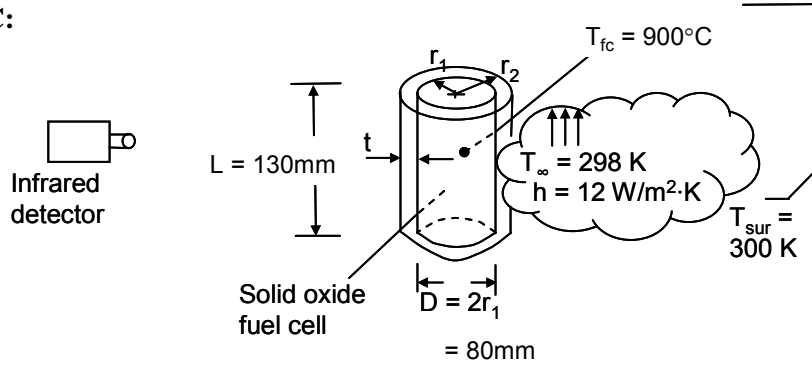# PROBLEM 3.23

KNOWN: Dimensions and temperature of a canister containing a solid oxide fuel cell. Surroundings and ambient temperature.

FIND: (a) Required insulation thickness to keep the equivalent blackbody temperature below 305 K, (b) Canister surface temperature for four cases, (c) Heat flux through the cylindrical walls for four cases.
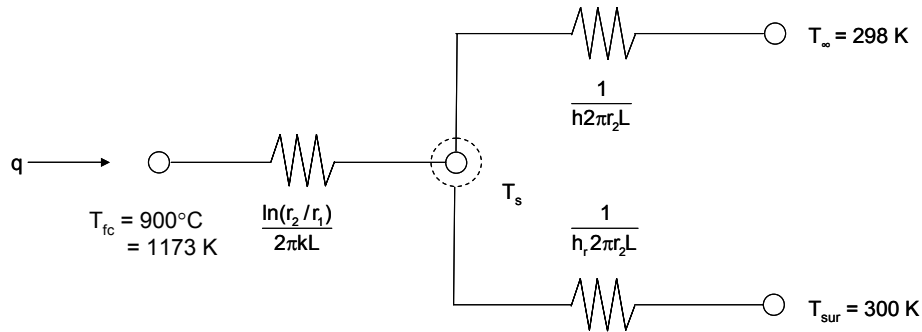
SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer, (4) Large surroundings.

ANALYSIS: The maximum allowable surface temperature may be found by relating the actual and inferred surface temperatures through the relation

$$E_s = E_b = \varepsilon_s \sigma T_s^4 = \sigma T_b^4 \quad \text{or} \quad T_s = (T_b^4/\varepsilon_s)^{1/4} \tag{1}$$

The thermal circuit is

where, from Eq. 1.9,

$$h_r = \varepsilon_s \sigma (T_s + T_{sur})(T_s^2 + T_{sur}^2) \tag{2}$$

Summing currents at the T_s node yields

Continued…

$$\frac{T_{fc} - T_s}{\left[\dfrac{\ln(r_2 / r_1)}{2\pi k L}\right]} = \frac{T_s - T_\infty}{\left[\dfrac{1}{h2\pi r_2 L}\right]} + \frac{T_s - T_{sur}}{\left[\dfrac{1}{h_r 2\pi r_2 L}\right]} \tag{3}$$

where

$$q = \frac{T_{fc} - T_s}{\left[\dfrac{\ln(r_2 / r_1)}{2\pi k L}\right]} \tag{4}$$

Noting that the insulation thickness is $t = r_2 - r_1$, solving Eqs. (2) and (3) simultaneously, and then solving Eq. (4) yields the following results.

ε_s	k (W/m·K)	t(m)	T_s(K)	q(W)
0.08	0.09	0.0126	573.5	161
0.9	0.09	0.151	313.1	40.4
0.08	0.006	0.00095	573.5	125
0.9	0.006	0.0168	313.1	12.0

COMMENTS: (1) Use of the low emissivity surface allows surface temperatures to be high without the fuel cell being detected. (2) The high surface temperature is not safe to the touch. (3) The low thermal conductivity of the aerogel allows the use of a small insulation thickness relative to the calcium silicate. (4) Small heat losses and low surface temperatures are desired. The $\varepsilon_s = 0.9$, k = 0.006 case offers the best performance, and the surface need not be kept in a polished condition to avoid detection.