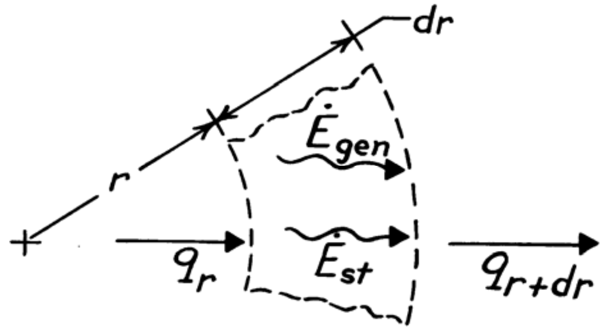
PROBLEM 2.45

KNOWN: Heat diffusion with internal heat generation for one-dimensional spherical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has the volume, $V = A_r \cdot dr = 4\pi r^2 dr$. Using the conservation of energy requirement, Eq. 1.12c,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this coordinate system has the form

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}.$$

At the outer surface, r + dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 4\pi r^2 dr = \rho \cdot 4\pi r^2 dr \cdot c_p \frac{\partial T}{\partial t}.$$

Dividing by the factor $4\pi r^2 dr$, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2 \frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$ <

COMMENTS: (1) Note how the result compares with Eq. 2.29 when the terms for the θ,ϕ directions are eliminated.

(2) Recognize that we did not require $\dot{q}$ and k to be independent of the coordinate r.