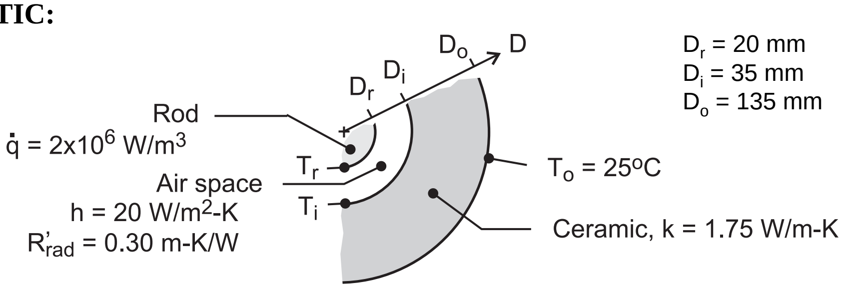
PROBLEM 3.55

KNOWN: Long rod experiencing uniform volumetric generation of thermal energy, $\dot{q}$, concentric with a hollow ceramic cylinder creating an enclosure filled with air. Thermal resistance per unit length due to radiation exchange between enclosure surfaces is R'_{rad}. The free convection coefficient for the enclosure surfaces is h = 20 W/m^2·K.

FIND: (a) Thermal circuit of the system that can be used to calculate the surface temperature of the rod, T_r; label all temperatures, heat rates and thermal resistances; evaluate the thermal resistances; and (b) Calculate the surface temperature of the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction through the hollow cylinder, (3) The enclosure surfaces experience free convection and radiation exchange.

ANALYSIS: (a) The thermal circuit is shown below. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

Enclosure, radiation exchange (given):

$$R'_{rad} = 0.30 \text{ m}\cdot\text{K/W}$$

Enclosure, free convection:

$$R'_{cv,rod} = \frac{1}{h\pi D_r} = \frac{1}{20 \text{ W/m}^2\cdot\text{K}\times\pi\times 0.020\text{m}} = 0.80 \text{ m}\cdot\text{K/W}$$

$$R'_{cv,cer} = \frac{1}{h\pi D_i} = \frac{1}{20 \text{ W/m}^2\cdot\text{K}\times\pi\times 0.035\text{m}} = 0.4547 \text{ m}\cdot\text{K/W}$$

Ceramic cylinder, conduction:

$$R'_{cd} = \frac{\ell n\left(D_o/D_i\right)}{2\pi k} = \frac{\ell n\left(0.135/0.035\right)}{2\pi\times 1.75 \text{ W/m}\cdot\text{K}} = 0.122 \text{ m}\cdot\text{K/W}$$

The thermal resistance between the enclosure surfaces (r-i) due to convection and radiation exchange is

$$\frac{1}{R'_{enc}} = \frac{1}{R'_{rad}} + \frac{1}{R'_{cv,rod} + R'_{cv,cer}}$$

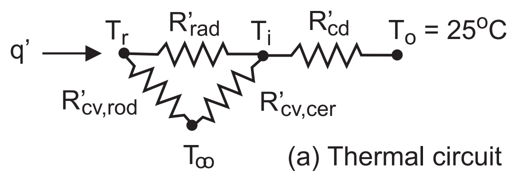
$$R'_{enc} = \left[\frac{1}{0.30} + \frac{1}{0.80 + 0.4547}\right]^{-1} \text{m}\cdot\text{K/W} = 0.242 \text{ m}\cdot\text{K/W}$$

The total resistance between the rod surface (r) and the outer surface of the cylinder (o) is

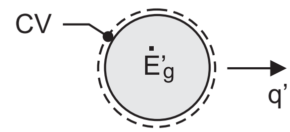
$$R'_{tot} = R'_{enc} + R'_{cd} = \left(0.242 + 0.123\right)\text{m}\cdot\text{K/W} = 0.365 \text{ m}\cdot\text{K/W}$$

Continued …

PROBLEM 3.55 (Cont.)



(a) Thermal circuit



(b) Overall energy balance on rod

(b) From an energy balance on the rod (see schematic) find T_r.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q + \dot{q}\forall = 0$$

$$-(T_r - T_i)/R'_{tot} + \dot{q}\left(\pi D_r^2/4\right) = 0$$

$$-(T_r - 25)\,K/0.365\ m \cdot K/W + 2 \times 10^6 W/m^3\left(\pi \times 0.020 m^2/4\right) = 0$$

$$T_r = 253°C$$ <

COMMENTS: In evaluating the convection resistance of the air space, it was necessary to define an average air temperature (T_∞) and consider the convection coefficients for each of the space surfaces. As you'll learn later in Chapter 9, correlations are available for directly estimating the convection coefficient (h_{enc}) for the enclosure so that $q_{cv} = h_{enc}(T_r - T_i)$.