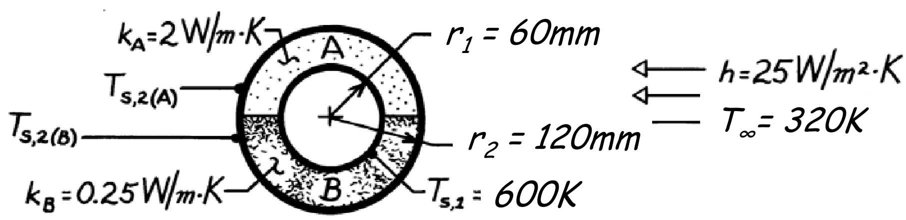
PROBLEM 3.61

KNOWN: Inner surface temperature of insulation blanket comprised of two semi-cylindrical shells of different materials. Ambient air conditions.

FIND: (a) Equivalent thermal circuit, (b) Total heat loss and material outer surface temperatures.

SCHEMATIC:



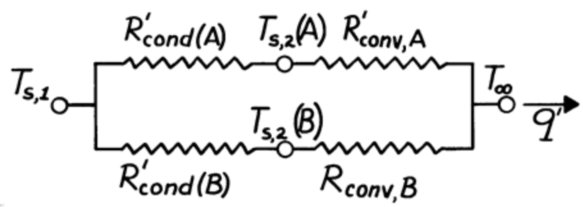
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction, (3) Infinite contact resistance between materials, (4) Constant properties.

ANALYSIS: (a) The thermal circuit is,

$R'_{conv,A} = R'_{conv,B} = 1/\pi\, r_2 h$

$$R'_{cond(A)} = \frac{\ln(r_2/r_1)}{\pi\, k_A}$$

$$R'_{cond(\mathbf{B})} = \frac{\ln(r_2/r_i)}{\pi\, k_B}$$



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The conduction resistances follow from Section 3.3.1 and Eq. 3.33. Each resistance is larger by a factor of 2 than the result of Eq. 3.28 due to the reduced area.

(b) Evaluating the thermal resistances and the heat rate $(q' = q'_A + q'_B)$,

$$R'_{conv} = \left(\pi \times 0.12\text{m} \times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1} = 0.106\ \text{m}\cdot\text{K/W}$$

$$R'_{cond(A)} = \frac{\ln(0.12\text{m}/0.06\text{m})}{\pi \times 2\ \text{W/m}\cdot\text{K}} = 0.1103\ \text{m}\cdot\text{K/W} \quad R'_{cond(B)} = 8\, R'_{cond(A)} = 0.8825\ \text{m}\cdot\text{K/W}$$

$$q' = \frac{T_{s,1} - T_\infty}{R'_{cond(A)} + R'_{conv}} + \frac{T_{s,1} - T_\infty}{R'_{cond(B)} + R'_{conv}}$$

$$q' = \frac{(600-320)\,\text{K}}{(0.1103+0.106)\,\text{m}\cdot\text{K/W}} + \frac{(600-320)\,\text{K}}{(0.8825+0.106)\,\text{m}\cdot\text{K/W}} = (1294+283)\,\text{W/m} = 1577\ \text{W/m}.$$ <

Hence, the temperatures are

$$T_{s,2(A)} = T_{s,1} - q'_A R'_{cond(A)} = 600\text{K} - 1294\frac{\text{W}}{\text{m}} \times 0.1103\frac{\text{m}\cdot\text{K}}{\text{W}} = 457\text{K}$$ <

$$T_{s,2(B)} = T_{s,1} - q'_B R'_{cond(B)} = 600\text{K} - 283\frac{\text{W}}{\text{m}} \times 0.8825\frac{\text{m}\cdot\text{K}}{\text{W}} = 350\text{K}.$$ <

COMMENTS: The total heat loss can also be computed from $q' = (T_{s,1} - T_\infty)/R_{equiv}$,

where $R_{equiv} = \left[\left(R'_{cond(A)} + R'_{conv,A}\right)^{-1} + \left(R'_{cond(B)} + R'_{conv,B}\right)^{-1}\right]^{-1} = 0.1775\ \text{m}\cdot\text{K/W}.$

Hence $q' = (600-320)\,\text{K}/0.1775\ \text{m}\cdot\text{K/W} = 1577\ \text{W/m}.$