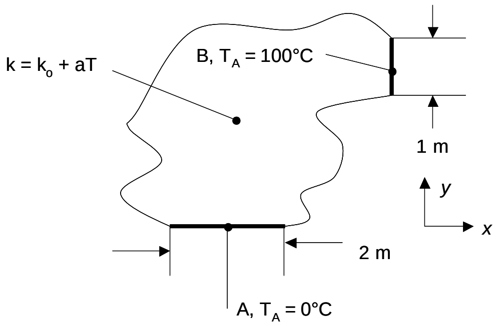
PROBLEM 2.15

KNOWN: Temperature, size and orientation of Surfaces A and B in a two-dimensional geometry. Thermal conductivity dependence on temperature.

FIND: Temperature gradient $\partial T/\partial y$ at surface A.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No volumetric generation, (3) Two-dimensional conduction.

ANALYSIS: At Surface A, $k_A = k_o + aT_A = 10$ W/m·K – 10^{-3} W/m·K^2 × 273 K = 9.73 W/m·K while at Surface B, $k_B = k_o + aT_B = 10$ W/m·K – 10^{-3} W/m·K^2 × 373 K = 9.63 W/m·K. For steady-state conditions, $\dot{E}_{in} = \dot{E}_{out}$ which may be written in terms of Fourier's law as

$$-k_B \left.\frac{\partial T}{\partial x}\right|_B A_B = -k_A \left.\frac{\partial T}{\partial y}\right|_A A_A$$

or

$$\left.\frac{\partial T}{\partial y}\right|_A = \left.\frac{\partial T}{\partial x}\right|_B \frac{k_B A_B}{k_A A_A} = 30\text{K/m} \times \frac{9.63}{9.73} \times \frac{1}{2} = 14.85 \text{ K/m}$$ <

COMMENTS: (1) If the thermal conductivity is not temperature-dependent, then the temperature gradient at A is 15 K/m. (2) Surfaces A and B are both isothermal. Hence, $\partial T/\partial x|_A = \partial T/\partial y|_B = 0$.