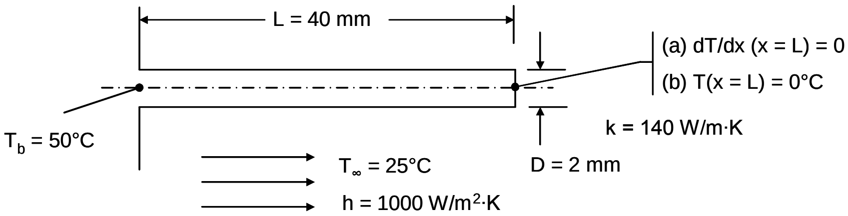
# PROBLEM 3.140

**KNOWN:** Dimension and length of an aluminum pin fin. Base and ambient temperatures, value of the convection heat transfer coefficient.

**FIND:** (a) Fin heat transfer rate with an adiabatic tip, (b) Fin heat transfer rate when the fin tip is cooled below the ambient temperature, (c) Temperature distribution along the fin for parts (a) and (b), (d) Fin heat rates for $0 \le h \le 1000\ W/m^2\cdot K$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Constant properties, (2) Steady-state conditions, (3) One-dimensional heat transfer, (4) Negligible radiation heat transfer.

**ANALYSIS:**

(a) The fin heat transfer rate is given by Eq. 3.81; $q_f = M \tanh ml$ where

$$M = \sqrt{hPkA_c}\ \theta_b$$
$$= \sqrt{1000 W/m^2 \cdot K \times \pi \times 2 \times 10^{-3}\ m \times 140\ W/m \cdot K \times \pi \times (2 \times 10^{-3}\ m)^2/4} \times (50 - 25)°C$$
$$= 1.314\ W$$

and

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{1000\ W/m^2 \cdot K \times \pi \times 2 \times 10^{-3}\ m}{140\ W/m^2 \cdot K \times \pi \times (2 \times 10^{-3}\ m)^2/4}} = 119.5\ m^{-1}$$

Therefore, $q_f = 1.314\ W \tanh (119.5\ m^{-1} \times 40 \times 10^{-3}\ m) = 1.314\ W$ <

(b) For the case where T(x = L) = 0°C, the fin heat transfer rate is

$$q_f = M \frac{(\cosh ml - \theta_L/\theta_b)}{\sinh ml}$$

$$= 1.314\ W \times \frac{\cosh (119.5\ m^{-1} \times 40 \times 10^{-3}\ m) - (0 - 25)°C / (50 - 25)°C}{\sinh (119.5\ m^{-1} \times 40 \times 10^{-3}\ m)} = 1.336\ W$$ <

(c) The temperature distributions are found by plotting Eqs. 3.80 and 3.82 over the range $0 \le x \le 40$ mm. Note, that for the adiabatic tip case, the tip temperature is nearly equal to the

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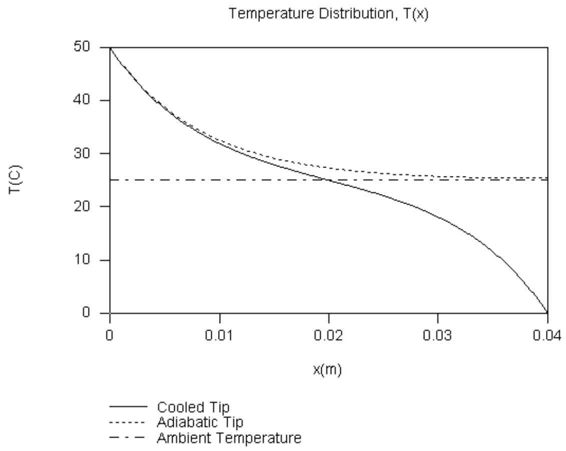
## PROBLEM 3.140 (Cont.)

ambient temperature. For the cooled tip, the temperature distribution is anti-symmetric about x = ½ L. For the cooled tip case and h = 0, the temperature distribution in the fin would be linear, corresponding to one-dimensional conduction in the fin. <




(d) The fin heat rate distributions are shown below. For adiabatic tip and h = 0, $q_f = 0$. For the case of the cooled tip and negligible convection, the fin heat rate is

$$q_f = kA_c(T(x{=}L) - T_b) / L = \left(140\ W/m^2 \cdot K \times \pi \times (2 \times 10^{-3}\ m)^2/ 4\right) \times \left((0 - 50)°C / 40 \times 10^{-3}\ m\right)$$
$= 0.549$ W.

As the convection coefficient increases, the temperatures at x = ½ L approach T(x = ½ L) = 25°C for both the adiabatic and cooled tip cases, resulting in nearly the same fin heat transfer rates. Equations 3.76 and 3.78 would yield the same result for the cooled tip case since heat lost by convection over the range $0 \le x \le 20$ mm would be exactly offset by the heat gain by convection over the range $20\ mm \le x \le 40$ mm, and heat loss at x = L by conduction is equal to heat gain at x = 0 by conduction. <