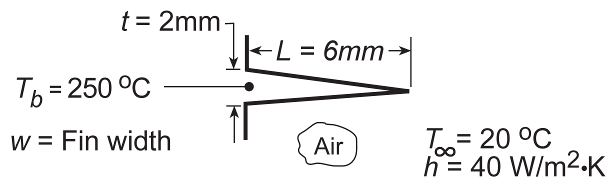
PROBLEM 3.157

KNOWN: Dimensions, base temperature and environmental conditions associated with a triangular, aluminum fin.

FIND: (a) Fin efficiency and effectiveness, (b) Heat dissipation per unit width.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and base contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) With $L_c = L = 0.006$ m, find

$$A_p = Lt/2 = (0.006\,\text{m})(0.002\,\text{m})/2 = 6\times10^{-6}\,\text{m}^2,$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.006\,\text{m})^{3/2}\left(\frac{40\,\text{W/m}^2\cdot\text{K}}{240\,\text{W/m}\cdot\text{K}\times6\times10^{-6}\,\text{m}^2}\right)^{1/2} = 0.077$$

and from Fig. 3.19, the fin efficiency is

$$\eta_f \approx 0.99\,.$$ <

From Eq. 3.91 and Table 3.5, the fin heat rate is

$$q_f = \eta_f q_{max} = \eta_f hA_{f\,(tri)}\theta_b = 2\eta_f hw\left[L^2+(t/2)^2\right]^{1/2}\theta_b\,.$$

From Eq. 3.86, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{2\eta_f hw\left[L^2+(t/2)^2\right]^{1/2}\theta_b}{h(w\cdot t)\theta_b} = \frac{2\eta_f\left[L^2+(t/2)^2\right]^{1/2}}{t}$$

$$\varepsilon_f = \frac{2\times0.99\left[(0.006)^2+(0.002/2)^2\right]^{1/2}\,\text{m}}{0.002\,\text{m}} = 6.02$$ <

(b) The heat dissipation per unit width is

$$q_f' = (q_f/w) = 2\eta_f h\left[L^2+(t/2)^2\right]^{1/2}\theta_b$$

$$q_f' = 2\times0.99\times40\,\text{W/m}^2\cdot\text{K}\left[(0.006)^2+(0.002/2)^2\right]^{1/2}\,\text{m}\times(250-20)^\circ\text{C} = 110.8\,\text{W/m}\,.$$ <

COMMENTS: The parabolic profile is known to provide the maximum heat dissipation per unit fin mass.