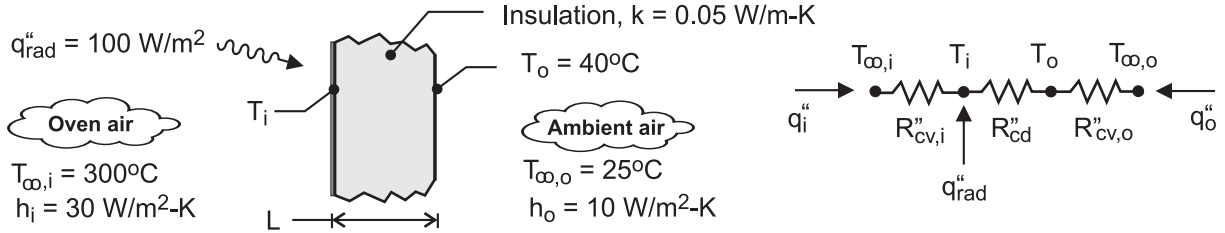# PROBLEM 3.19

KNOWN: Drying oven wall having material with known thermal conductivity sandwiched between thin metal sheets. Radiation and convection conditions prescribed on inner surface; convection conditions on outer surface.

FIND: (a) Thermal circuit representing wall and processes and (b) Insulation thickness required to maintain outer wall surface at $T_o = 40°C$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Thermal resistance of metal sheets negligible,(4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) Perform energy balances on the i- and o- nodes finding

$$\frac{T_{\infty,i} - T_i}{R''_{cv,i}} + \frac{T_o - T_i}{R''_{cd}} + q''_{rad} = 0 \tag{1}$$

$$\frac{T_i - T_o}{R''_{cd}} + \frac{T_{\infty,o} - T_o}{R''_{cv,o}} = 0 \tag{2}$$

where the thermal resistances are

$$R''_{cv,i} = 1/h_i = 0.0333 \text{ m}^2 \cdot K/W \tag{3}$$

$$R''_{cd} = L/k = L/0.05 \text{ m}^2 \cdot K/W \tag{4}$$

$$R''_{cv,o} = 1/h_o = 0.100 \text{ m}^2 \cdot K/W \tag{5}$$

Substituting numerical values, and solving Eqs. (1) and (2) simultaneously, find

$$L = 86 \text{ mm} \qquad\qquad <$$

COMMENTS: (1) The temperature at the inner surface can be found from an energy balance on the i-node using the value found for L.

$$\frac{T_{\infty,i} - T_i}{R''_{cv,o}} + \frac{T_{\infty,o} - T_i}{R''_{cd} + R''_{cv,i}} + q''_{rad} = 0 \qquad\qquad T_i = 298.3°C$$

It follows that T_i is close to $T_{\infty,i}$ since the wall represents the dominant resistance of the system.

(2) Verify that $q''_i = 50 \text{ W}/m^2$ and $q''_o = -150 \text{ W}/m^2$. Is the overall energy balance on the system satisfied?