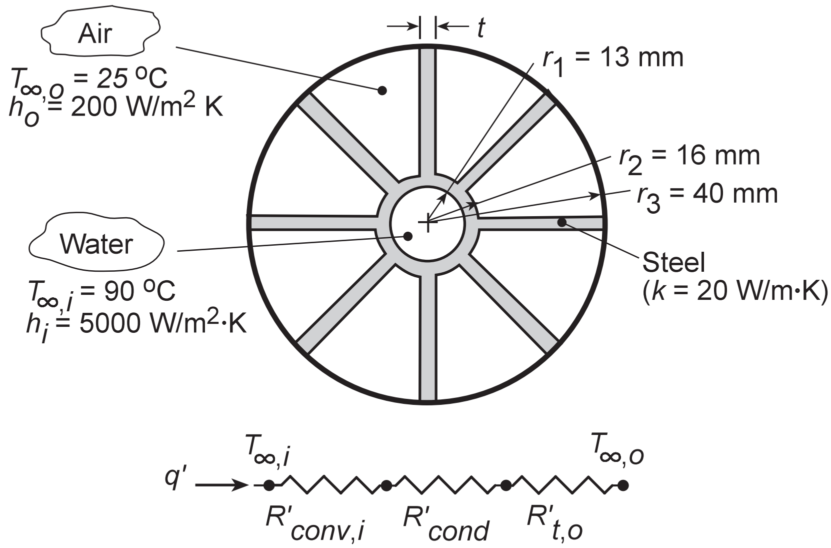
PROBLEM 3.154

KNOWN: Geometrical and convection conditions of internally finned, concentric tube air heater.

FIND: (a) Thermal circuit, (b) Heat rate per unit tube length, (c) Effect of changes in fin array.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in radial direction, (3) Constant k, (4) Adiabatic outer surface.

ANALYSIS: (a) For the thermal circuit shown schematically,

$$R'_{conv,i} = \left(h_i 2\pi r_1\right)^{-1}, \qquad R'_{cond} = \ln\left(r_2/r_1\right)/2\pi k \text{, and} \qquad R'_{t,o} = \left(\eta_o h_o A'_t\right)^{-1},$$

where

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1-\eta_f\right), \quad A'_f = 2L = 2\left(r_3 - r_2\right), \quad A'_t = NA'_f + \left(2\pi r_2 - Nt\right), \text{ and} \quad \eta_f = \frac{\tanh mL}{mL}.$$

(b)
$$q' = \frac{\left(T_{\infty,i} - T_{\infty,o}\right)}{R'_{conv,i} + R'_{cond} + R'_{t,o}}$$

Substituting the known conditions, it follows that

$$R'_{conv,i} = \left(5000\,W/m^2\cdot K \times 2\pi \times 0.013m\right)^{-1} = 2.45\times10^{-3} m\cdot K/W$$

$$R'_{cond} = \ln\left(0.016m/0.013m\right)/2\pi\left(20\,W/m\cdot K\right) = 1.65\times10^{-3} m\cdot K/W$$

$$R'_{t,o} = \left(0.575\times 200\,W/m^2\cdot K \times 0.461m\right)^{-1} = 18.86\times10^{-3} m\cdot K/W$$

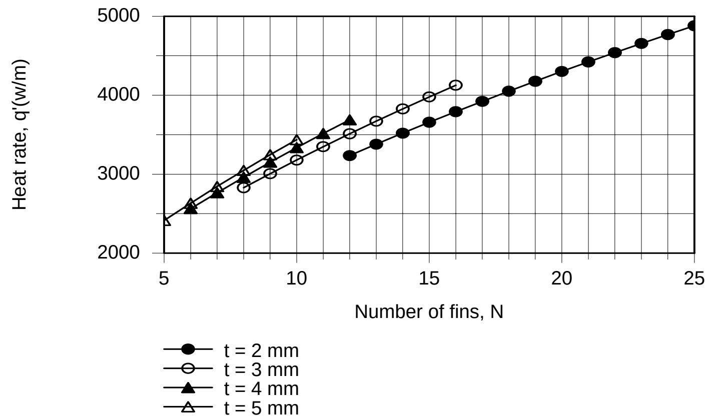
where $\eta_f = 0.490$. Hence,

$$q' = \frac{\left(90-25\right)^\circ C}{\left(2.45+1.65+18.86\right)\times10^{-3} m\cdot K/W} = 2831\,W/m$$ <

(c) The small value of η_f suggests that some benefit may be gained by increasing t, as well as by increasing N. With the requirement that Nt ≤ 50 mm, we use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to consider the following range of conditions: t = 2 mm, 12 ≤ N ≤ 25; t = 3 mm, 8 ≤ N ≤ 16; t = 4 mm, 6 ≤ N ≤ 12; t = 5 mm, 5 ≤ N ≤ 10. Calculations based on the foregoing model are plotted as follows.

Continued...

PROBLEM 3.154 (Cont.)



By increasing t from 2 to 5 mm, η_f increases from 0.410 to 0.598. Hence, for fixed N, q' increases with increasing t. However, from the standpoint of maximizing q_t', it is clearly preferable to use the larger number of thinner fins. Hence, subject to the prescribed constraint, we would choose t = 2 mm and N = 25, for which q' = 4880 W/m.

COMMENTS: (1) The air side resistance makes the dominant contribution to the total resistance, and efforts to increase q' by reducing $R_{t,o}'$ are well directed. (2) A fin thickness any smaller than 2 mm would be difficult to manufacture.