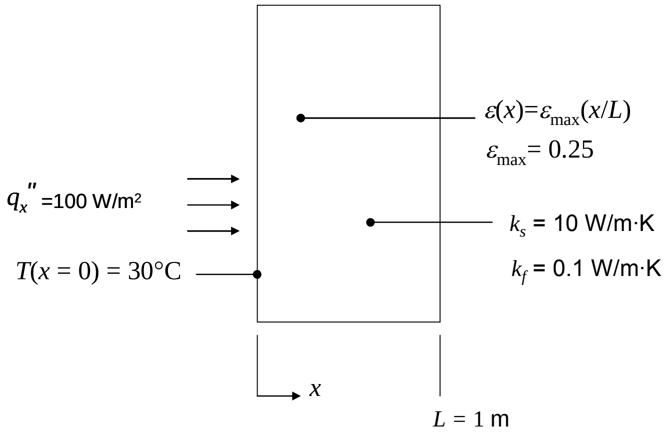
PROBLEM 3.38

KNOWN: Porosity distribution in a one-dimensional plane wall, $\varepsilon(x)$, thermal conductivities of solid and fluid, wall thickness, temperature at $x = 0$, and heat flux.

FIND: Plot of the temperature distribution using the expressions for the maximum and minimum effective thermal conductivities, Maxwell's expression, and for the constant property case; $k_{\text{eff}}(x) = k_s$.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties within the two phases, (2) Negligible radiation, (3) No thermal energy generation, (4) Steady state, one-dimensional heat transfer.

PROPERTIES: *Given*: k_s = 10 W/m·K, k_f = 0.1 W/m·K.

ANALYSIS: Fourier's law may be expressed as

$$\frac{dT}{dx} = -\frac{q_x''}{k_{\text{eff}}} \tag{1}$$

We note that the heat flux is constant. The effective thermal conductivity may be evaluated from the various formulae as follows.

Maximum effective thermal conductivity:

$$k_{\text{eff}} = k_{\text{eff,max}} = \varepsilon k_f + (1-\varepsilon)k_s \tag{2}$$

Minimum effective thermal conductivity:

$$k_{\text{eff}} = k_{\text{eff,min}} = \frac{1}{(1-\varepsilon)/k_s + \varepsilon/k_f} \tag{3}$$

Maxwell's expression:

$$k_{\text{eff}} = k_{\text{eff,Max}} = \left[\frac{k_f + 2k_s - 2\varepsilon(k_s - k_f)}{k_f + 2k_s + \varepsilon(k_s - k_f)}\right]k_s \tag{4}$$

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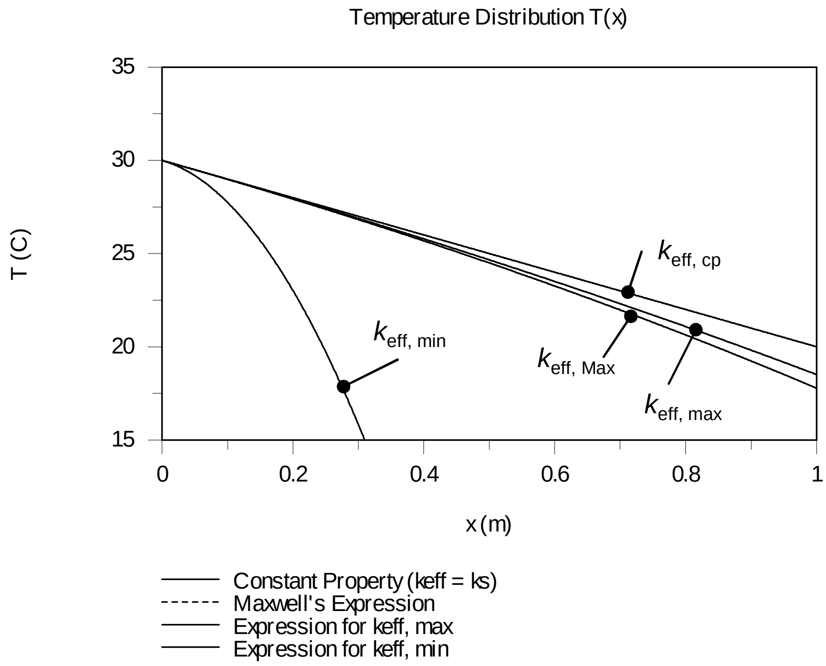
PROBLEM 3.38 (Cont.)

No dispersed phase:

$$k_{\text{eff}} = k_s \tag{5}$$

Equations 2, 3, 4, or 5 may be substituted into Equation 1 and the expression may be integrated numerically using a commercial code. IHT was used to generate the following temperature distributions.



The constant property solution exhibits a linear temperature distribution.

The introduction of the low thermal conductivity matter within the medium decreases its effective thermal conductivity. From Equation (1), the temperature gradient must become larger in order to sustain the imposed heat flux as the effective thermal conductivity decreases. The increased temperature gradients are evident in the plot.

Predictions using Maxwell's expression, and the expression for the maximum effective thermal conductivity are in relatively good agreement. This is because Maxwell's expression describes conduction within a porous medium that is characterized by a contiguous solid phase; thermal energy can be transferred across the entire thickness of the plane wall within the solid phase only. The concept of a contiguous solid phase is also embedded in the assumptions that were made in deriving the expression for $k_{\text{eff,max}}$. In contrast, the predictions using the expression for the minimum effective thermal conductivity are not consistent with the other predictions. In deriving the expression for $k_{\text{eff,min}}$, it is assumed that thermal energy *must* be transferred through the low thermal conductivity fluid phase as it propagates through the porous wall. The non-contiguous solid phase associated with the minimum thermal conductivity expression manifests itself as very large temperature gradients through the plane wall.

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COMMENTS: (1) It is important to be cognizant of the morphology of the porous medium before selecting the appropriate expression for the effective thermal conductivity. (2) The IHT code is shown below.

```
//Input phase properties, dimensions, thermal boundary condition, and porosity parameter.

ks = 10            //W/mK
kf = 0.1           //W/mK
L = 1              //m
emax = 0.25        //dimensionless
qflux = 100        //W/m^2
T1 = 30            //Degrees C
eps = emax*(x/L)

//Constant Property Solution
Der(Tcp,x) = -qflux/ks

//Minimum Effective Thermal Conductivity Solution
Der(Tmin,x) = -qflux/keffmin
keffmin = 1/dena
dena=(1-eps)/ks + eps/kf

//Maximum Effective Thermal Conductivity Solution
Der(Tmax,x) = -qflux/keffmax
keffmax = eps*kf + (1 - eps)*ks

//Maxwell's Expression
Der(TMax,x) = -qflux/keffMax
keffMax = ks*num/denb
num = kf+2*ks-2*eps*(ks-kf)
denb = kf+2*ks+eps*(ks-kf)
```