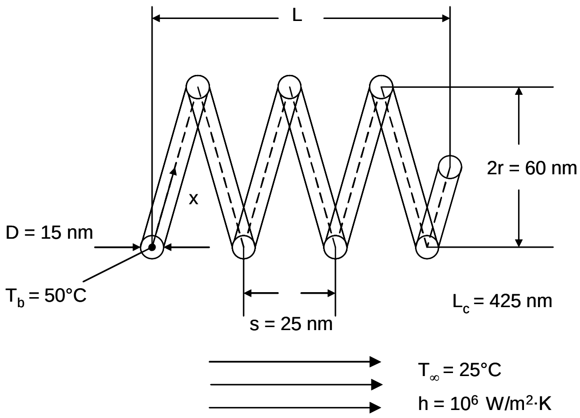
PROBLEM 3.138

KNOWN: Dimensions of a nanospring, dependence of pitch upon temperature.

FIND: Actuation distance of the spring in response to heating of its end, accuracy to which the actuation length can be controlled.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Steady-state conditions, (3) One-dimensional heat transfer, (4) Adiabatic tip, (5) Negligible radiation heat transfer, (6) Negligible impact of nanoscale heat transfer effects.

PROPERTIES: Table A.2, silicon carbide (300 K): k = 490 W/m·K.

ANALYSIS: When the nanospring is at T_i = 25°C, the spring length is

$$L_i = \frac{s}{2\pi}\frac{Lc}{\sqrt{r^2 + (S/2\pi)^2}} = \frac{25 \times 10^{-9}\ \text{m}}{2\pi} \times \frac{425 \times 10^{-9}\ \text{m}}{\sqrt{(30 \times 10^{-9}\ \text{m})^2 + \left(25 \times 10^{-9}\ \text{m})/2\pi\right)^2}}$$

$$= 55.9 \times 10^{-9}\ \text{m} = 55.9\ \text{nm}$$

Since the average spring pitch varies linearly with the average temperature, the average pitch of the heated spring is

$$\overline{s} = s_i + \frac{d\overline{s}}{d\overline{T}}(\overline{T} - T_i) \tag{1}$$

The average excess temperature is

$$\overline{\theta} = \overline{T} - T_\infty = \frac{1}{L_c}\int_{x=0}^{L_c} \theta(x)dx \quad \text{where, from Eq. 3.80,}$$

Continued…

PROBLEM 3.138 (Cont.)

$$\bar{\theta} = \frac{\theta_b}{L_c}\int_{x=0}^{L_c}\frac{\cosh m(L - x)}{\cosh ml}\,dx = -\frac{\theta_b}{mL_c(\cosh mL_c)}\sinh m(L_c - x)\Big|_0^{L_c}$$

$$\bar{\theta} = \frac{\theta_b}{mL_c\ (\cosh mL_c)} \times (0 - \sinh mL_c) = \frac{\theta_b}{mL_c}\tanh(mL_c)$$

For a particular spring,

$$mL_c = \left(\frac{hP}{kA_c}\right)^{1/2} L_c = \left(\frac{4h}{kD}\right)^{1/2} L_c = \left(\frac{4 \times 10^6\ \text{W/m}^2 \cdot \text{K}}{490\ \text{W/m} \cdot \text{K} \times 15 \times 10^{-9}\ \text{m}}\right)^{1/2} \times 425 \times 10^{-9}\ \text{m} = 0.314$$

Therefore $\bar{\theta} = \dfrac{(50 - 25)°\text{C}}{0.314}\tanh(0.314) = 24.2°\text{C}$

and $\quad \bar{T} = \bar{\theta} + T_\infty = 24.2°\text{C} + 25°\text{C} = 49.2°\text{C}$

From Eq. (1),

$$\bar{s} = 25 \times 10^{-9}\ \text{m} + 0.1 \times 10^{-9}\ \text{m/K} \times (49.2 - 25)°\text{C} = 27.4 \times 10^{-9}\ \text{m}$$

Therefore,

$$L_2 = \frac{27.4 \times 10^{-9}\ \text{m}}{2\pi} \times \frac{425 \times 10^{-9}\ \text{m}}{\sqrt{(30 \times 10^{-9}\ \text{m})^2 + (27.4 \times 10^{-9}\ \text{m}/2\pi)^2}} = 61.1 \times 10^{-9}\ \text{m} = 61.1\ \text{nm}$$

and the actuation length is

$$\Delta L = L_2 - L_i = 61.1\ \text{nm} - 55.9\ \text{nm} = 5.2\ \text{nm} \qquad <$$

If the base temperature can be controlled to within 1 degree Celsius, the resolution of the actuation length is: $R = \Delta L \times \dfrac{1\ \text{degree C}}{25\ \text{degree C}} = 0.2\ \text{nm}$ $\qquad <$

COMMENTS: (1) The actuation distance and its resolution are extremely small. (2) Application of other tip conditions will lead to different predictions of the actuation distance.