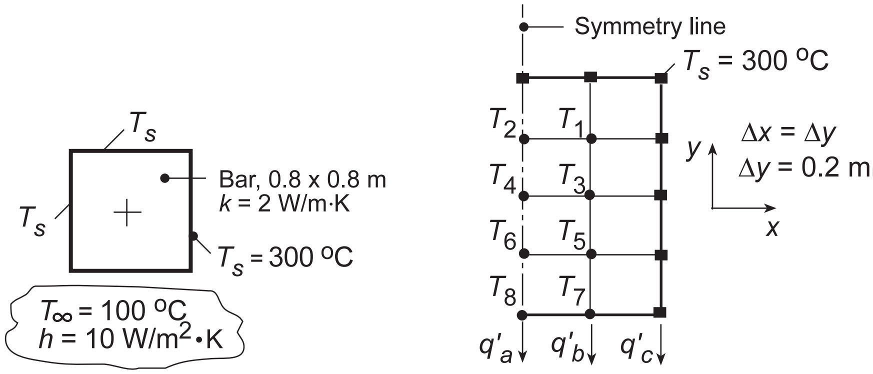
PROBLEM 4.65

KNOWN: Long bar of square cross section, three sides of which are maintained at a constant temperature while the fourth side is subjected to a convection process.

FIND: (a) The mid-point temperature and heat transfer rate between the bar and fluid; a numerical technique with grid spacing of 0.2 m is suggested, and (b) Reducing the grid spacing by a factor of 2, find the midpoint temperature and the heat transfer rate. Also, plot temperature distribution across the surface exposed to the fluid.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Considering symmetry, the nodal network is shown above. The matrix inversion method of solution will be employed. The finite-difference equations are:

Nodes 1, 3, 5 -	Interior nodes, Eq. 4.29; written by inspection.
Nodes 2, 4, 6 -	Also can be treated as interior points, considering symmetry.
Nodes 7, 8 -	On a plane with convection, Eq. 4.42; noting that $h\Delta x/k$ = 10 W/m^2·K × 0.2 m/2W/m·K = 1, find
	Node 7: $(2T_5 + 300 + T_8) + 2\times1\cdot100 - 2(1+2)T_7 = 0$
	Node 8: $(2T_6 + T_7 + T_7) + 2\times1\cdot100 - 2(1+2)T_8 = 0$

The solution matrix [T] can be found using a stock matrix program using the [A] and [C] matrices shown below to obtain the solution matrix [T] (Eq. 4.48). Alternatively, the set of equations could be entered into the IHT workspace and solved for the nodal temperatures.

$$A = \begin{bmatrix} -4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 & -6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & -6 \end{bmatrix} \qquad C = \begin{bmatrix} -600 \\ -300 \\ -300 \\ 0 \\ -300 \\ 0 \\ -500 \\ -200 \end{bmatrix} \qquad T = \begin{bmatrix} 292.2 \\ 289.2 \\ 279.7 \\ 272.2 \\ 254.5 \\ 240.1 \\ 198.1 \\ 179.4 \end{bmatrix}$$

From the solution matrix, [T], find the mid-point temperature as

T_4 = 272.2°C <

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The heat rate by convection between the bar and fluid is given as,

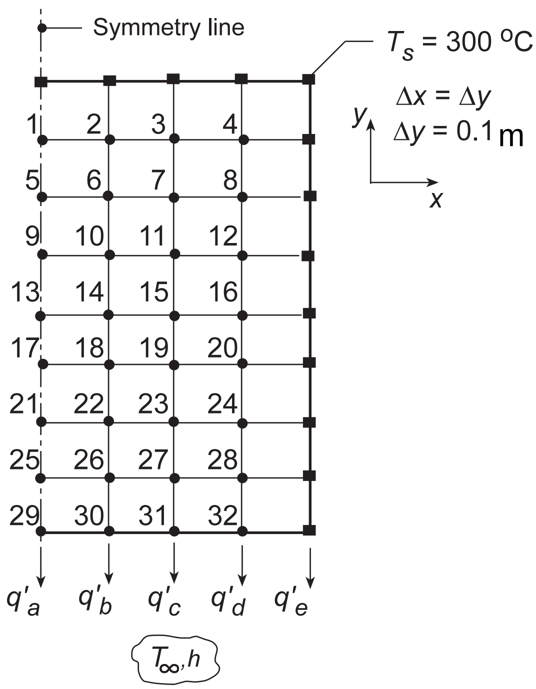
$$q'_{conv} = 2\left(q'_a + q'_b + q'_c\right)$$

$$q'_{conv} = 2\left[h\left(\Delta x/2\right)\left(T_8 - T_\infty\right) + h\left(\Delta x\right)\left(T_7 - T_\infty\right) + h\left(\Delta x/2\right)\left(300 - T_\infty\right)\right]$$

$$q'_{conv} = 2\left[10\,\mathrm{W/m^2\cdot K} \times \left(0.2\,\mathrm{m}/2\right)\left[\left(179.4-100\right) + 2\left(198.1-100\right) + \left(300-100\right)\right]\mathrm{K}\right]$$

$$q'_{conv} = 952\,\mathrm{W/m}\,.$$ <

(b) Reducing the grid spacing by a factor of 2, the nodal arrangement will appear as shown. The finite-difference equation for the interior and centerline nodes were written by inspection and entered into the IHT workspace. The *IHT Finite-Difference Equations Tool* for 2-D, SS conditions, was used to obtain the FDE for the nodes on the exposed surface.



The midpoint temperature T_{13} and heat rate for the finer mesh are

T_{13} = 271.0°C $\qquad\qquad$ q' = 834 W/m <

COMMENTS: The midpoint temperatures for the coarse and finer meshes agree closely, T_4 = 272°C vs. T_{13} = 271.0°C, respectively. However, the estimate for the heat rate is substantially influenced by the mesh size; q' = 952 vs. 834 W/m for the coarse and finer meshes, respectively.