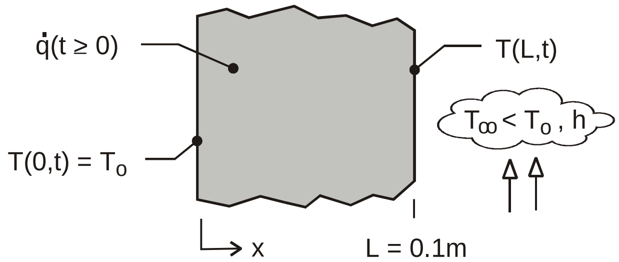
## PROBLEM 2.61

**KNOWN:** Plane wall, initially at a uniform temperature $T_o$, has one surface (x = L) suddenly exposed to a convection process ($T_\infty < T_o$, h), while the other surface (x = 0) is maintained at $T_o$. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed $T_\infty$.

**FIND:** (a) Sketch temperature distribution (T vs. x) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; identify key features of the distributions, (b) Sketch the heat flux $\left(q''_x \text{ vs. } t\right)$ at the boundaries x = 0 and L; identify key features of the distributions.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_\infty < T_o$ and $\dot{q}$ large enough that $T(x,\infty) > T_o$.

**ANALYSIS:** (a) The initial and boundary conditions for the wall can be written as

*Initial ($t \le 0$):* $T(x,0) = T_o$ Uniform temperature

*Boundary:* $x = 0 \quad T(0,t) = T_o$ Constant temperature

$$x = L \qquad -k\left.\frac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right] \qquad \text{Convection process.}$$

The temperature distributions are shown on the T-x coordinates below. Note that the maximum temperature occurs under steady-state conditions not at the midplane, but to the right toward the surface experiencing convection. The temperature gradients at x = L increase for t > 0 since the convection heat rate from the surface increases as the surface temperature increases.

(b) The heat flux as a function of time at the boundaries, $q''_x(0,t)$ and $q''_x(L,t)$, can be inferred from the temperature distributions using Fourier's law. At the surface x = L, the convection heat flux at t = 0 is $q''_x(L,0) = h(T_o - T_\infty)$. Because the surface temperature dips slightly at early times, the convection heat flux decreases slightly, and then increases until the steady-state condition is reached. For the steady-state condition, heat transfer at both boundaries must be out of the wall. It follows from an overall energy balance on the wall that $+q''_x(0,\infty) - q''_x(L,\infty) + \dot{q}L = 0$.