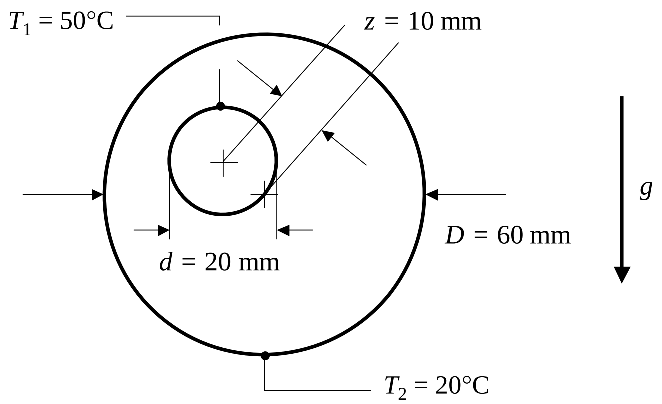
PROBLEM 4.7

KNOWN: Diameters and temperatures of horizontal circular cylinders. Eccentricity factor. Heat transfer rate per unit length. Fluid thermal conductivity.

FIND: Effective thermal conductivity.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Steady state conditions.

PROPERTIES: Given: k = 0.27 W/m·K.

ANALYSIS: In the absence of free convection the conduction heat transfer per unit length may be found by using the shape factor expression and applying Case 7 of Table 4.1. Hence

$$q'_{\text{cond}} = \frac{S}{L}k\left(T_1 - T_2\right) = \frac{2\pi k\left(T_1 - T_2\right)}{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)} = \frac{2\pi \times 0.27\ \text{W/m}\cdot\text{K}\left(50 - 20\right)°\text{C}}{\cosh^{-1}\left(\dfrac{\left(60\times10^{-3}\text{m}\right)^2 + \left(20\times10^{-3}\ \text{m}\right)^2 - 4\times\left(10\times10^{-3}\ \text{m}\right)^2}{2\times60\times10^{-3}\ \text{m}\times20\times10^{-3}\ \text{m}}\right)}$$

$$= 53\ \text{W/m}$$

The free convection heat transfer rate is

$$q'_{\text{conv}} = \frac{S}{L}k_{\text{eff}}\left(T_1 - T_2\right) = 89\ \text{W/m}$$

Therefore the effective thermal conductivity is

$$k_{\text{eff}} = k\frac{q_{\text{conv}}}{q_{\text{cond}}} = 0.27\ \text{W/m}\cdot\text{K}\cdot\frac{89\ \text{W/m}}{53\ \text{W/m}} = 0.45\ \text{W/m}\cdot\text{K}$$ <

COMMENTS: Buoyancy-induced fluid motion increases the heat transfer rate between the cylinders by 68%.