PROBLEM 3.168

KNOWN: Thermoelectric module properties and performance, as given in Example 3.13.

FIND: (a) The thermodynamic efficiency, $\eta_{therm} \equiv P_{M=1}/q_1$, (b) the figure of merit $Z\overline{T}$ for one module, and the thermoelectric efficiency, η_{TE}. (c) the Carnot efficiency, $\eta_{Carnot} = 1 - T_2/T_1$, (d) the value of η_{TE} based upon the inappropriate use of $T_{\infty,1}$ and $T_{\infty,2}$ (e) the thermoelectric efficiency based upon the correct usage of T_1 and T_2 in Equation 3.128, and the Carnot efficiency for the case where $h_1 = h_2 \rightarrow \infty$.

ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Negligible contact resistances, (3) Negligible radiation exchange and gas phase conduction inside the module, (4) Negligible conduction resistance due to metallic contacts and ceramic insulators, (5) The properties of the two semiconductors are identical and $S_p = - S_n$.

ANALYSIS: (a) From Example 3.13 the electrical power per module is $P_{M=1} = P_{tot}/M = 46.9$ W/48 = 0.9773 W. The heat input to one module may be evaluated from Equation 3 of the solution to the example problem as

$$q_1 = h_1 W^2 (T_{\infty,1} - T_1) = 40 \text{ W/m}^2 \cdot \text{K} \times (0.054\text{m})^2 \times \left[\left(550 + 273)\text{K} - (173 + 273)\text{K} \right) \right] = 43.92 \text{ W}$$

Therefore, the thermodynamic efficiency is $\eta_{therm} = P_{M=1}/q_1 = 0.9773 \text{ W}/43.92 \text{ W} = 0.022$ <

(b) From Equations 3.121 and 3.125 (or 3.122 and 3.126), we note that

$$S_{p-n} = S_{p-n,\text{eff}} / N = 0.1435 \text{ volts/K} / 100 = 0.001435 \text{ volts/K}$$

and

$$\rho_{e,s} = \frac{R_{e,\text{eff}} A_s}{2NL} = \frac{4\Omega \times 1.2 \times 10^{-5} \text{ m}^2}{2 \times 100 \times 2.5 \times 10^{-3} \text{ m}} = 9.6 \times 10^{-5} \ \Omega \cdot \text{m}$$

From Section 3.8, $S = S_p = - S_n$ and for $S_p = - S_n$, $S = S_p = S_{p-n}/2 = 0.0007175$ volts/K,

$$Z = \frac{S^2}{\rho_{e,s} k} = \frac{(0.0007175 \text{ volts/K})^2}{9.6 \times 10^{-5} \ \Omega \cdot \text{m} \times 1.2 \text{ W/m} \cdot \text{K}} = 0.004469 \text{ K}^{-1}$$

For $T_1 = 173°\text{C} + 273 \text{ K} = 446 \text{ K}$ and $T_2 = 134°\text{C} + 273 \text{ K} = 407 \text{ K}$, as determined in the example problem, the average module temperature is $\overline{T} = (T_1 + T_2)/2 = (446 \text{ K} + 407 \text{ K})/2 = 426.5 \text{ K}$. Therefore, the figure of merit is

$$Z\overline{T} = 0.004469 \text{ K}^{-1} \times 426.5 \text{ K} = 1.908.$$ <

The thermoelectric efficiency is therefore,

Continued...

PROBLEM 3.168 (Cont.)

$$\eta_{TE} = \left(1 - \frac{T_2}{T_1}\right) \frac{\sqrt{1 + Z\overline{\overline{T}}} - 1}{\sqrt{1 + Z\overline{\overline{T}}} + T_2 / T_1} = \left(1 - \frac{407 \text{ K}}{446 \text{ K}}\right) \frac{\sqrt{1 + 1.908\text{K}^{-1}} - 1}{\sqrt{1 + 1.908\text{K}^{-1}} + 407 \text{ K} / 446 \text{ K}} = 0.024$$ <

We note that the thermodynamic efficiency is less than the thermoelectric efficiency based uo the figure of merit and the surface temperatures of the module. The thermoelectric efficiency is the maximum possible efficiency for the case when the load resistance is optimized.

(c) The Carnot efficiency is η_{Carnot} = 1 - 407K /446 K = 0.087 <

(d) The value of η_{TE} based upon $T_{\infty,1}$ = 550°C + 273 K = 823 K, $T_{\infty,2}$ = 105°C + 273 K = 378 K, and $Z\overline{T} = 0.004469 \text{ K}^{-1} \times 600.5 \text{ K} = 2.684$ is

$$\eta_{TE} = \left(1 - \frac{T_{\infty,2}}{T_{\infty,1}}\right) \frac{\sqrt{1 + Z\overline{T}} - 1}{\sqrt{1 + Z\overline{\overline{T}}} + T_{\infty,2} / T_{\infty,1}} = \left(1 - \frac{378 \text{ K}}{823 \text{ K}}\right) \frac{\sqrt{1 + 2.684} - 1}{\sqrt{1 + 2.684} + 378 \text{ K} / 823 \text{ K}} = 0.21$$ <

(e) For $h_1 = h_2 \rightarrow \infty$, the surface temperatures of the module are $T_1 = T_{\infty,1}$ = 823 K and $T_2 = T_{\infty,2}$ = 378 K, respectively. Therefore, from part (d) η_{TE} = 0.21. The Carnot efficiency is η_{Carnot} = 1 - 378 K/823 K = 0.54. <

COMMENTS: (1) The conversion efficiency for the thermoelectric modules of Example 3.13 is quite small, approximately 2%. (2) The conversion efficiency can be increased by an order of magnitude (to 21%) by utilizing thermal management approaches that will increase the temperature difference across the module. (c) The incorrect usage of $T_{\infty,1}$ and $T_{\infty,2}$ in the expression for the thermoelectric efficiency as in part (d) provides an efficiency (21%) that far exceeds the Carnot efficiency of 8.7% found in part (c). Hence, use of the incorrect temperatures in the thermoelectric efficiency expression can lead to grossly exaggerated levels of thermodynamic performance that violate the second law of thermodynamics. Reporting the efficiency of a thermoelectric module or thermoelectric material based upon fluid (or surrounding) temperatures is meaningless.