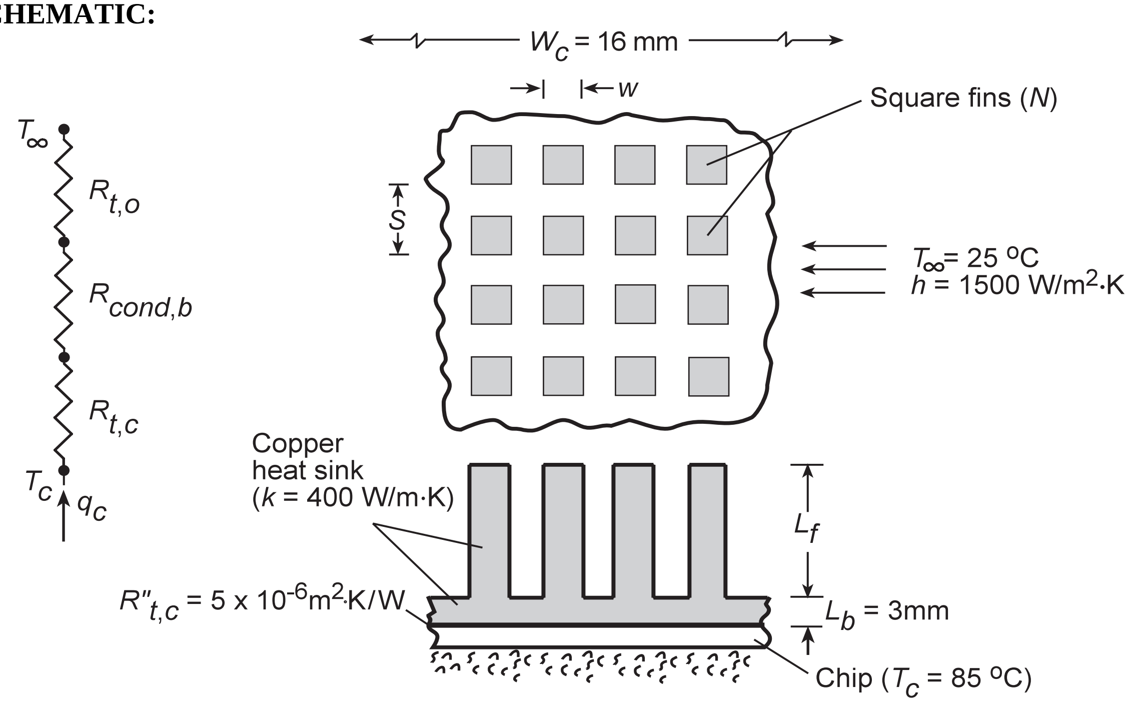
PROBLEM 3.150

KNOWN: Copper heat sink dimensions and convection conditions.

FIND: (a) Maximum allowable heat dissipation for a prescribed chip temperature and interfacial chip/heat-sink contact resistance, (b) Effect of fin length and width on heat dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-heat sink assembly, (3) Constant k, (4) Negligible chip thermal resistance, (5) Negligible heat transfer from back of chip, (6) Uniform chip temperature.

ANALYSIS: (a) For the prescribed system, the chip power dissipation may be expressed as

$$q_c = \frac{T_c - T_\infty}{R_{t,c} + R_{cond,b} + R_{t,o}}$$

where $$R_{t,c} = \frac{R''_{t,c}}{W_c^2} = \frac{5\times10^{-6}\,m^2\cdot K/W}{(0.016m)^2} = 0.0195\,K/W$$

$$R_{cond,b} = \frac{L_b}{kW_c^2} = \frac{0.003m}{400\,W/m\cdot K(0.016m)^2} = 0.0293\,K/W$$

The thermal resistance of the fin array is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

and $$A_t = NA_f + A_b = N(4wL_c) + \left(W_c^2 - Nw^2\right)$$

Continued...

With w = 0.25 mm, S = 0.50 mm, L_f = 6 mm, N = 1024, and $L_c \approx L_f + w/4 = 6.063\times10^{-3}$ m, it follows that $A_f = 6.06\times10^{-6}\,m^2$ and $A_t = 6.40\times10^{-3}\,m^2$. The fin efficiency is

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

where $m = (hP/kA_c)^{1/2} = (4h/kw)^{1/2}$ = 245 m^{-1} and mL_c = 1.49. It follows that η_f = 0.608 and η_o = 0.619, in which case

$$R_{t,o} = \left(0.619\times1500\,W/m^2\cdot K\times6.40\times10^{-3}\,m^2\right) = 0.168\,K/W$$

and the maximum allowable heat dissipation is

$$q_c = \frac{(85-25)^\circ C}{(0.0195+0.0293+0.168)K/W} = 276W$$

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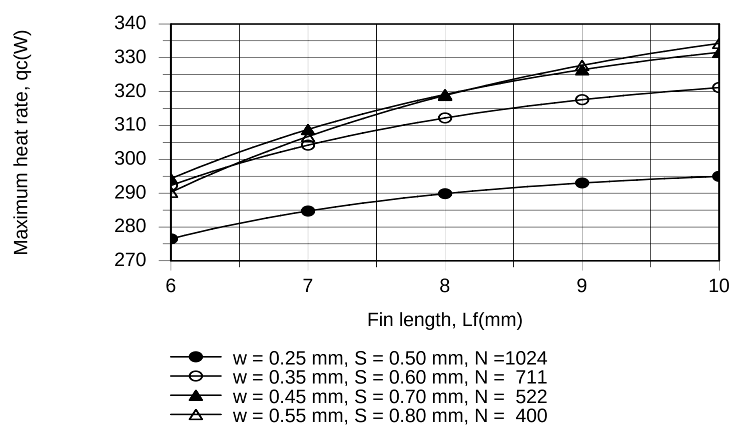
(b) The IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array* has been used to determine q_c as a function of L_f for four different cases, each of which is characterized by the closest allowable fin spacing of (S - w) = 0.25 mm.

Case	w (mm)	S (mm)	N
A	0.25	0.50	1024
B	0.35	0.60	711
C	0.45	0.70	522
D	0.55	0.80	400



With increasing w and hence decreasing N, there is a reduction in the total area A_t associated with heat transfer from the fin array. However, for Cases A through C, the reduction in A_t is more than balanced by an increase in η_f (and η_o), causing a reduction in $R_{t,o}$ and hence an increase in q_c. As the fin efficiency approaches its limiting value of η_f = 1, reductions in A_t due to increasing w are no longer balanced by increases in η_f, and q_c begins to decrease. Hence there is an optimum value of w, which depends on L_f. For the conditions of this problem, L_f = 10 mm and w = 0.55 mm provide the largest heat dissipation.