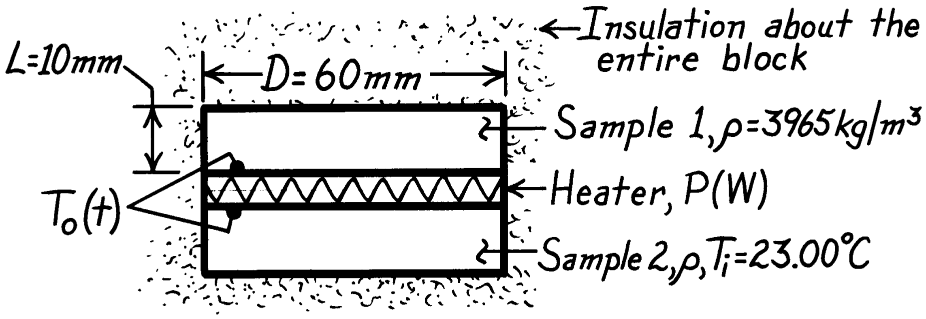
PROBLEM 2.23

KNOWN: Identical samples of prescribed diameter, length and density initially at a uniform temperature T_i, sandwich an electric heater which provides a uniform heat flux q''_o for a period of time Δt_o. Conditions shortly after energizing and a long time after de-energizing heater are prescribed.

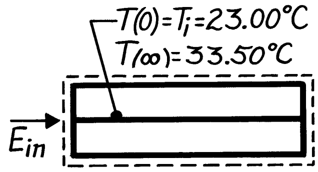
FIND: Specific heat and thermal conductivity of the test sample material. From these properties, identify type of material using Table A.1 or A.2.

SCHEMATIC:



ASSUMPTIONS: (1) One dimensional heat transfer in samples, (2) Constant properties, (3) Negligible heat loss through insulation, (4) Negligible heater mass.

ANALYSIS: Consider a control volume about the samples and heater, and apply conservation of energy over the time interval from t = 0 to ∞



$$E_{in} - E_{out} = \Delta E = E_f - E_i$$

$$P\Delta t_o - 0 = Mc_p\left[T(\infty) - T_i\right]$$

where energy inflow is prescribed by the power condition and the final temperature T_f is known. Solving for c_p,

$$c_p = \frac{P\Delta t_o}{M\left[T(\infty) - T_i\right]} = \frac{15\text{ W} \times 120\text{ s}}{2 \times 3965\text{ kg/m}^3\left(\pi \times 0.060^2/4\right)\text{m}^2 \times 0.010\text{ m}\left[33.50\text{-}23.00\right]^\circ\text{C}}$$

$$c_p = 765\text{ J/kg}\cdot\text{K}$$ <

where M = ρV = 2ρ(πD^2/4)L is the mass of both samples. The transient thermal response of the heater is given by

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PROBLEM 2.23 (Cont.)

$$T_o(t) - T_i = 2q_o''\left[\frac{t}{\pi\rho c_p k}\right]^{1/2}$$

$$k = \frac{t}{\pi\rho c_p}\left[\frac{2q_o''}{T_o(t) - T_i}\right]^2$$

$$k = \frac{30\ \text{s}}{\pi \times 3965\ \text{kg/m}^3 \times 765\ \text{J/kg}\cdot\text{K}}\left[\frac{2 \times 2653\ \text{W/m}^2}{(24.57 - 23.00)^\circ\ \text{C}}\right]^2 = 36.0\ \text{W/m}\cdot\text{K}$$ <

where

$$q_o'' = \frac{P}{2A_s} = \frac{P}{2\left(\pi D^2/4\right)} = \frac{15\ \text{W}}{2\left(\pi \times 0.060^2/4\right)\text{m}^2} = 2653\ \text{W/m}^2.$$

With the following properties now known,

ρ = 3965 kg/m^3 $\qquad$ c_p = 765 J/kg·K $\qquad$ k = 36 W/m·K

entries in Table A.1 are scanned to determine whether these values are typical of a metallic material. Consider the following,

- metallics with low ρ generally have higher thermal conductivities,
- specific heats of both types of materials are of similar magnitude,
- the low k value of the sample is typical of poor metallic conductors which generally have much higher specific heats,
- more than likely, the material is nonmetallic.

From Table A.2, the second entry, polycrystalline aluminum oxide, has properties at 300 K corresponding to those found for the samples. <