pi k C_1 L$$

Hence, q_r is independent of r. <

COMMENTS: The requirement that q_r is invariant with r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r.