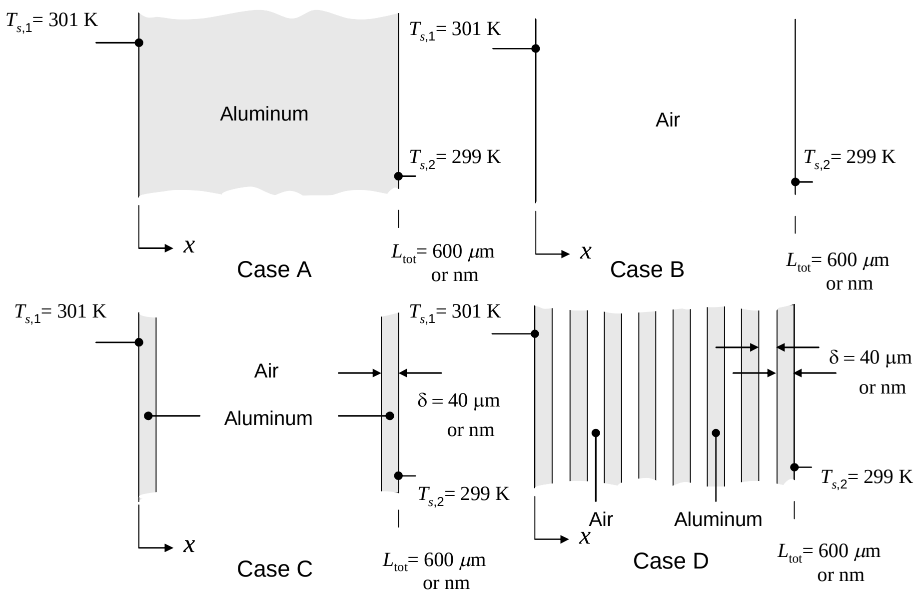
PROBLEM 3.175

KNOWN: Thickness of of parallel aluminum plates and air layers. Wall surface temperatures.

FIND: The conduction heat flux through (a) aluminum wall, (b) air layer, (c) air layer contained between two aluminum sheets and (d) composite wall consisting of 8 aluminum sheets and 7 air layers.

SCHEMATIC:



ASSUMPTIONS: (1) Ideal gas behavior. (2) Nanoscale effects within the solid are not important.

PROPERTIES: Table A.4 (T = 300 K): Air; c_p = 1007 J/kg·K, k_{Air} = 0.0263 W/m·K. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol, $d = 0.372 \times 10^{-9}$ m. Table A.1 (T = 300 K): Pure Aluminum, k_{Al} = 237 W/m·K.

ANALYSIS:

(a) Case A: Aluminum Wall

For L_{tot} = 600 μm, the heat flux is

$$q_x'' = \frac{k_{Al}\left(T_{s,1} - T_{s,2}\right)}{L_{tot}} = \frac{237 \text{ W/m}\cdot\text{K}\left(301 \text{ K} - 299 \text{ K}\right)}{600 \times 10^{-6} \text{ m}} = 7.9 \times 10^5 \text{ W/m}^2 \qquad <$$

Similarly, for L_{tot} = 600 nm, the heat flux is $q_x'' = 7.9 \times 10^8$ W/m^2 <

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(b) Case B: Air Layer

For L_{tot} = 600 μm, the heat flux is

$$q_x'' = \frac{k_{\text{Air}}\left(T_{s,1} - T_{s,2}\right)}{L_{\text{tot}}} = \frac{0.0263 \text{ W/m}\cdot\text{K}\left(301 \text{ K} - 299 \text{ K}\right)}{600\times10^{-6} \text{ m}} = 87.7 \text{ W/m}^2 \qquad <$$

Similarly, for L_{tot} = 600 nm, the heat flux is q_x'' = 87.7 × 10^3 W/m^2 <

(c) Case C: Air Layer between two Aluminum Sheets

This case involves a resistance due to molecule-molecule interactions, as well as molecule-surface collisions. For air, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315 \text{ kJ/kmol}\cdot\text{K}}{28.97 \text{ kg/kmol}} = 0.287 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.007\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.720\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad \gamma = \frac{c_p}{c_v} = \frac{1.007}{0.720} = 1.399$$

From Equation 2.11 the mean free path of air is

$$\lambda_{\text{mfp}} = \frac{k_B T}{\sqrt{2}\pi d^2 p} = \frac{1.381\times10^{-23} \text{ J/K}\times300 \text{ K}}{\sqrt{2}\pi\left(0.372\times10^{-9} \text{ m}\right)^2\left(1.0133\times10^5 \text{ N/m}^2\right)} = 66.5\times10^{-9} \text{ m} = 66.5 \text{ nm}$$

For L_{tot} = 600 μm, the air layer is $L = L_{\text{tot}} - 2\delta$ = 600 μm - 2 × 40 μm = 520 μm thick.

$$R''_{t,m-m} = \frac{L}{k_{\text{Air}}} = \frac{520\times10^{-6}\text{m}}{0.0263 \text{ W/m}\cdot\text{K}} = 0.0198 \text{ K}\cdot\text{m}^2\text{/W}$$

$$R''_{t,m-s} = \frac{\lambda_{\text{mfp}}}{k_{\text{Air}}}\left[\frac{2-\alpha_t}{\alpha_t}\right]\left[\frac{9\gamma-5}{\gamma+1}\right] = \frac{66.5\times10^{-9} \text{ m}}{0.0263 \text{ W/m}\cdot\text{K}}\left[\frac{2-0.92}{0.92}\right]\left[\frac{9\times1.399-5}{1.399+1}\right]$$
$$= 9.39\times10^{-6} \text{ K}\cdot\text{m}^2\text{/W}$$

In addition, the aluminum sheets pose a cumulative thermal resistance of

$$R''_{t,\text{cond}} = \frac{2\delta}{k_{\text{Al}}} = \frac{2\times40\times10^{-6} \text{ m}}{237 \text{ W/m}\cdot\text{K}} = 3.38\times10^{-7} \text{ K}\cdot\text{m}^2\text{/W}$$

Hence, the conduction heat flux is

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$$q_x'' = \frac{T_{s,1} - T_{s,2}}{\left(R_{t,m-m}'' + R_{t,m-s}'' + R_{t,\text{cond}}''\right)}$$

$$= \frac{301\text{ K} - 299\text{ K}}{\left(0.0198\text{ K}\cdot\text{m}^2/\text{W} + 9.39\times10^{-6}\text{ K}\cdot\text{m}^2/\text{W} + 3.38\times10^{-7}\text{ K}\cdot\text{m}^2/\text{W}\right)}$$ <

$$= 101\text{ W/m}^2$$

<u>For L_{tot} = 600 nm</u>, the air layer is $L = L_{\text{tot}} - 2\delta$ = 600 nm - 2 × 40 nm = 520 nm thick.

$$R_{t,m-m}'' = \frac{L}{k_{\text{Air}}} = \frac{520\times10^{-9}\text{m}}{0.0263\text{ W/m}\cdot\text{K}} = 19.8\times10^{-6}\text{ K}\cdot\text{m}^2/\text{W}$$

The aluminum sheets pose a cumulative thermal resistance of

$$R_{t,\text{cond}}'' = \frac{2\delta}{k_{\text{Al}}} = \frac{2\times40\times10^{-9}\text{ m}}{237\text{ W/m}\cdot\text{K}} = 3.38\times10^{-10}\text{ K}\cdot\text{m}^2/\text{W}$$

Hence, the conduction heat flux is

$$q_x'' = \frac{T_{s,1} - T_{s,2}}{\left(R_{t,m-m}'' + R_{t,m-s}'' + R_{t,\text{cond}}''\right)}$$

$$= \frac{301\text{ K} - 299\text{ K}}{\left(19.8\times10^{-6}\text{ K}\cdot\text{m}^2/\text{W} + 9.39\times10^{-6}\text{ K}\cdot\text{m}^2/\text{W} + 3.38\times10^{-10}\text{ K}\cdot\text{m}^2/\text{W}\right)}$$ <

$$= 68.6\times10^3\text{ W/m}^2$$

<u>(d) Case D: Seven Air Layers between Eight Aluminum Sheets</u>

This case involves multiple resistances due to molecule-molecule interactions, as well as molecule-surface collisions at multiple surfaces.

<u>For L_{tot} = 600 μm</u>, each air layer is $L = L_{\text{tot}} \times (1/15)$= 600 μm × (1/15) = 40 μm thick. Hence, for each air layer

$$R_{t,m-m}'' = \frac{L}{k_{\text{Air}}} = \frac{40\times10^{-6}\text{m}}{0.0263\text{ W/m}\cdot\text{K}} = 1.52\times10^{-3}\text{ K}\cdot\text{m}^2/\text{W}$$

In addition, the aluminum sheets pose a cumulative thermal resistance of

$$R_{t,\text{cond}}'' = \frac{8\delta}{k_{\text{Al}}} = \frac{8\times40\times10^{-6}\text{ m}}{237\text{ W/m}\cdot\text{K}} = 1.35\times10^{-6}\text{ K}\cdot\text{m}^2/\text{W}$$

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Problem 3.175 (Cont.)

Hence, the conduction heat flux is

$$q_x'' = \frac{T_{s,1} - T_{s,2}}{\left(7R_{t,m-m}'' + 7R_{t,m-s}'' + R_{t,\text{cond}}''\right)}$$

$$= \frac{301\text{ K} - 299\text{ K}}{\left(7\times 1.52\times 10^{-3}\text{ K}\cdot\text{m}^2/\text{W} + 7\times 9.39\times 10^{-6}\text{ K}\cdot\text{m}^2/\text{W} + 1.35\times 10^{-6}\text{ K}\cdot\text{m}^2/\text{W}\right)} \quad <$$

$$= 186.7\text{ W/m}^2$$

For L_{tot} = 600 nm, each air layer is $L = L_{\text{tot}} \times (1/15)$= 600 nm × (1/15) = 40 nm thick. Hence, for each air layer

$$R_{t,m-m}'' = \frac{L}{k_{\text{Air}}} = \frac{40\times 10^{-9}\text{m}}{0.0263\text{ W/m}\cdot\text{K}} = 1.52\times 10^{-6}\text{ K}\cdot\text{m}^2/\text{W}$$

In addition, the aluminum sheets pose a cumulative thermal resistance of

$$R_{t,\text{cond}}'' = \frac{8\delta}{k_{\text{Al}}} = \frac{8\times 40\times 10^{-9}\text{ m}}{237\text{ W/m}\cdot\text{K}} = 1.35\times 10^{-9}\text{ K}\cdot\text{m}^2/\text{W}$$

Hence, the conduction heat flux is

$$q_x'' = \frac{T_{s,1} - T_{s,2}}{\left(7R_{t,m-m}'' + 7R_{t,m-s}'' + R_{t,\text{cond}}''\right)}$$

$$= \frac{301\text{ K} - 299\text{ K}}{\left(7\times 1.52\times 10^{-6}\text{ K}\cdot\text{m}^2/\text{W} + 7\times 9.39\times 10^{-6}\text{ K}\cdot\text{m}^2/\text{W} + 1.35\times 10^{-9}\text{ K}\cdot\text{m}^2/\text{W}\right)} \quad <$$

$$= 26.2\times 10^3\text{ W/m}^2$$

The predicted heat fluxes are summarized below.

Case	L_{tot} = 600 μm	L_{tot} = 600 nm
A	$q_x'' = 7.9\times 10^5\text{ W/m}^2\cdot\text{K}$	$q_x'' = 7.9\times 10^8\text{ W/m}^2\cdot\text{K}$
B	$q_x'' = 87.7\text{ W/m}^2\cdot\text{K}$	$q_x'' = 87.7\times 10^3\text{ W/m}^2\cdot\text{K}$
C	$q_x'' = 101\text{ W/m}^2\cdot\text{K}$	$q_x'' = 68.6\times 10^3\text{ W/m}^2\cdot\text{K}$
D	$q_x'' = 186.7\text{ W/m}^2\cdot\text{K}$	$q_x'' = 26.2\times 10^3\text{ W/m}^2\cdot\text{K}$

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PROBLEM 3.175 (Cont.)

COMMENTS: (1) For the L_{tot} = 600 μm cases, it is readily evident that the highest heat flux corresponds to Case A in which conduction occurs exclusively through the high thermal conductivity aluminum. The lowest heat flux is associated with conduction through the pure air layer (Case B). For the case involving two aluminum sheets (Case C) the heat flux is increased relative to Case B primarily in response to replacing some low thermal conductivity air with high thermal conductivity metal. However, as more aluminum sheets are added, the thermal resistance across the entire layer is reduced, leading to increases in the heat flux for Case D. In each case, the resistance posed by molecule-surface interactions is not significant. Specifically, heat transfer rates for Cases C and D, calculated without accounting for the molecule-surface collisions, are 101 W/m^2 and 187.8 W/m^2, respectively.

(2) For the L_{tot} = 600 nm cases, we again observe that the largest heat flux is associated with conduction exclusively within the aluminum (Case A). *However, consideration of the other three cases reveals nanoscale behavior that would be unexpected from the macroscale point-of-view.* Specifically, Case B involving conduction through pure air is no longer characterized by the lowest heat flux. Rather, we observe that as more sheets of high thermal conductivity metal are added to the composite layer, the heat flux is *reduced*, with the minimum heat flux associated with the most aluminum sheets, Case D. The reduction in the conduction resistance due to the replacement of low thermal conductivity air with high thermal conductivity metal is more than offset with the increase in the total thermal resistance that is associated with molecule-surface interactions at the interfaces between the aluminum sheets and the air. The molecule-surface interactions can have a profound effect on nanoscale heat transfer.

(3) Nanoscale effects could become important in the solid as the thickness of the solid approaches the mean free path. See Table 2.1.