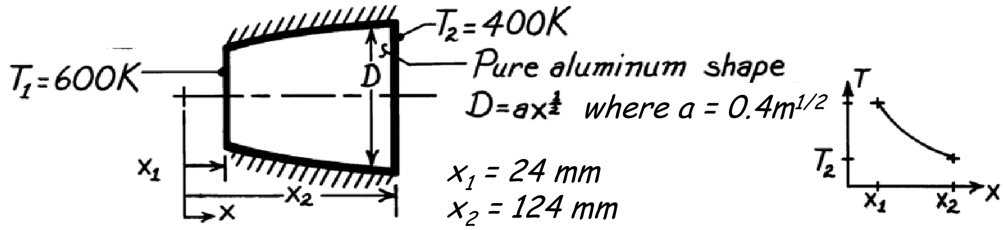
PROBLEM 3.39

KNOWN: Conduction in a conical section with prescribed diameter, D, as a function of x in the form $D = ax^{1/2}$.

FIND: (a) Temperature distribution, T(x), (b) Heat transfer rate, q_x.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation, (4) Constant properties.

PROPERTIES: *Table A-1*, Pure Aluminum (500K): k= 236 W/m·K.

ANALYSIS: (a) Based upon the assumptions, and following the same methodology of Example 3.4, q_x is a constant independent of x. Accordingly,

$$q_x = -kA\frac{dT}{dx} = -k\left[\pi\left(ax^{1/2}\right)^2/4\right]\frac{dT}{dx} \qquad (1)$$

using $A = \pi D^2/4$ where $D = ax^{1/2}$. Separating variables and identifying limits,

$$\frac{4q_x}{\pi a^2 k}\int_{x_1}^{x}\frac{dx}{x} = -\int_{T_1}^{T} dT. \qquad (2)$$

Integrating and solving for T(x) and then for T_2,

$$T(x) = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x}{x_1} \qquad T_2 = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x_2}{x_1}. \qquad (3,4)$$

Solving Eq. (4) for q_x and then substituting into Eq. (3) gives the results,

$$q_x = -\frac{\pi}{4}a^2 k\left(T_1 - T_2\right)/\ln\left(x_1/x_2\right) \qquad (5)$$

$$T(x) = T_1 + \left(T_1 - T_2\right)\frac{\ln\left(x/x_1\right)}{\ln\left(x_1/x_2\right)}. \qquad <$$

From Eq. (1) note that (dT/dx)·x = Constant. It follows that T(x) has the distribution shown above.

(b) The heat rate follows from Eq. (5),

$$q_x = -\frac{\pi}{4}\times\left(0.4^2\right)m\times 236\frac{W}{m\cdot K}\left(600-400\right)K/\ln\frac{24}{124} = 3.61kW. \qquad <$$