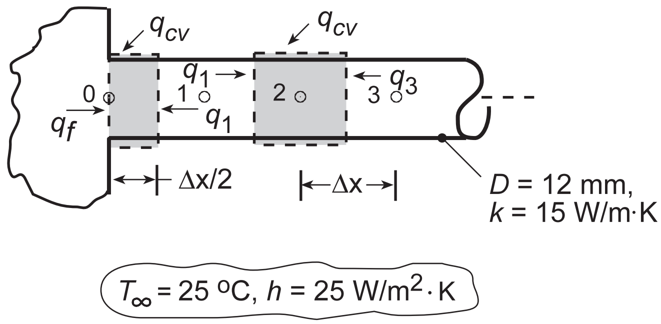
PROBLEM 4.62

KNOWN: Nodal temperatures from a steady-state finite-difference analysis for a cylindrical fin of prescribed diameter, thermal conductivity and convection conditions (T_∞, h).

FIND: (a) The fin heat rate, q_f, and (b) Temperature at node 3, T_3.

SCHEMATIC:

$T_0 = 100.0°C$
$T_1 = 93.4°C$
$T_2 = 89.5°C$



ASSUMPTIONS: (a) The fin heat rate, q_f, is that of conduction at the base plane, x = 0, and can be found from an energy balance on the control volume about node 0, $\dot{E}_{in} - \dot{E}_{out} = 0$,

$$q_f + q_1 + q_{conv} = 0 \qquad \text{or} \qquad q_f = -q_1 - q_{conv}.$$

Writing the appropriate rate equation for q_1 and q_{conv}, with $A_c = \pi D^2/4$ and $P = \pi D$,

$$q_f = -kA_c \frac{T_1 - T_0}{\Delta x} - hP(\Delta x/2)(T_\infty - T_0) = -\frac{\pi k D^2}{4\Delta x}(T_1 - T_0) - (\pi/2)Dh\Delta x(T_\infty - T_0)$$

Substituting numerical values, with Δx = 0.010 m, find

$$q_f = -\frac{\pi \times 15\,\text{W/m}\cdot\text{K}(0.012\,\text{m})^2}{4 \times 0.010\,\text{m}}(93.4 - 100)^\circ\text{C}$$
$$-\frac{\pi}{2} \times 0.012\,\text{m} \times 25\,\text{W/m}^2\cdot\text{K} \times 0.010\,\text{m}(25 - 100)^\circ\text{C}$$

$$q_f = (1.120 + 0.353)\,\text{W} = 1.473\,\text{W}.$$ <

(b) To determine T_3, derive the finite-difference equation for node 3, perform an energy balance on the control volume shown above, $\dot{E}_{in} - \dot{E}_{out} = 0$,

$$q_{cv} + q_3 + q_1 = 0$$

$$hP\Delta x(T_\infty - T_2) + kA_c \frac{T_3 - T_2}{\Delta x} + kA_c \frac{T_1 - T_2}{\Delta x} = 0$$

$$T_3 = -T_1 + 2T_2 - \frac{hP\Delta x^2}{kA_c}[T_\infty - T_2]$$

Substituting numerical values, find

$$T_3 = 89.2^\circ\text{C}$$ <

COMMENTS: Note that in part (a), the convection heat rate from the outer surface of the control volume is significant (25%). It would have been a poor approximation to ignore this term.