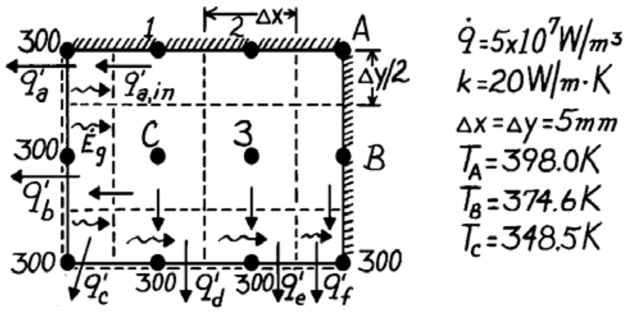
PROBLEM 4.55

KNOWN: Steady-state temperatures (K) at three nodes of a long rectangular bar.

FIND: (a) Temperatures at remaining nodes and (b) heat transfer per unit length from the bar using nodal temperatures; compare with result calculated using knowledge of $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equations for the nodes (1,2,3,A,B,C) can be written by inspection using Eq. 4.35 and recognizing that the adiabatic boundary can be represented by a symmetry plane.

$$\sum T_{neighbors} - 4T_i + \dot{q}\Delta x^2 / k = 0 \quad \text{and} \quad \frac{\dot{q}\Delta x^2}{k} = \frac{5\times10^7\ \text{W/m}^3 (0.005\text{m})^2}{20\ \text{W/m}\cdot\text{K}} = 62.5\text{K}.$$

Node A (to find T_2*):* $\quad 2T_2 + 2T_B - 4T_A + \dot{q}\Delta x^2 / k = 0$

$$T_2 = \frac{1}{2}(-2\times374.6 + 4\times398.0 - 62.5)\text{K} = 390.2\text{K}$$ <

Node 3 (to find T_3*):* $\quad T_c + T_2 + T_B + 300\text{K} - 4T_3 + \dot{q}\Delta x^2 / k = 0$

$$T_3 = \frac{1}{4}(348.5 + 390.2 + 374.6 + 300 + 62.5)\text{K} = 369.0\text{K}$$ <

Node 1 (to find T_1*):* $\quad 300 + 2T_C + T_2 - 4T_1 + \dot{q}\Delta x^2 / k = 0$

$$T_1 = \frac{1}{4}(300 + 2\times348.5 + 390.2 + 62.5) = 362.4\text{K}$$ <

(b) The heat rate out of the bar is determined by calculating the heat rate out of each control volume around the 300 K nodes. Consider the node in the upper left-hand corner; from an energy balance

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \quad \text{or} \quad q'_a = q'_{a,in} + \dot{E}_g \quad \text{where} \quad \dot{E}_g = \dot{q}V.$$

Hence, for the entire bar $\quad q'_{bar} = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$, or

$$q'_{bar} = \left[k\frac{\Delta y}{2}\frac{T_1 - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_a + \left[k\Delta y\frac{T_C - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\Delta y\right]\right]_b + \left[\dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_c +$$
$$\left[k\Delta x\frac{T_C - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_d + \left[k\Delta x\frac{T_3 - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_e + \left[k\frac{\Delta x}{2}\frac{T_B - 300}{\Delta y} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_f.$$

Substituting numerical values, find $q'_{bar} = 7,502.5\ \text{W/m}$. From an overall energy balance on the bar,

$$q'_{bar} = \dot{E}'_g = \dot{q}V/\ell = \dot{q}(3\Delta x\cdot2\Delta y) = 5\times10^7\ \text{W/m}^3\times6(0.005\text{m})^2 = 7,500\ \text{W/m}.$$ <

As expected, the results of the two methods agree. Why must that be?