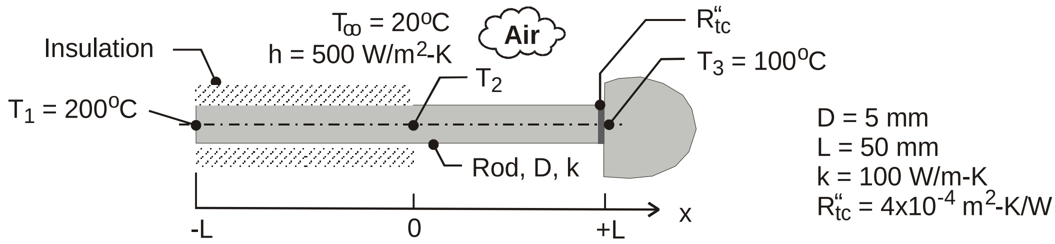
PROBLEM 3.119

KNOWN: Rod (D, k, 2L) that is perfectly insulated over the portion of its length $-L \le x \le 0$ and experiences convection (T_∞, h) over the portion $0 \le x \le +L$. One end is maintained at T_1 and the other is separated from a heat sink at T_3 with an interfacial thermal contact resistance R''_{tc}.

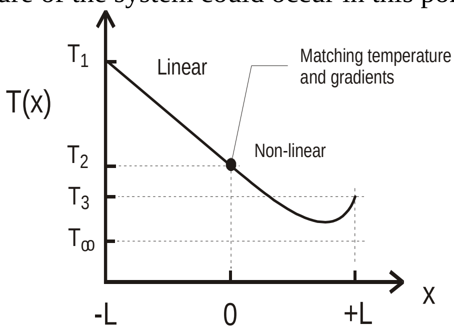
FIND: (a) Sketch the temperature distribution T vs. x and identify key features; assume $T_1 > T_3 > T_2$; (b) Derive an expression for the mid-point temperature T_2 in terms of thermal and geometric parameters of the system, (c) Using numerical values, calculate T_2 and plot the temperature distribution. Describe key features and compare to your sketch of part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod for $-L \le x \le 0$, (3) Rod behaves as one-dimensional extended surface for $0 \le x \le +L$, (4) Constant properties.

ANALYSIS: (a) The sketch for the temperature distribution is shown below. Over the insulated portion of the rod, the temperature distribution is linear. A temperature drop occurs across the thermal contact resistance at x = +L. The distribution over the exposed portion of the rod is non-linear. The minimum temperature of the system could occur in this portion of the rod.



(b) To derive an expression for T_2, begin with the general solution from the conduction analysis for a fin of uniform cross-sectional area, Eq. 3.71.

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad 0 \le x \le +L \tag{1}$$

where $m = (hP/kA_c)^{1/2}$ and $\theta = T(x) - T_\infty$. The arbitrary constants are determined from the boundary conditions.

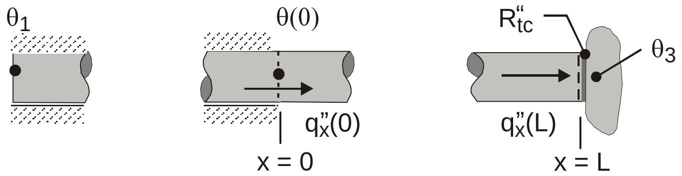
At x = 0, thermal resistance of rod

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right|_{x=0} = kA_c \frac{\theta_1 - \theta(0)}{L} \qquad \theta_1 = T_1 - T_\infty$$

$$-\left[m C_1 e^0 - m C_2 e^0 \right] = \frac{1}{L}\left[\theta_1 - \left(C_1 e^0 + C_2 e^0 \right) \right] \tag{2}$$

Continued …

PROBLEM 3.119 (Cont.)



At x=L, thermal contact resistance

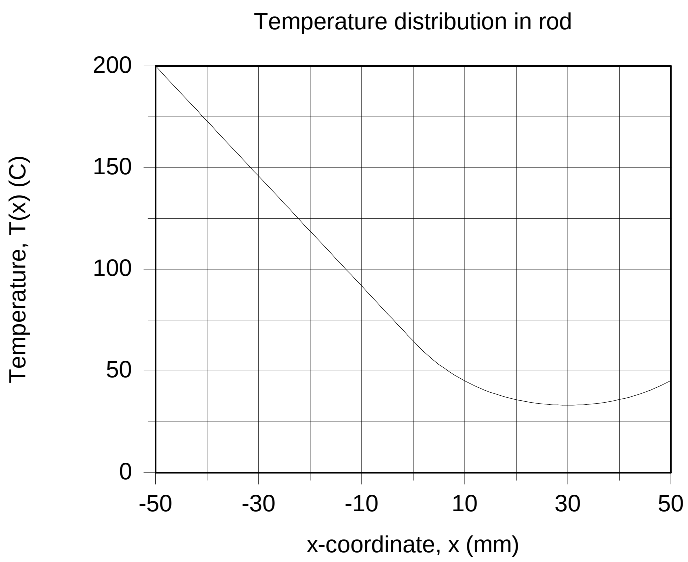
$$q_x(+L) = -kA_c \left.\frac{d\theta}{dx}\right|_{x=L} = \frac{\theta(L)-\theta_3}{R''_{tc}/A_c} \qquad \theta_3 = T_3 - T_\infty$$

$$-k\left[mC_1e^{mL} - mC_2e^{-mL}\right] = \frac{1}{R''_{tc}}\left[C_1e^{mL} + C_2e^{-mL} - \theta_3\right] \tag{3}$$

Eqs. (2) and (3) cannot be rearranged easily to find explicit forms for C_1 and C_2. The constraints will be evaluated numerically in part (c). Knowing C_1 and C_2, Eq. (1) gives

$$\theta_2 = \theta(0) = T_2 - T_\infty = C_1 e^0 + C_2e^0 \tag{4}$$

(c) With Eqs. (1-4) in the *IHT Workspace* using numerical values shown in the schematic, find T_2 = 62.1°C. The temperature distribution is shown in the graph below.



COMMENTS: (1) The purpose of asking you to sketch the temperature distribution in part (a) was to give you the opportunity to identify the relevant thermal processes and come to an understanding of the system behavior.

(2) Sketch the temperature distributions for the following conditions and explain their key features: (a) $R''_{tc} = 0$, (b) $R''_{tc} \to \infty$, and (c) the exposed portion of the rod behaves as an infinitely long fin; that is, k is very large.