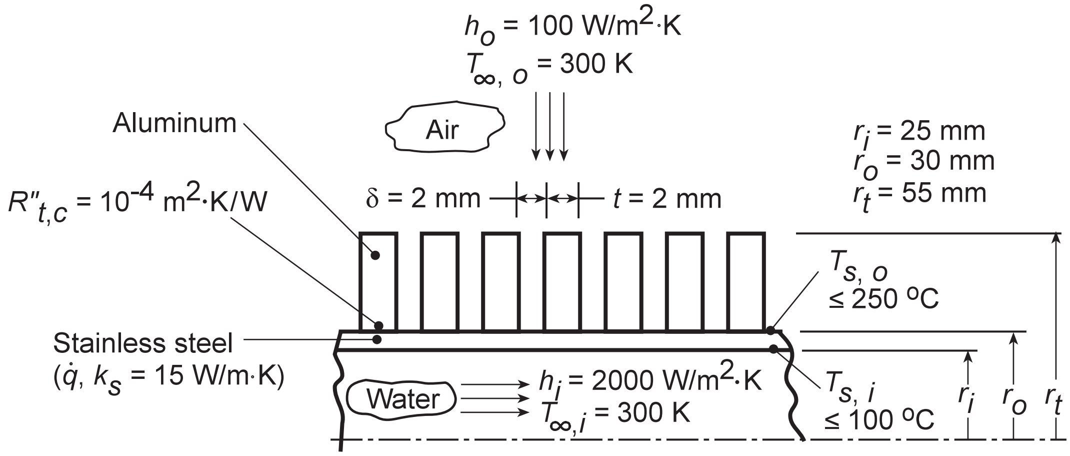
## PROBLEM 3.164

**KNOWN:** Design and operating conditions of a tubular, air/water heater.

**FIND:** (a) Expressions for heat rate per unit length at inner and outer surfaces, (b) Expressions for inner and outer surface temperatures, (c) Surface heat rates and temperatures as a function of volumetric heating $\dot{q}$ for prescribed conditions. Upper limit to $\dot{q}$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Constant properties, (3) One-dimensional heat transfer.

**PROPERTIES:** Table A-1: Aluminum, T = 300 K, $k_a$ = 237 W/m·K.

**ANALYSIS:** (a) Applying Equation C.8 to the inner and outer surfaces, it follows that

$$q'(r_i) = \dot{q}\pi r_i^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right] \quad <$$

$$q'(r_o) = \dot{q}\pi r_o^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right] \quad <$$

(b) From Equations C.16 and C.17, energy balances at the inner and outer surfaces are of the form

$$h_i\left(T_{\infty,i}-T_{s,i}\right) = \frac{\dot{q}r_i}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]}{r_i\ln(r_o/r_i)} \quad <$$

$$U_o\left(T_{s,o}-T_{\infty,o}\right) = \frac{\dot{q}r_o}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]}{r_o\ln(r_o/r_i)} \quad <$$

Accounting for the fin array and the contact resistance, Equation 3.109 may be used to cast the overall heat transfer coefficient $U_o$ in the form

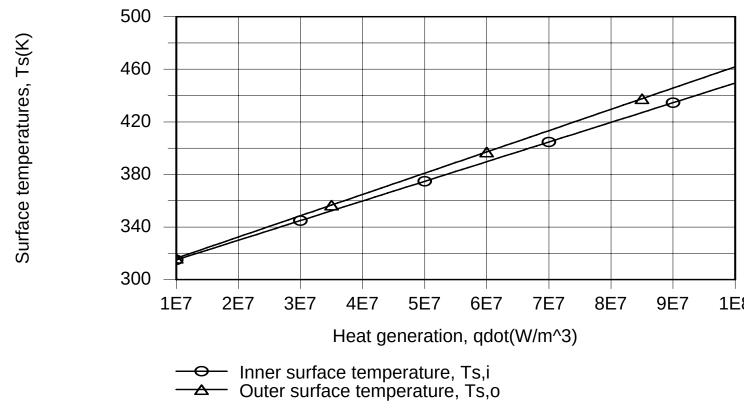
$$U_o = \frac{q'(r_o)}{A'_w\left(T_{s,o}-T_{\infty,o}\right)} = \frac{1}{A'_w R'_{t,o(c)}} = \frac{A'_t}{A'_w}\eta_{o(c)}h_o$$

where $\eta_{o(c)}$ is determined from Equations 3.110a,b and $A'_w = 2\pi r_o$.

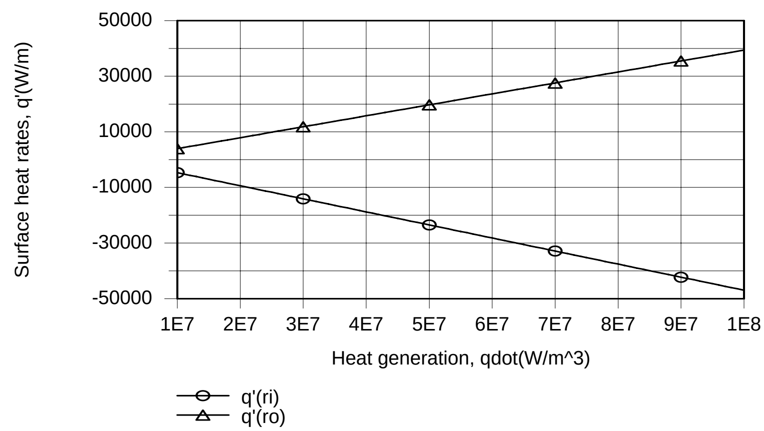
Continued...

## PROBLEM 3.164 (Cont.)

(c) For the prescribed conditions and a representative range of $10^7 \le \dot{q} \le 10^8$ W/m$^3$, use of the relations of part (b) with the capabilities of the IHT *Performance Calculation Extended Surface Model* for a *Circular Fin Array* yields the following graphical results.




It is in this range that the upper limit of $T_{s,i}$ = 373 K is exceeded for $\dot{q}$ = 4.9 × $10^7$ W/m$^3$, while the corresponding value of $T_{s,o}$ = 379 K is well below the prescribed upper limit. The expressions of part (a) yield the following results for the surface heat rates, where heat transfer in the negative r direction corresponds to $q'(r_i) < 0$.




For $\dot{q}$ = 4.9 × $10^7$ W/m$^3$, $q'(r_i)$ = -2.30 × $10^4$ W/m and $q'(r_o)$ = 1.93 × $10^4$ W/m.

**COMMENTS:** The foregoing design provides for comparable heat transfer to the air and water streams. This result is a consequence of the nearly equivalent thermal resistances associated with heat transfer from the inner and outer surfaces. Specifically, $R'_{conv,i} = (h_i 2\pi r_i)^{-1}$ = 0.00318 m·K/W is slightly smaller than $R'_{t,o(c)}$ = 0.00411 m·K/W, in which case $|q'(r_i)|$ is slightly larger than $q'(r_o)$, while $T_{s,i}$ is slightly smaller than $T_{s,o}$. Note that the solution must satisfy the energy conservation requirement, $\pi(r_o^2 - r_i^2)\dot{q} = |q'(r_i)| + q'(r_o)$.