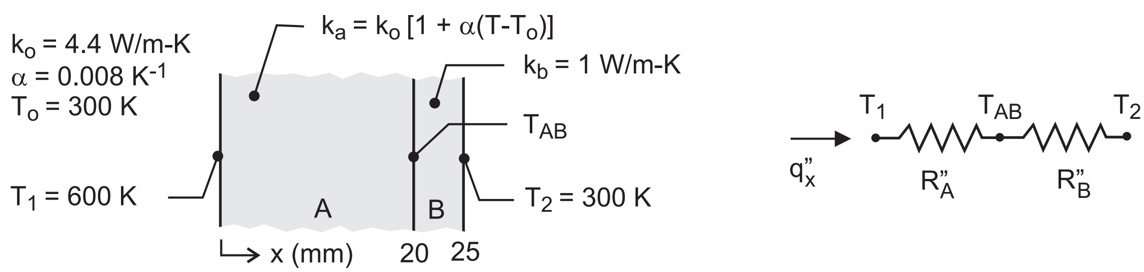
PROBLEM 4.94

KNOWN: Plane composite wall with exposed surfaces maintained at fixed temperatures. Material A has temperature-dependent thermal conductivity.

FIND: Heat flux through the wall (a) assuming a uniform thermal conductivity in material A evaluated at the average temperature of the section, and considering the temperature-dependent thermal conductivity of material A using (b) a finite-difference method of solution in IHT with a space increment of 1 mm and (c) the finite-element method of FEHT.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) No thermal contact resistance between the materials, and (3) No internal generation.

ANALYSIS: (a) From the thermal circuit in the above schematic, the heat flux is

$$q''_x = \frac{T_1 - T_2}{R''_A + R''_B} = \frac{T_{AB} - T_2}{R''_B} \qquad (1, 2)$$

and the thermal resistances of the two sections are

$$R''_A = L_A / k_A \qquad\qquad R''_B = L_B / k_B \qquad (3, 4)$$

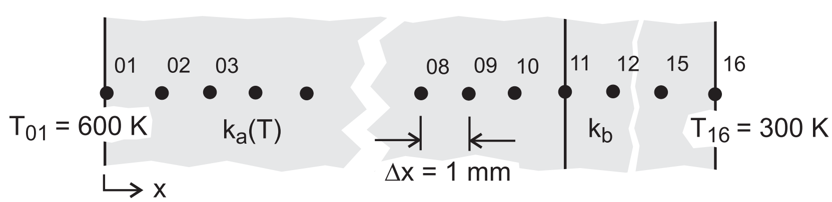
The thermal conductivity of material A is evaluated at the average temperature of the section

$$k_A = k_o \left\{1 + \alpha\left[\left(T_1 + T_{AB}\right)/2 - T_o\right]\right\} \qquad (5)$$

Substituting numerical values and solving the system of equations simultaneously in IHT, find

$$T_{AB} = 563.2 \text{ K} \qquad\qquad q''_x = 52.64 \text{ kW/m}^2 \qquad <$$

(b) The nodal arrangement for the finite-difference method of solution is shown in the schematic below. FDEs must be written for the internal nodes (02 – 10, 12 – 15) and the A-B interface node (11) considering in section A, the temperature-dependent thermal conductivity.



Interior Nodes, Section A (m = 02, 03 … 10)
Referring to the schematic below, the energy balance on node *m* is written in terms of the heat fluxes at the control surfaces using Fourier's law with the thermal conductivity based upon the average temperature of adjacent nodes. The heat fluxes into node *m* are

Continued …

$$q''_c = k_a(m, m+1)\frac{T_{m+1} - T_m}{\Delta x} \tag{1}$$

$$q''_d = k_a(m-1, m)\frac{T_{m-1} - T_m}{\Delta x} \tag{2}$$

and the FDEs are obtained from the energy balance written as

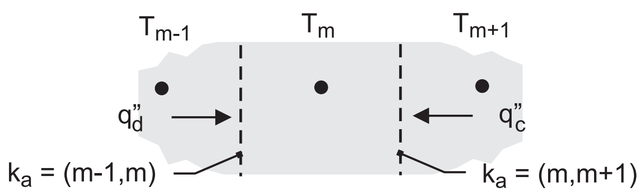
$$q''_c + q''_d = 0 \tag{3}$$

$$k_a(m, m+1)\frac{T_{m+1} - T_m}{\Delta x} + k_a(m-1, m)\frac{T_{m-1} - T_m}{\Delta x} = 0 \tag{4}$$

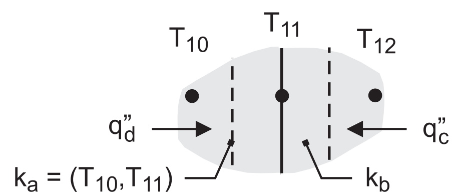
where the thermal conductivities averaged over the path between the nodes are expressed as

$$k_a(m-1, m) = k_o\left\{1 + \alpha\left[\left(T_{m-1} + T_m\right)/2 - T_o\right]\right\} \tag{5}$$

$$k_a(m, m+1) = k_o\left\{1 + \alpha\left[\left(T_m + T_{m+1}\right)/2 - T_o\right]\right\} \tag{6}$$



Interior nodes, Section A



A-B interface node

A-B Interface Node 11

Referring to the above schematic, the energy balance on the interface node, $q''_c + q''_d = 0$, has the form

$$k_b\frac{T_{12} - T_{11}}{\Delta x} + k_a(10, 11)\frac{T_{10} - T_{11}}{\Delta x} = 0 \tag{7}$$

where the thermal conductivity in the section A path is

$$k(10, 11) = k_o\left\{1 + \left[\left(T_{10} + T_{11}\right)/2 - T_o\right]\right\} \tag{8}$$

Interior Nodes, Section B (n = 12 …15)

Since the thermal conductivity in Section B is uniform, the FDEs have the form

$$T_n = \left(T_{n-1} + T_{n+1}\right)/2 \tag{9}$$

And the heat flux in the x-direction is

$$q''_x = k_b\frac{T_n - T_{n+1}}{\Delta x} \tag{10}$$

Finite-Difference Method of Solution

The foregoing FDE equations for section A nodes (m = 02 to 10), the AB interface node and their respective expressions for the thermal conductivity, k (m, m +1), and for section B nodes are entered into the IHT workspace and solved for the temperature distribution. The heat flux can be evaluated using Eq. (2) or (10). A portion of the IHT code is contained in the Comments, and the results of the analysis are tabulated below.

$T_{11} = T_{AB} = 563.2\text{ K}$ $\qquad q''_x = 52.64\text{ kW/m}^2$ $\qquad <$

Continued …

PROBLEM 4.94 (Cont.)

(c) The finite-element method of FEHT can be used readily to obtain the heat flux considering the temperature-dependent thermal conductivity of section A. Draw the composite wall outline with properly scaled section thicknesses in the x-direction with an arbitrary y-direction dimension. In the *Specify* | *Materials Properties* box for the thermal conductivity, specify k_a as 4.4*[1 + 0.008*(T – 300)] having earlier selected *Set* | *Temperatures in K*. The results of the analysis are

$$T_{AB} = 563\text{ K} \qquad q''_x = 52.6\text{ kW/m}^2 \qquad <$$

COMMENTS: (1) The results from the three methods of analysis compare very well. Because the thermal conductivity in section A is linear, and moderately dependent on temperature, the simplest method of using an overall section average, part (a), is recommended. This same method is recommended when using tabular data for temperature-dependent properties.

(2) For the finite-difference method of solution, part (b), the heat flux was evaluated at several nodes within section A and in section B with identical results. This is a consequence of the technique for averaging k_a over the path between nodes in computing the heat flux into a node.

(3) To illustrate the use of IHT in solving the finite-difference method of solution, lines of code for representative nodes are shown below.

```
// FDEs – Section A
k01_02 * (T01-T02)/deltax + k02_03 * (T03-T02)/deltax = 0
k01_02 = ko * (1+ alpha * ((T01 + T02)/2 – To))
k02_03 = ko * (1 + alpha * ((T02 + T03)/2 – To))

k02_03 * (T02 – T03)/deltax + k03_04 * (T04 – T03)/deltax = 0
k03_04 = ko * (1 + alpha * ((T03 + T04)/2 – To))

// Interface, node 11
k11 * (T10 –T11)/deltax + kb * (T12 –T11)/deltax =0
k11 = ko * (1 + alpha * ((T10 + T11)/2 – To))

// Section B (using Tools/FDE/One-dimensional/Steady-state)
/* Node 12: interior node; */
0.0 = fd_1d_int(T12, T13, T11, kb, qdot, deltax)
```