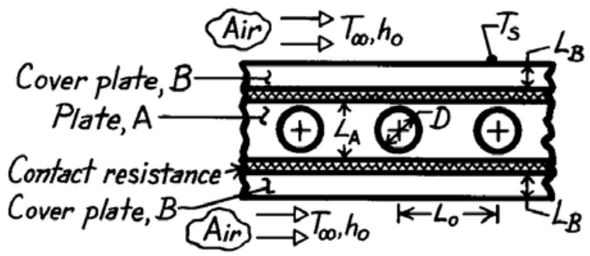
PROBLEM 4.27

KNOWN: Platen heated by passage of hot fluid in poor thermal contact with cover plates exposed to cooler ambient air.

FIND: (a) Heat rate per unit thickness from each channel, q_i', (b) Surface temperature of cover plate, T_s, (c) q_i' and T_s if lower surface is perfectly insulated, (d) Effect of changing centerline spacing on q_i' and T_s

SCHEMATIC:



$D = 15$ mm $\quad L_o = 60$ mm

$L_A = 30$ mm $\quad L_B = 7.5$ mm

$T_i = 150°C \quad h_i = 1000\ W/m^2 \cdot K$

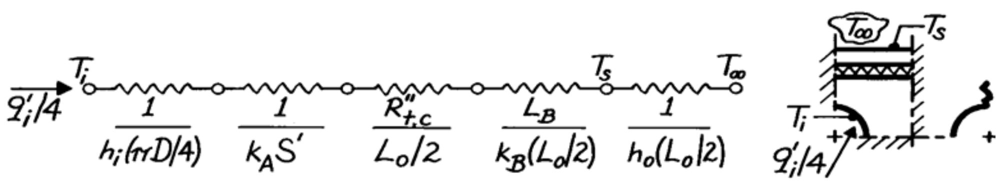
$T_\infty = 25°C \quad h_o = 200\ W/m^2 \cdot K$

$k_A = 20\ W/m \cdot K \quad k_B = 75\ W/m \cdot K$

$R''_{t,c} = 2.0 \times 10^{-4} m^2 \cdot K/W$

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction in platen, but one-dimensional in coverplate, (3) Temperature of interfaces between A and B is uniform, (4) Constant properties.

ANALYSIS: (a) The heat rate per unit thickness from each channel can be determined from the following thermal circuit representing the quarter section shown.



The value for the shape factor is $S' = 1.06$ as determined from the flux plot shown on the next page. Hence, the heat rate is

$$q_i' = 4\left(T_i - T_\infty\right)/R'_{tot} \tag{1}$$

$$\begin{aligned} R'_{tot} = [&1/1000\ W/m^2 \cdot K\left(\pi 0.015m/4\right) + 1/20\ W/m \cdot K \times 1.06 \\ &+ 2.0\times10^{-4} m^2 \cdot K/W/\left(0.060m/2\right) + 0.0075m/75\ W/m \cdot K\left(0.060m/2\right) \\ &+ 1/200\ W/m^2 \cdot K\left(0.060m/2\right)] \end{aligned}$$

$$R'_{tot} = \left[0.085 + 0.047 + 0.0067 + 0.0033 + 0.1667\right] m \cdot K/W$$
$$R'_{tot} = 0.309\ m \cdot K/W$$

$$q_i' = 4\left(150 - 25\right) K/0.309\ m \cdot K/W = 1.62\ kW/m. \quad <$$

(b) The surface temperature of the cover plate also follows from the thermal circuit as

$$q_i'/4 = \frac{T_s - T_\infty}{1/h_o\left(L_o/2\right)} \tag{2}$$

Continued …

PROBLEM 4.27 (Cont.)

$$T_s = T_\infty + \frac{q_i'}{4}\frac{1}{h_o(L_o/2)} = 25^\circ C + \frac{1.62\ \text{kW}}{4} \times 0.167\ \text{m} \cdot \text{K/W}$$

$$T_s = 25^\circ C + 67.6^\circ C \approx 93^\circ C.$$ <

(c,d) The effect of the centerline spacing on q_i' and T_s can be understood by examining the relative magnitudes of the thermal resistances. The dominant resistance is that due to the ambient air convection process which is inversely related to the spacing L_o. Hence, from Equation (1), the heat rate will increase nearly linearly with an increase in L_o,

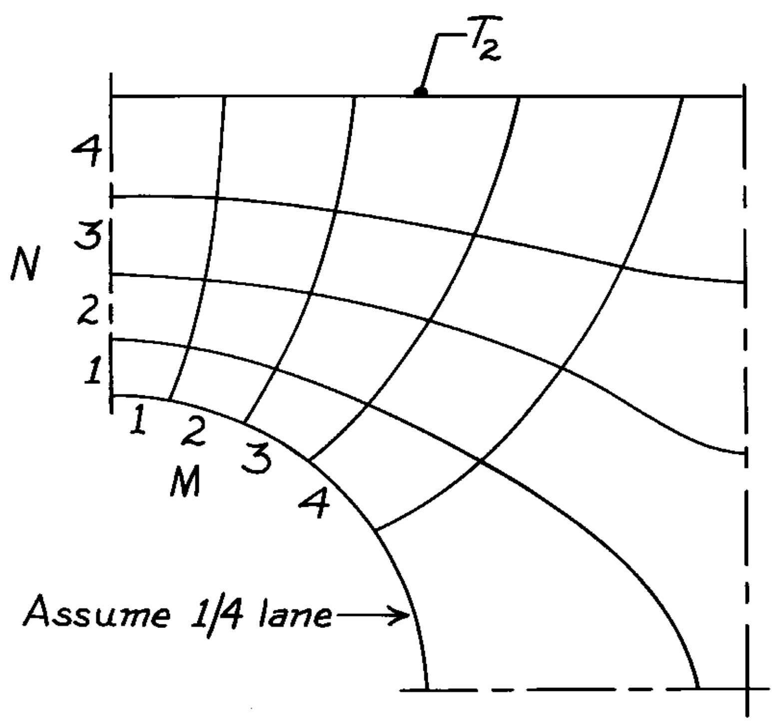
$$q_i' \sim \frac{1}{R_{tot}'} \approx \frac{1}{1/h_o(L_o/2)} \sim L_o.$$

From Eq. (2), find

$$\Delta T = T_s - T_\infty = \frac{q_i'}{4}\frac{1}{h_o(L_o/2)} \sim q_i' \cdot L_o^{-1} \sim L_o \cdot L_o^{-1} \approx 1.$$

Hence we conclude that ΔT will not increase with a change in L_o. Does this seem reasonable? What effect does L_o have on Assumptions (2) and (3)?

If the lower surface were insulated, the heat rate would be decreased nearly by half. This follows again from the fact that the overall resistance is dominated by the surface convection process. The temperature difference, T_s - T_∞, would only increase slightly.



$S' = S/l = M/N = 4.25/4 = 1.06$