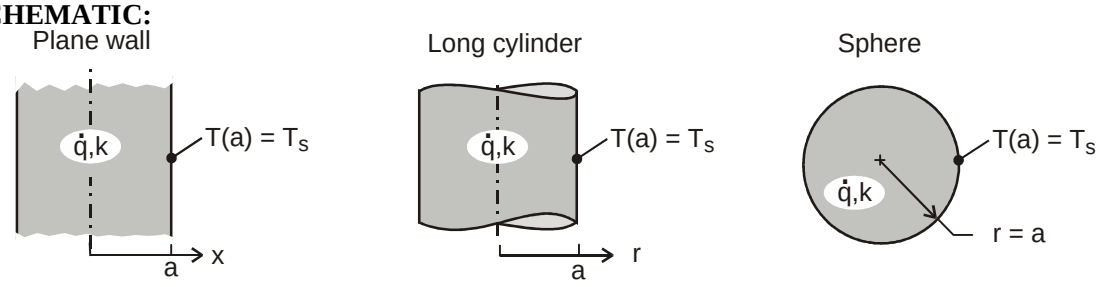
PROBLEM 3.106

KNOWN: Plane wall, long cylinder and sphere, each with characteristic length a, thermal conductivity k and uniform volumetric energy generation rate $\dot{q}$.

FIND: (a) On the same graph, plot the dimensionless temperature, $[T(x \text{ or } r) - T(a)]/[\dot{q}\, a^2/2k]$, *vs.* the dimensionless characteristic length, x/a or r/a, for each shape; (b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area; and (c) Which shape would be preferred for use as a nuclear fuel element? Explain why?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties and (4) Uniform volumetric generation.

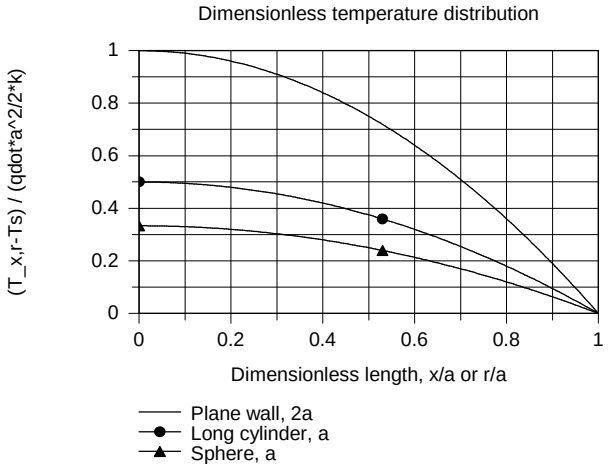
ANALYSIS: (a) For each of the shapes, with $T(a) = T_s$, the dimensionless temperature distributions can be written by inspection from results in Appendix C.3.

Plane wall, Eq. C.22
$$\frac{T(x) - T_s}{\dot{q}a^2/2k} = 1 - \left(\frac{x}{a}\right)^2$$

Long cylinder, Eq. C.23
$$\frac{T(r) - T_s}{\dot{q}a^2/2k} = \frac{1}{2}\left[1 - \left(\frac{r}{a}\right)^2\right]$$

Sphere, Eq. C.24
$$\frac{T(r) - T_s}{\dot{q}a^2/2k} = \frac{1}{3}\left[1 - \left(\frac{r}{a}\right)^2\right]$$

The dimensionless temperature distributions using the foregoing expressions are shown in the graph below.



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PROBLEM 3.106 (Cont.)

(b) The sphere shape has the smallest temperature difference between the center and surface, T(0) – T(a). The ratio of volume-to-surface-area, $\forall/A_s$, for each of the shapes is

Plane wall
$$\frac{\forall}{A_s} = \frac{a(1\times 1)}{(1\times 1)} = a$$

Long cylinder
$$\frac{\forall}{A_s} = \frac{\pi a^2 \times 1}{2\pi a \times 1} = \frac{a}{2}$$

Sphere
$$\frac{\forall}{A_s} = \frac{4\pi a^3/3}{4\pi a^2} = \frac{a}{3}$$

The smaller the $\forall/A_s$ ratio, the smaller the temperature difference, T(0) – T(a).

(c) The sphere would be the preferred element shape since, for a given $\forall/A_s$ ratio, which controls the generation and transfer rates, the sphere will operate at the lowest temperature.