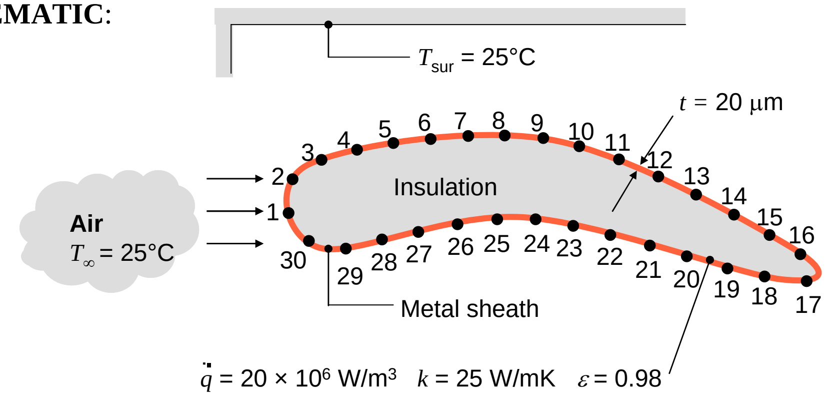
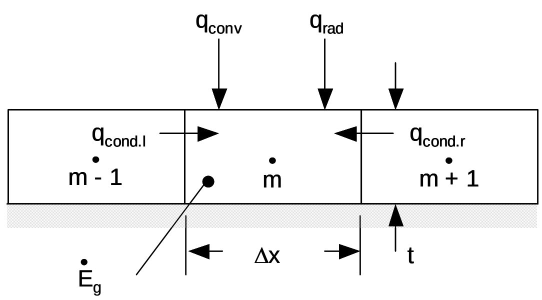
PROBLEM 4.71

KNOWN: Geometry of long airfoil shape, thickness of metal sheathing, volumetric generation rate within the sheathing, thermal conductivity of sheathing, emissivity of the sheathing, and measured temperatures at discrete locations.

FIND: Local convection coefficient at discrete locations accounting for conduction along the sheathing and radiation. Determine effects of conduction and radiation on the calculated convection heat transfer coefficients.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform internal generation, (4) Metal sheathing is very thin relative to cylinder diameter, (5) One-dimensional conduction, (6) Large surroundings.

ANALYSIS: We apply Newton's law of cooling, Fourier's law and Eq. 1.7 to a general control volume about the thin sheathing to find the following general finite-difference formula.

$$q'_{\text{cond,l}} + q'_{\text{cond,r}} + q'_{\text{conv}} + q'_{\text{rad}} + \dot{E}'_{\text{g}} = 0$$

or

$$k\frac{(T_{m-1}-T_m)}{\Delta x}t + k\frac{(T_{m+1}-T_m)}{\Delta x}t + h_m(\Delta x)(T_\infty - T_m) + \varepsilon\sigma(\Delta x)(T_{\text{sur}}^4 - T_m^4) + \dot{q}(\Delta x)t = 0$$

Recognizing that for $m = 1$, $m - 1 = 30$, we may substitute values of T_{m-1}, T_m, and T_{m+1} and $\Delta x = 2$ mm into the preceding formula and solve for h_m. The results for case A (inclusion of convection, conduction and radiation), case B (inclusion of convection and conduction only) and case C (inclusion of convection only) are tabulated below.

Continued…

PROBLEM 4.71 (Cont.)

m	T_m (°C)	$h_{m,A}$ ($W/m^2 \cdot K$)	$h_{m,B}$ ($W/m^2 \cdot K$)	$h_{m,C}$ ($W/m^2 \cdot K$)
1	27.77	162.8	<u>168.8</u>	*144.4*
2	27.67	150.4	<u>156.4</u>	*149.8*
3	27.71	*145.3*	<u>151.3</u>	147.6
4	27.83	*140.2*	<u>146.2</u>	141.3
5	28.06	132.1	<u>138.1</u>	*130.7*
6	28.47	*112.9*	<u>118.9</u>	115.3
7	28.98	*100.1*	<u>106.2</u>	100.5
8	29.67	87.66	<u>93.68</u>	*85.65*
9	30.66	76.32	<u>82.38</u>	*70.67*
10	32.18	59.88	<u>65.98</u>	*55.71*
11	34.29	*42.01*	<u>48.17</u>	43.06
12	36.78	*27.93*	<u>34.17</u>	33.96
13	39.29	*19.14*	25.45	<u>27.99</u>
14	41.51	*9.89*	16.28	<u>24.23</u>
15	42.68	*9.06*	15.48	<u>22.62</u>
16	42.84	*4.01*	10.44	<u>22.42</u>
17	41.29	*3.98*	10.36	<u>24.55</u>
18	37.89	*24.95*	<u>31.23</u>	31.03
19	34.51	52.06	<u>58.23</u>	*42.06*
20	32.36	63.87	<u>69.97</u>	*54.35*
21	31.13	74.28	<u>80.34</u>	*65.25*
22	30.64	74.84	<u>80.90</u>	*70.92*
23	30.60	*70.08*	<u>76.12</u>	71.43
24	30.77	*68.04*	74.09	69.32
25	31.16	*58.26*	64.33	<u>64.94</u>
26	31.52	*54.70*	60.77	<u>61.35</u>
27	31.85	*40.08*	46.17	<u>58.39</u>
28	31.51	*31.17*	37.25	<u>61.44</u>
29	29.91	*78.24*	<u>84.27</u>	81.47
30	28.42	141.7	<u>147.7</u>	*117.0*

Note: Maximum predicted values are <u>underlined</u>, while minimum predicted values are *italicized*.

COMMENTS: (1) The heat transfer coefficient distribution is non-uniform. Such non-uniformity is typical of situations involving convection around complex geometries. The largest heat transfer coefficients exist at the leading edge of the object, while the smallest values of h are near the trailing edge. (2) Differences in the measured values of the heat transfer coefficient evolve when different analyses are used to interpret the measured temperatures. (3) Inclusion of radiation in Analysis B always results in lower heat transfer coefficients (relative to Analysis A) since the object is hot relative to the large surroundings.