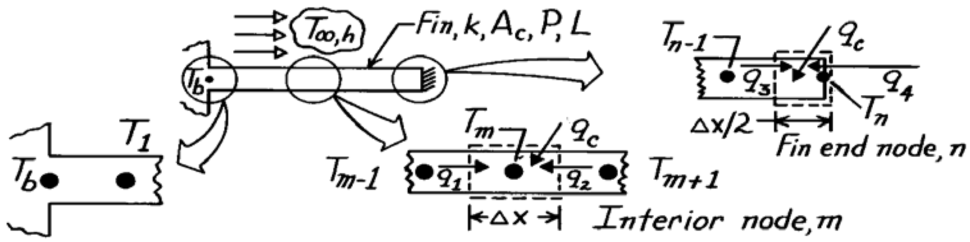
PROBLEM 4.49

KNOWN: One-dimensional fin of uniform cross section insulated at one end with prescribed base temperature, convection process on surface, and thermal conductivity.

FIND: Finite-difference equation for these nodes: (a) Interior node, m and (b) Node at end of fin, n, where x = L.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction.

ANALYSIS: (a) The control volume about node m is shown in the schematic; the node spacing and control volume length in the x direction are both Δx. The uniform cross-sectional area and fin perimeter are A_c and P, respectively. The heat transfer process on the control surfaces, q_1 and q_2, represent conduction while q_c is the convection heat transfer rate between the fin and ambient fluid. Performing an energy balance, find

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_c = 0$$

$$kA_c \frac{T_{m-1} - T_m}{\Delta x} + kA_c \frac{T_{m+1} - T_m}{\Delta x} + hP\Delta x \left(T_\infty - T_m\right) = 0.$$

Multiply the expression by $\Delta x / kA_c$ and regroup to obtain

$$T_{m-1} + T_{m+1} + \frac{hP}{kA_c} \cdot \Delta x^2 T_\infty - \left[2 + \frac{hP}{kA_c} \Delta x^2\right] T_m = 0 \qquad 1<m<n \qquad <$$

Considering now the special node m = 1, then the m-1 node is T_b, the base temperature. The finite-difference equation would be

$$T_b + T_2 + \frac{hP}{kA_c} \Delta x^2 T_\infty - \left[2 + \frac{hP}{kA_c} \Delta x^2\right] T_1 = 0 \qquad m=1 \qquad <$$

(b) The control volume of length $\Delta x/2$ about node n is shown in the schematic. Performing an energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_3 + q_4 + q_c = 0$$

$$kA_c \frac{T_{n-1} - T_n}{\Delta x} + 0 + hP \frac{\Delta x}{2} \left(T_\infty - T_n\right) = 0.$$

Note that $q_4 = 0$ since the end (x = L) is insulated. Multiplying by $\Delta x / kA_c$ and regrouping,

$$T_{n-1} + \frac{hP}{kA_c} \cdot \frac{\Delta x^2}{2} T_\infty - \left[\frac{hP}{kA_c} \cdot \frac{\Delta x^2}{2} + 1\right] T_n = 0. \qquad <$$

COMMENTS: The value of Δx will be determined by the selection of n; that is, $\Delta x = L/n$. Note that the grouping, hP/kA_c, appears in the finite-difference and differential forms of the energy balance.