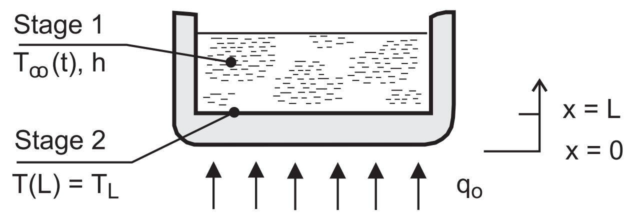
PROBLEM 2.27

KNOWN: Diameter D, thickness L and initial temperature T_i of pan. Heat rate from stove to bottom of pan. Convection coefficient h and variation of water temperature $T_\infty(t)$ during Stage 1. Temperature T_L of pan surface in contact with water during Stage 2.

FIND: Form of heat equation and boundary conditions associated with the two stages.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in pan bottom, (2) Heat transfer from stove is uniformly distributed over surface of pan in contact with the stove, (3) Constant properties.

ANALYSIS:

Stage 1

Heat Equation:
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions:
$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_o'' = \frac{q_o}{\left(\pi D^2/4\right)}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty(t)\right]$$

Initial Condition: $T(x,0) = T_i$

Stage 2

Heat Equation:
$$\frac{d^2 T}{dx^2} = 0$$

Boundary Conditions:
$$-k\left.\frac{dT}{dx}\right|_{x=0} = q_o''$$

$$T(L) = T_L$$

COMMENTS: Stage 1 is a transient process for which $T_\infty(t)$ must be determined separately. As a first approximation, it could be estimated by neglecting changes in thermal energy storage by the pan bottom and assuming that all of the heat transferred from the stove acted to increase thermal energy storage within the water. Hence, with $q \approx Mc_p\, dT_\infty/dt$, where M and c_p are the mass and specific heat of the water in the pan, $T_\infty(t) \approx (q/Mc_p)\, t$.