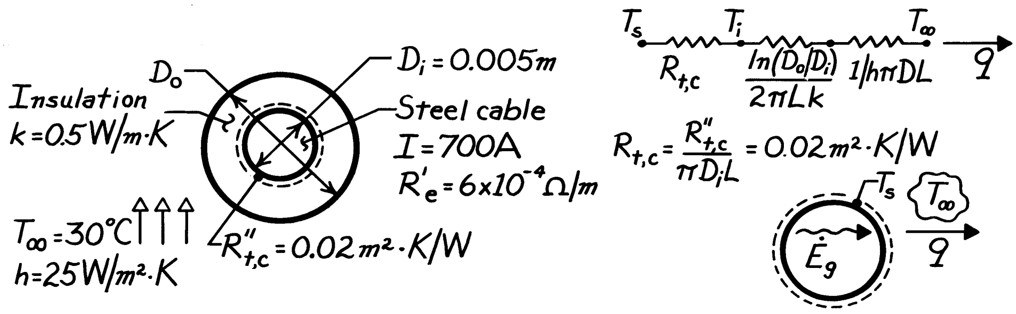
PROBLEM 3.58

KNOWN: Electric current flow, resistance, diameter and environmental conditions associated with a cable.

FIND: (a) Surface temperature of bare cable, (b) Cable surface and insulation temperatures for a thin coating of insulation, (c) Insulation thickness which provides the lowest value of the maximum insulation temperature. Corresponding value of this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: (a) The rate at which heat is transferred to the surroundings is fixed by the rate of heat generation in the cable. Performing an energy balance for a control surface about the cable, it follows that $\dot{E}_g = q$ or, for the bare cable, $I^2R'_eL=h(\pi\ D_iL)(T_s - T_\infty)$. With $q'=I^2R'_e = (700A)^2\left(6\times10^{-4}\Omega/m\right) = 294\ W/m$, it follows that

$$T_s = T_\infty + \frac{q'}{h\pi\ D_i} = 30°C + \frac{294\ W/m}{\left(25\ W/m^2\cdot K\right)\pi(0.005m)}$$

$$T_s = 778.7°C.$$ <

(b) With a thin coating of insulation, there exist contact and convection resistances to heat transfer from the cable. The heat transfer rate is determined by heating within the cable, however, and therefore remains the same.

$$q=\frac{T_s - T_\infty}{R_{t,c} + \dfrac{1}{h\pi\ D_iL}} = \frac{T_s - T_\infty}{\dfrac{R''_{t,c}}{\pi\ D_iL} + \dfrac{1}{h\pi\ D_iL}}$$

$$q'=\frac{\pi\ D_i(T_s - T_\infty)}{R''_{t,c} + 1/h}$$

and solving for the surface temperature, find

$$T_s = \frac{q'}{\pi\ D_i}\left[R''_{t,c} + \frac{1}{h}\right] + T_\infty = \frac{294\ W/m}{\pi(0.005m)}\left[0.02\frac{m^2\cdot K}{W} + 0.04\frac{m^2\cdot K}{W}\right] + 30°C$$

$$T_s = 1153°C.$$ <

Continued …

The insulation temperature is then obtained from

$$q = \frac{T_s - T_i}{R_{t,c}}$$

or

$$T_i = T_s - qR_{t,c} = 1153^\circ C - q\frac{R''_{t,c}}{\pi D_i L} = 1153^\circ C - \frac{294\frac{W}{m} \times 0.02\frac{m^2 \cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 778.7^\circ C. \qquad <$$

(c) The maximum insulation temperature could be reduced by reducing the resistance to heat transfer from the outer surface of the insulation. Such a reduction is possible if $D_i < D_{cr}$. From Example 3.6,

$$r_{cr} = \frac{k}{h} = \frac{0.5\ W/m \cdot K}{25\ W/m^2 \cdot K} = 0.02m.$$

Hence, $D_{cr} = 0.04m > D_i = 0.005m$. To minimize the maximum temperature, which exists at the inner surface of the insulation, add insulation in the amount

$$t = \frac{D_o - D_i}{2} = \frac{D_{cr} - D_i}{2} = \frac{(0.04 - 0.005)m}{2}$$

$$t = 0.0175m. \qquad <$$

The cable surface temperature may then be obtained from

$$q' = \frac{T_s - T_\infty}{\frac{R''_{t,c}}{\pi D_i} + \frac{\ln(D_{cr}/D_i)}{2\pi k} + \frac{1}{h\pi D_{cr}}} = \frac{T_s - 30^\circ C}{\frac{0.02\ m^2 \cdot K/W}{\pi(0.005m)} + \frac{\ln(0.04/0.005)}{2\pi(0.5\ W/m \cdot K)} + \frac{1}{25\frac{W}{m^2 \cdot K}\pi(0.04m)}}$$

Hence,

$$294\frac{W}{m} = \frac{T_s - 30^\circ C}{(1.27+0.66+0.32)m \cdot K/W} = \frac{T_s - 30^\circ C}{2.25\ m \cdot K/W}$$

$$T_s = 692.5^\circ C$$

Recognizing that $q = (T_s - T_i)/R_{t,c}$, find

$$T_i = T_s - qR_{t,c} = T_s - q\frac{R''_{t,c}}{\pi D_i L} = 692.5^\circ C - \frac{294\frac{W}{m} \times 0.02\frac{m^2 \cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 318.2^\circ C. \qquad <$$

COMMENTS: Use of the critical insulation thickness in lieu of a thin coating has the effect of reducing the maximum insulation temperature from 778.7°C to 318.2°C. Use of the critical insulation thickness also reduces the cable surface temperature to 692.5°C from 778.7°C with no insulation or from 1153°C with a thin coating.