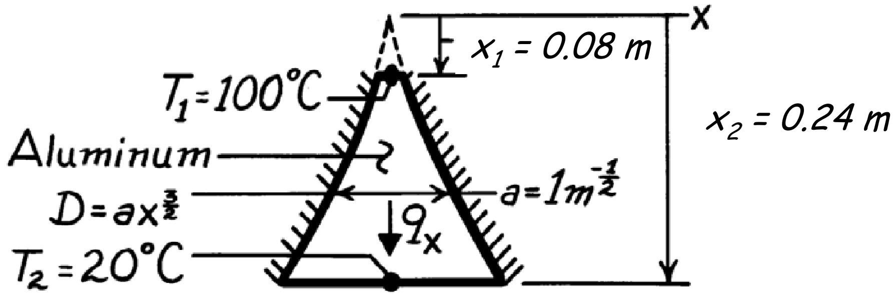
PROBLEM 3.40

KNOWN: Geometry and surface conditions of a truncated solid cone.

FIND: (a) Temperature distribution, (b) Rate of heat transfer across the cone.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum (333K): k = 238 W/m·K.

ANALYSIS: (a) From Fourier's law, Eq. 2.1, with $A=\pi D^2/4=\left(\pi a^2/4\right)x^3$, it follows that

$$\frac{4q_x dx}{\pi a^2 x^3} = -k dT.$$

Hence, since q_x is independent of x,

$$\frac{4q_x}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^3} = -k\int_{T_1}^{T} dT$$

or

$$\frac{4q_x}{\pi a^2}\left[-\frac{1}{2x^2}\right]\Bigg|_{x_1}^{x} = -k\left(T - T_1\right).$$

Hence

$$T = T_1 + \frac{2q_x}{\pi a^2 k}\left[\frac{1}{x^2}-\frac{1}{x_1^2}\right]. \qquad <$$

(b) From the foregoing expression, it also follows that

$$q_x = \frac{\pi a^2 k}{2}\,\frac{T_2 - T_1}{\left[1/x_2^2 - 1/x_1^2\right]}$$

$$q_x = \frac{\pi\left(1m^{-1}\right)238\ W/m\cdot K}{2}\times\frac{\left(20-100\right)^\circ C}{\left[\left(0.240\right)^{-2}-\left(0.08\right)^{-2}\right]m^{-2}}$$

$$q_x = 215\ W. \qquad <$$

COMMENTS: The foregoing results are approximate due to use of a one-dimensional model in treating what is inherently a two-dimensional problem.