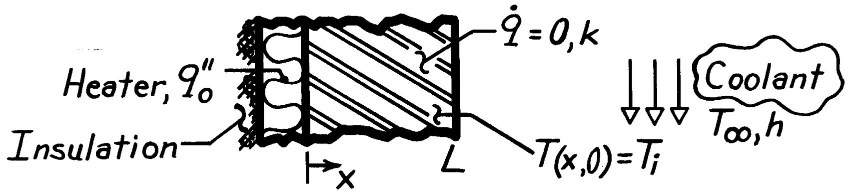
PROBLEM 2.59

KNOWN: Plane wall, initially at a uniform temperature T_i, is suddenly exposed to convection with a fluid at T_∞ at one surface, while the other surface is exposed to a constant heat flux q_o''.

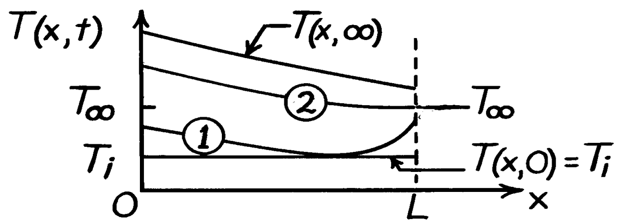
FIND: (a) Temperature distributions, T(x,t), for initial, steady-state and two intermediate times, (b) Corresponding heat fluxes on $q_x'' - x$ coordinates, (c) Heat flux at locations x = 0 and x = L as a function of time, (d) Expression for the steady-state temperature of the heater, T(0,∞), in terms of q_o'', T_∞, k, h and L.

SCHEMATIC:



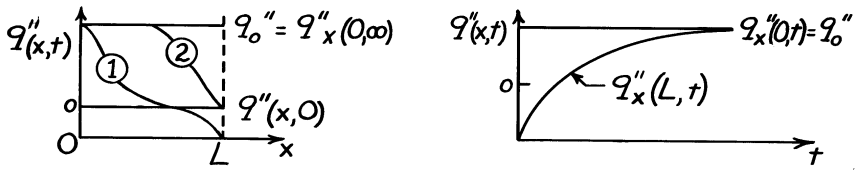
ASSUMPTIONS: (1) One-dimensional conduction, (2) No heat generation, (3) Constant properties.

ANALYSIS: (a) For $T_i < T_\infty$, the temperature distributions are



Note the constant gradient at x = 0 since $q_x''(0) = q_o''$.

(b) The heat flux distribution, $q_x''(x,t)$, is determined from knowledge of the temperature gradients, evident from Part (a), and Fourier's law.

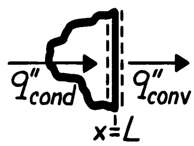


(c) On $q_x''(x,t) - t$ coordinates, the heat fluxes at the boundaries are shown above.

(d) Perform a surface energy balance at x = L and an energy balance on the wall:

$$q''_{cond} = q''_{conv} = h\left[T(L,\infty) - T_\infty\right] \quad (1), \qquad q''_{cond} = q''_o. \quad (2)$$

For the wall, under steady-state conditions, Fourier's law gives



$$q_o'' = -k\frac{dT}{dx} = k\frac{T(0,\infty) - T(L,\infty)}{L}. \qquad (3)$$

Combine Eqs. (1), (2), (3) to find:

$$T(0,\infty) = T_\infty + \frac{q_o''}{1/h + L/k}.$$