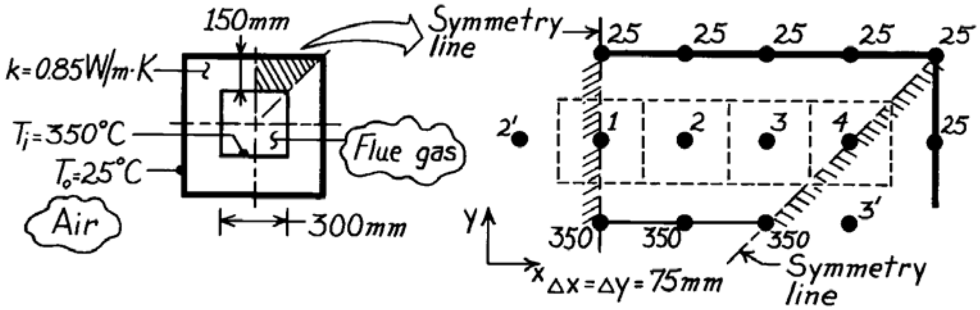
PROBLEM 4.54

KNOWN: Flue of square cross section with prescribed geometry, thermal conductivity and inner and outer surface temperatures.

FIND: Heat loss per unit length from the flue, q'.

SCHEMATIC:



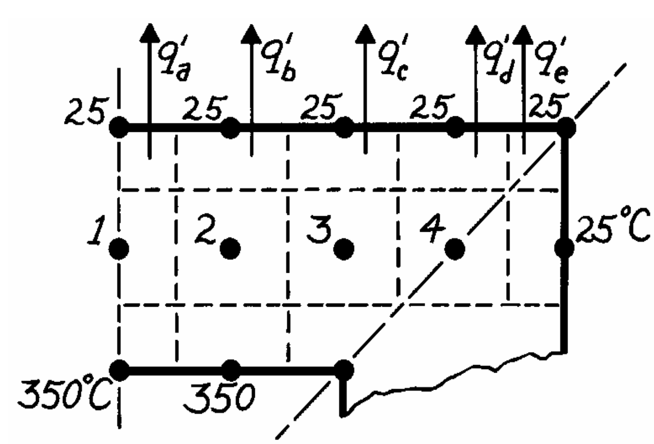
ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) No internal generation.

ANALYSIS: Taking advantage of symmetry, the nodal network using the suggested 75mm grid spacing is shown above. To obtain the heat rate, we first need to determine the unknown temperatures T_1, T_2, T_3 and T_4. Recognizing that these nodes may be treated as interior nodes, the nodal equations from Eq. 4.29 are

$$(T_2 + 25 + T_2 + 350) - 4T_1 = 0$$
$$(T_1 + 25 + T_3 + 350) - 4T_2 = 0$$
$$(T_2 + 25 + T_4 + 350) - 4T_3 = 0$$
$$(T_3 + 25 + 25 + T_3) - 4T_4 = 0.$$

Simultaneous solution yields $T_1 = 183.9°C$, $T_2 = 180.3°C$, $T_3 = 162.2°C$, $T_4 = 93.6°C$ <

From knowledge of the temperature distribution, the heat rate may be obtained by summing the heat rates across the nodal control volume surfaces, as shown in the sketch.



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The heat rate leaving the outer surface of this flue section is,

$$q' = q'_a + q'_b + q'_c + q'_d + q'_e$$
$$q' = k\frac{\Delta x}{\Delta y}\left[\frac{1}{2}(T_1 - 25) + (T_2 - 25) + (T_3 - 25) + (T_4 - 25) + 0\right]$$
$$q' = 0.85\frac{W}{m \cdot K}\left[\frac{1}{2}(183.9 - 25) + (180.3 - 25) + (162.2 - 26) + (93.6 - 25)\right]$$
$$q' = 374.5 \text{ W/m.}$$

Since this flue section is 1/8 the total cross section, the total heat loss from the flue is

$$q' = 8 \times 374.5 \text{ W/m} = 3.00 \text{ kW/m.}$$ <

COMMENTS: (1) The heat rate could have been calculated at the inner surface, and from the above sketch has the form

$$q' = k\frac{\Delta x}{\Delta y}\left[\frac{1}{2}(350 - T_1) + (350 - T_2) + (350 - T_3)\right] = 374.5 \text{ W/m.}$$

This result should compare very closely with that found for the outer surface since the conservation of energy requirement must be satisfied in obtaining the nodal temperatures.
(2) The Gauss-Seidel iteration method can be used to find the nodal temperatures. Following the procedures of Appendix D,

$$T_1^k = 0.50\ T_2^{k-1} + 93.75$$
$$T_2^k = 0.25\ T_1^k + 0.25\ T_3^{k-1} + 93.75$$
$$T_3^k = 0.25\ T_2^k + 0.25\ T_4^{k-1} + 93.75$$
$$T_4^k = 0.50\ T_3^k + 12.5.$$

The iteration procedure is implemented in the table on the following page, one row for each iteration k. The initial estimates, for k = 0, are all chosen as (350 + 25)/2 ≈ 185°C. Iteration is continued until the maximum temperature difference is less than 0.2°C, i.e., $\varepsilon < 0.2$°C.

Note that if the system of equations were organized in matrix form, Eq. 4.48, diagonal dominance would exist. Hence there is no need to reorder the equations since the magnitude of the diagonal element is greater than that of other elements in the same row.

k	T_1(°C)	T_2(°C)	T_3(°C)	T_4(°C)	
0	185	185	185	185	← initial estimate
1	186.3	186.6	186.6	105.8	
2	187.1	187.2	167.0	96.0	
3	187.4	182.3	163.3	94.2	
4	184.9	180.8	162.5	93.8	
5	184.2	180.4	162.3	93.7	
6	184.0	180.3	162.3	93.6	
7	183.9	180.3	162.2	93.6	← ε <0.2°C

The nodal temperatures are the same as those calculated using the simultaneous solution.