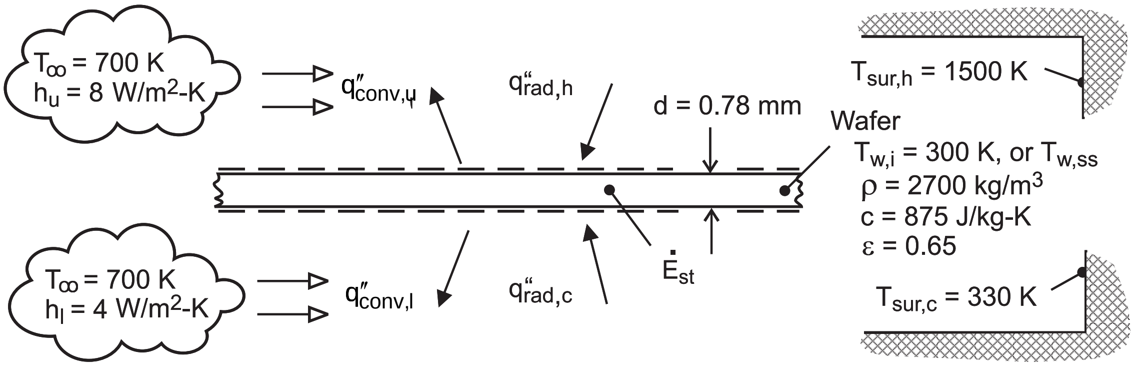
PROBLEM 1.57

KNOWN: Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of the wafer temperature corresponding to the wafer temperature $T_{w,i} = 300\,\text{K}$, and (b) Steady-state temperature reached if the wafer remains in this position. How significant is convection for this situation? Sketch how you'd expect the wafer temperature to vary as a function of vertical distance.

SCHEMATIC:



ASSUMPTIONS: (1) Wafer temperature is uniform, (2) Transient conditions when wafer is initially positioned, (3) Hot and cool zones have uniform temperatures, (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone), and (4) Negligible heat losses from wafer to mounting pin holder.

ANALYSIS: The energy balance on the wafer illustrated in the schematic above includes convection from the upper (u) and lower (l) surfaces with the ambient gas, radiation exchange with the hot- and cool-zone (chamber) surroundings, and the rate of energy storage term for the transient condition.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$q''_{rad,h} + q''_{rad,c} - q''_{conv,u} - q''_{conv,l} = \rho c d \frac{dT_w}{dt}$$

$$\varepsilon\sigma\left(T^4_{sur,h} - T^4_w\right) + \varepsilon\sigma\left(T^4_{sur,c} - T^4_w\right) - h_u\left(T_w - T_\infty\right) - h_l\left(T_w - T_\infty\right) = \rho c d \frac{dT_w}{dt}$$

(a) For the initial condition, the time rate of temperature change of the wafer is determined using the energy balance above with $T_w = T_{w,i} = 300\,\text{K}$,

$$0.65 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\left(1500^4 - 300^4\right)\text{K}^4 + 0.65 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\left(330^4 - 300^4\right)\text{K}^4$$

$$-8\,\text{W/m}^2 \cdot \text{K}\left(300 - 700\right)\text{K} - 4\,\text{W/m}^2 \cdot \text{K}\left(300 - 700\right)\text{K} =$$

$$2700\,\text{kg/m}^3 \times 875\,\text{J/kg} \cdot \text{K} \times 0.00078\,\text{m}\left(dT_w/dt\right)_i$$

$$\left(dT_w/dt\right)_i = 104\,\text{K/s} \qquad <$$

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, $T_w = T_{w,ss}$.

Continued …..

PROBLEM 1.57 (Cont.)

$$0.65\sigma\left(1500^4 - T_{w,ss}^4\right)K^4 + 0.65\sigma\left(330^4 - T_{w,ss}^4\right)K^4$$

$$-8\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K - 4\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K = 0$$

$$T_{w,ss} = 1251\,K$$ <

To determine the relative importance of the convection processes, re-solve the energy balance above ignoring those processes to find $\left(dT_w/dt\right)_i = 101\ K/s$ and $T_{w,ss} = 1262\ K$. We conclude that the radiation exchange processes control the initial time rate of temperature change and the steady-state temperature.

If the wafer were elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone chamber. Conversely, by lowering the wafer, the upper surface would experience less radiant exchange with the hot zone chamber, and its temperature would decrease. The temperature-distance trend might appear as shown in the sketch.