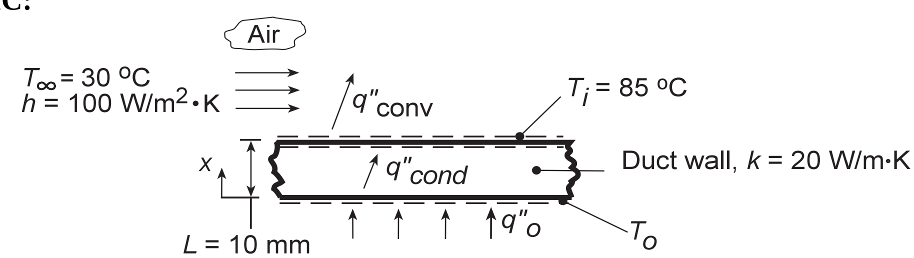
PROBLEM 1.78

KNOWN: Duct wall of prescribed thickness and thermal conductivity experiences prescribed heat flux q''_o at outer surface and convection at inner surface with known heat transfer coefficient.

FIND: (a) Heat flux at outer surface required to maintain inner surface of duct at T_i = 85°C, (b) Temperature of outer surface, T_o, (c) Effect of h on T_o and q''_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Constant properties, (4) Backside of heater perfectly insulated, (5) Negligible radiation.

ANALYSIS: (a) By performing an energy balance on the wall, recognize that $q''_o = q''_{cond}$. From an energy balance on the top surface, it follows that $q''_{cond} = q''_{conv} = q''_o$. Hence, using the convection rate equation,

$$q''_o = q''_{conv} = h\left(T_i - T_\infty\right) = 100\,\mathrm{W/m^2\cdot K}\left(85-30\right)^\circ\mathrm{C} = 5500\,\mathrm{W/m^2}.$$ <

(b) Considering the duct wall and applying Fourier's Law,

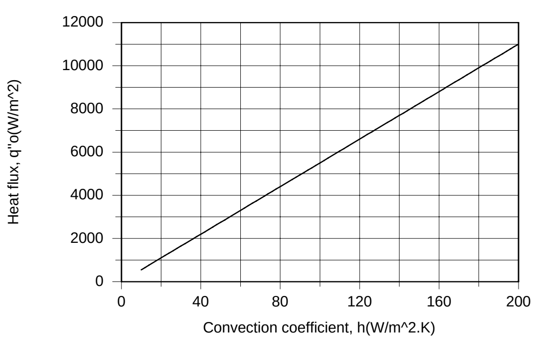
$$q''_o = k\frac{\Delta T}{\Delta X} = k\frac{T_o - T_i}{L}$$

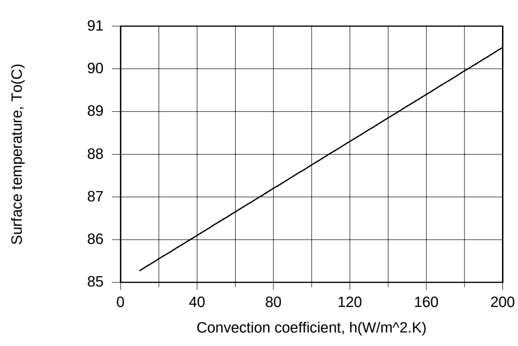
$$T_o = T_i + \frac{q''_o L}{k} = 85^\circ\mathrm{C} + \frac{5500\,\mathrm{W/m^2}\times 0.010\,\mathrm{m}}{20\,\mathrm{W/m\cdot K}} = \left(85+2.8\right)^\circ\mathrm{C} = 87.8^\circ\mathrm{C}.$$ <

(c) For T_i = 85°C, the desired results may be obtained by simultaneously solving the energy balance equations

$$q''_o = k\frac{T_o - T_i}{L} \qquad \text{and} \qquad k\frac{T_o - T_i}{L} = h\left(T_i - T_\infty\right)$$

Using the IHT *First Law* Model for a *Nonisothermal Plane Wall*, the following results are obtained.





Since q''_{conv} increases linearly with increasing h, the applied heat flux q''_o and q''_{cond} must also increase. An increase in q''_{cond}, which, with fixed k, T_i and L, necessitates an increase in T_o.

COMMENTS: The temperature difference across the wall is small, amounting to a maximum value of $\left(T_o - T_i\right)$ = 5.5°C for h = 200 W/m²·K. If the wall were thinner (L < 10 mm) or made from a material with higher conductivity (k > 20 W/m·K), this difference would be reduced.