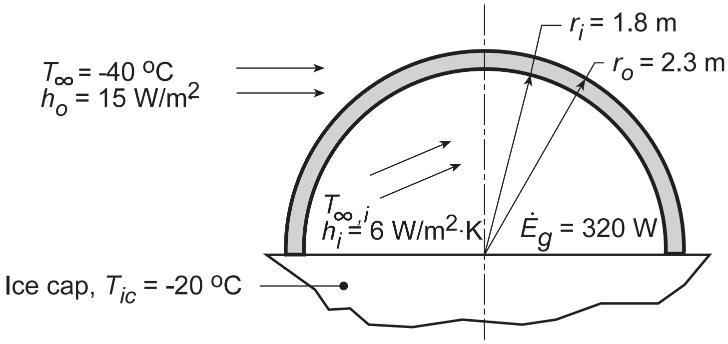
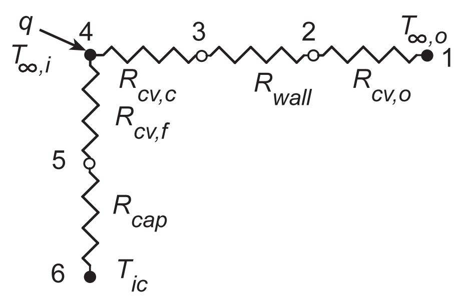
PROBLEM 4.33

KNOWN: Igloo constructed in hemispheric shape sits on ice cap; igloo wall thickness and inside/outside convection coefficients (h_i, h_o) are prescribed.

FIND: (a) Inside air temperature $T_{\infty,i}$ when outside air temperature is $T_{\infty,o}$ = -40°C assuming occupants provide 320 W within igloo, (b) Perform parameter sensitivity analysis to determine which variables have significant effect on T_i.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficient is the same on floor and ceiling of igloo, (3) Floor and ceiling are at uniform temperatures, (4) Floor-ice cap resembles disk on semi-infinite medium, (5) One-dimensional conduction through igloo walls.

PROPERTIES: Ice and compacted snow (given): k = 0.15 W/m·K.

ANALYSIS: (a) The thermal circuit representing the heat loss from the igloo to the outside air and through the floor to the ice cap is shown above. The heat loss is

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{cv,c} + R_{wall} + R_{cv,o}} + \frac{T_{\infty,i} - T_{ic}}{R_{cv,f} + R_{cap}}.$$

Convection, ceiling: $$R_{cv,c} = \frac{2}{h_i\left(4\pi r_i^2\right)} = \frac{2}{6\,\text{W/m}^2\cdot\text{K}\times 4\pi(1.8\,\text{m})^2} = 0.00819\,\text{K/W}$$

Convection, outside: $$R_{cv,o} = \frac{2}{h_o\left(4\pi r_o^2\right)} = \frac{2}{15\,\text{W/m}^2\cdot\text{K}\times 4\pi(2.3\,\text{m})^2} = 0.00201\,\text{K/W}$$

Convection, floor: $$R_{cv,f} = \frac{1}{h_i\left(\pi r_i^2\right)} = \frac{1}{6\,\text{W/m}^2\cdot\text{K}\times \pi(1.8\,\text{m})^2} = 0.01637\,\text{K/W}$$

Conduction, wall: $$R_{wall} = 2\left[\frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)\right] = 2\left[\frac{1}{4\pi\times 0.15\,\text{W/m}\cdot\text{K}}\left(\frac{1}{1.8} - \frac{1}{2.3}\right)\text{m}\right] = 0.1281\,\text{K/W}$$

Conduction, ice cap: $$R_{cap} = \frac{1}{kS} = \frac{1}{4kr_i} = \frac{1}{4\times 0.15\,\text{W/m}\cdot\text{K}\times 1.8\,\text{m}} = 0.9259\,\text{K/W}$$

where S was determined from the shape factor of Table 4.1. Hence,

$$q = 320\,\text{W} = \frac{T_{\infty,i} - (-40)^\circ\text{C}}{(0.00819 + 0.1281 + 0.00201)\,\text{K/W}} + \frac{T_{\infty,i} - (-20)^\circ\text{C}}{(0.01637 + 0.9259)\,\text{K/W}}$$

$320\text{ W} = 7.231(T_{\infty,i} + 40) + 1.06(T_{\infty,i} + 20)$ $\qquad T_{\infty,i} = 1.2°\text{C}$. <

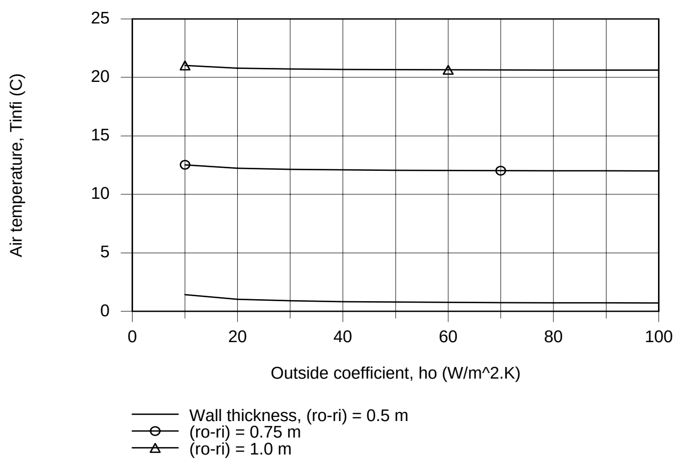
Continued...

(b) Begin the parameter sensitivity analysis to determine important variables which have a significant influence on the inside air temperature by examining the thermal resistances associated with the processes present in the system and represented by the network.

Process	Symbols		Value (K/W)
Convection, outside	$R_{cv,o}$	R21	0.0020
Conduction, wall	R_{wall}	R32	0.1281
Convection, ceiling	$R_{cv,c}$	R43	0.0082
Convection, floor	$R_{cv,f}$	R54	0.0164
Conduction, ice cap	R_{cap}	R65	0.9259

It follows that the convection resistances are negligible relative to the conduction resistance across the igloo wall. As such, only changes to the wall thickness will have an appreciable effect on the inside air temperature relative to the outside ambient air conditions. We don't want to make the igloo walls thinner and thereby allow the air temperature to dip below freezing for the prescribed environmental conditions.

Using the *IHT Thermal Resistance Network Model*, we used the circuit builder to construct the network and perform the energy balances to obtain the inside air temperature as a function of the outside convection coefficient for selected increased thicknesses of the wall.



COMMENTS: (1) From the plot, we can see that the influence of the outside air velocity which controls the outside convection coefficient h_o is negligible.

(2) The thickness of the igloo wall is the dominant thermal resistance controlling the inside air temperature.