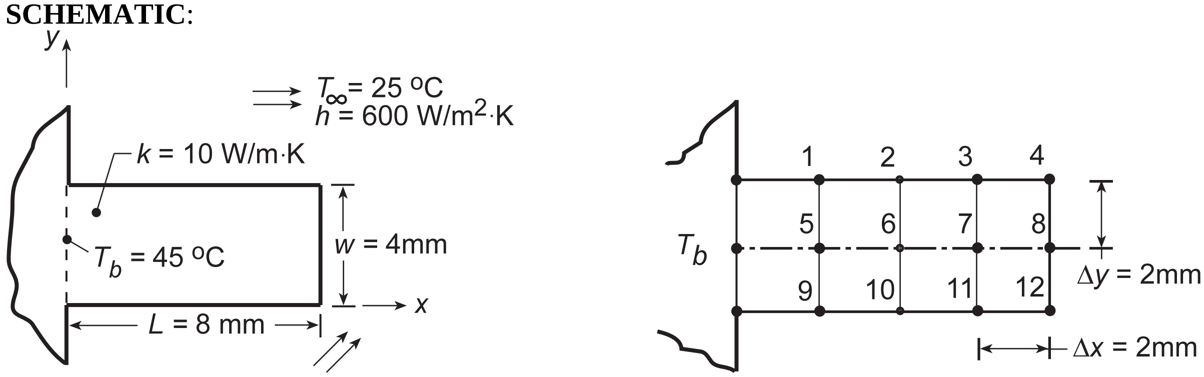
PROBLEM 4.82

KNOWN: Longitudinal rib (k = 10 W/m·K) with rectangular cross-section with length L= 8 mm and width w = 4 mm. Base temperature T_b and convection conditions, T_∞ and h, are prescribed.

FIND: (a) Temperature distribution and fin base heat rate using a finite-difference method with $\Delta x = \Delta y =$ 2 mm for a total of $5 \times 3 = 15$ nodal points and regions; compare results with those obtained assuming one-dimensional heat transfer in rib; and (b) The effect of grid spacing by reducing nodal spacing to $\Delta x = \Delta y = 1$ mm for a total of $9 \times 3 = 27$ nodal points and regions considering symmetry of the centerline; and (c) A criterion for which the one-dimensional approximation is reasonable; compare the heat rate for the range $1.5 \leq L/w \leq 10$, keeping L constant, as predicted by the two-dimensional, finite-difference method and the one-dimensional fin analysis.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, and (3) Convection coefficient uniform over rib surfaces, including tip.

ANALYSIS: (a) The rib is represented by a 5×3 nodal grid as shown above where the symmetry plane is an adiabatic surface. The *IHT Tool*, *Finite-Difference Equations*, for *Two-Dimensional*, *Steady-State* conditions is used to formulate the nodal equations (see Comment 2 below) which yields the following nodal temperatures (° C)

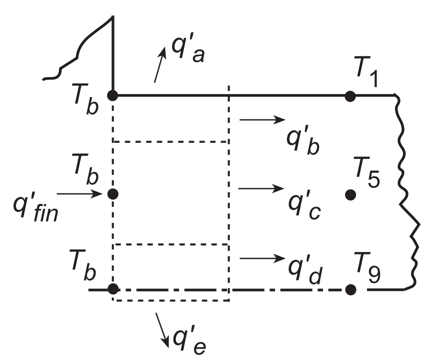
45	39.3	35.7	33.5	32.2
45	40.0	36.4	34.0	32.6
45	39.3	35.7	33.5	32.2

Note that the fin tip temperature is

$$T_{tip} = T_{12} = 32.6^\circ C$$ <

The fin heat rate per unit width normal to the page, q'_{fin}, can be determined from energy balances on the three base nodes as shown in the schematic below.

$$q'_{fin} = q'_a + q'_b + q'_c + q'_d + q'_e$$
$$q'_a = h(\Delta x/2)(T_b - T_\infty)$$
$$q'_b = k(\Delta y/2)(T_b - T_1)/\Delta x$$
$$q'_c = k(\Delta y)(T_b - T_5)/\Delta x$$
$$q'_d = k(\Delta y/2)(T_b - T_9)\Delta x$$
$$q'_3 = h(\Delta x/2)(T_b - T_\infty)$$



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PROBLEM 4.82 (Cont.)

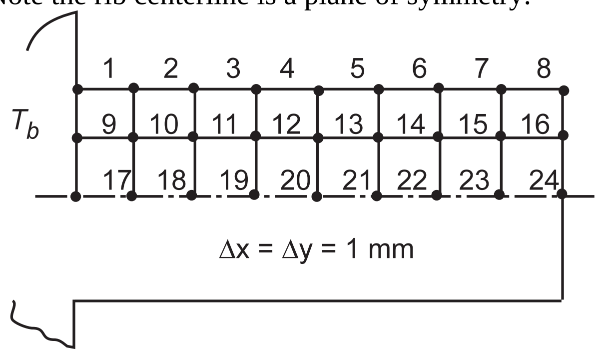
Substituting numerical values, find

$$q'_{fin} = (12.0 + 28.4 + 50.0 + 28.4 + 12.0)\,W/m = 130.8\,W/m$$ <

Using the *IHT Model*, *Extended Surfaces*, *Heat Rate* and *Temperature Distributions* for *Rectangular, Straight Fins*, with convection tip condition, the one-dimensional fin analysis yields

$q'_f = 131\,W/m$ $\qquad$ $T_{tip} = 32.2^\circ C$ <

(b) With Δx = L/8 = 1 mm and Δx = 1 mm, for a total of 9 × 3 = 27 nodal points and regions, the grid appears as shown below. Note the rib centerline is a plane of symmetry.

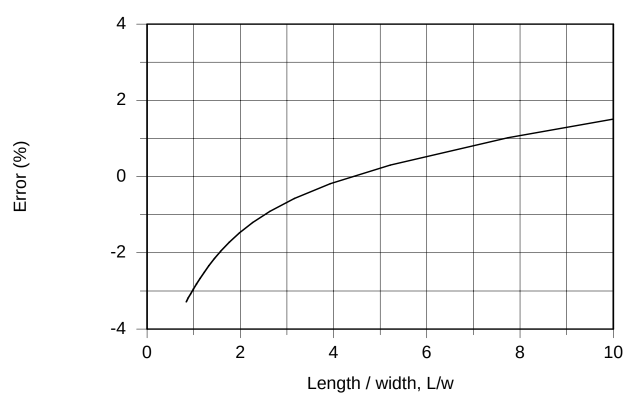


Using the same IHT FDE Tool as above with an appropriate expression for the fin heat rate, Eq. (1), the fin heat rate and tip temperature were determined.

	1-D analysis	2-D analysis (nodes) (5 × 3)	(9 × 3)	
T_{tip} (°C)	32.2	32.6	32.6	<
q'_{fin} (W/m)	131	131	129	<

(c) To determine when the one-dimensional approximation is reasonable, consider a rib of constant length, L = 8 mm, and vary the thickness w for the range 1.5 ≤ L/w ≤ 10. Using the above IHT model for the 27 node grid, the fin heat rates for 1-D, q'_{1d}, and 2-D, q'_{2d}, analysis were determined as a function of w with the error in the approximation evaluated as

$$\text{Error}(\%) = (q'_{2d} - q'_{1d}) \times 100 / q'_{1d}$$



Note that for small L/w, a thick rib, the 1-D approximation is poor. For large L/w, a thin rib which approximates a fin, we would expect the 1-D approximation to become increasingly more satisfactory. The discrepancy at large L/w must be due to discretization error; that is, the grid is too coarse to accurately represent the slender rib.