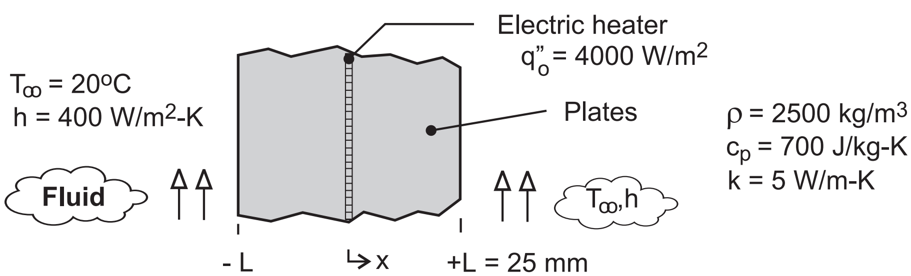
PROBLEM 2.53

KNOWN: Thin electrical heater dissipating 4000 W/m^2 sandwiched between two 25-mm thick plates whose surfaces experience convection.

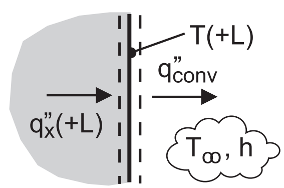
FIND: (a) On T-x coordinates, sketch the steady-state temperature distribution for $-L \le x \le +L$; calculate values for the surfaces x = L and the mid-point, x = 0; label this distribution as Case 1 and explain key features; (b) Case 2: sudden loss of coolant causing existence of adiabatic condition on the x = +L surface; sketch temperature distribution on same T-x coordinates as part (a) and calculate values for x = 0, ± L; explain key features; (c) Case 3: further loss of coolant and existence of adiabatic condition on the x = - L surface; situation goes undetected for 15 minutes at which time power to the heater is deactivated; determine the eventual ($t \to \infty$) uniform, steady-state temperature distribution; sketch temperature distribution on same T-x coordinates as parts (a,b); and (d) On T-t coordinates, sketch the temperature-time history at the plate locations x = 0, ± L during the transient period between the steady-state distributions for Case 2 and Case 3; at what location and when will the temperature in the system achieve a maximum value?

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal volumetric generation in plates, and (3) Negligible thermal resistance between the heater surfaces and the plates.

ANALYSIS: (a) Since the system is symmetrical, the heater power results in equal conduction fluxes through the plates. By applying a surface energy balance on the surface x = +L as shown in the schematic, determine the temperatures at the mid-point, x = 0, and the exposed surface, x + L.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(+L) - q''_{conv} = 0 \qquad \text{where} \qquad q''_x(+L) = q''_o/2$$

$$q''_o/2 - h\left[T(+L) - T_\infty\right] = 0$$

$$T_1(+L) = q''_o/2h + T_\infty = 4000\,\text{W/m}^2 / \left(2 \times 400\,\text{W/m}^2 \cdot \text{K}\right) + 20°\text{C} = 25°\text{C} \qquad <$$

From Fourier's law for the conduction flux through the plate, find T(0).

$$q''_x = q''_o/2 = k\left[T(0) - T(+L)\right]/L$$

$$T_1(0) = T_1(+L) + q''_o L/2k = 25°\text{C} + 4000\,\text{W/m}^2 \cdot \text{K} \times 0.025\text{m} / \left(2 \times 5\,\text{W/m} \cdot \text{K}\right) = 35°\text{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates below and labeled Case 1. The key features of the distribution are its symmetry about the heater plane and its linear dependence with distance.

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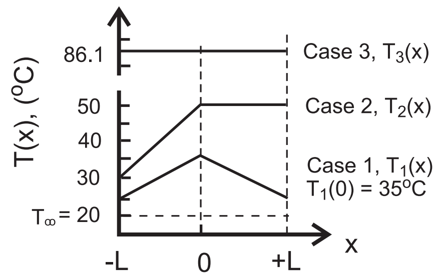
PROBLEM 2.53 (Cont.)



(b) Case 2: sudden loss of coolant with the existence of an adiabatic condition on surface x = +L. For this situation, all the heater power will be conducted to the coolant through the left-hand plate. From a surface energy balance and application of Fourier's law as done for part (a), find

$$T_2(-L) = q_o''/h + T_\infty = 4000\,\mathrm{W/m^2}/400\,\mathrm{W/m^2\cdot K} + 20°\mathrm{C} = 30°\mathrm{C} \quad <$$

$$T_2(0) = T_2(-L) + q_o'' L/k = 30°\mathrm{C} + 4000\,\mathrm{W/m^2} \times 0.025\ \mathrm{m}/5\,\mathrm{W/m\cdot K} = 50°\mathrm{C} \quad <$$

The temperature distribution is shown on the T-x coordinates above and labeled Case 2. The distribution is linear in the left-hand plate, with the maximum value at the mid-point. Since no heat flows through the right-hand plate, the gradient must zero and this plate is at the maximum temperature as well. The maximum temperature is higher than for Case 1 because the heat flux through the left-hand plate has increased two-fold.

(c) Case 3: sudden loss of coolant occurs at the x = -L surface also. For this situation, there is no heat transfer out of either plate, so that for a 15-minute period, Δt_o, the heater dissipates 4000 W/m^2 and then is deactivated. To determine the eventual, uniform steady-state temperature distribution, apply the conservation of energy requirement on a time-interval basis, Eq. 1.12b. The initial condition corresponds to the temperature distribution of Case 2, and the final condition will be a uniform, elevated temperature $T_f = T_3$ representing Case 3. We have used T_∞ as the reference condition for the energy terms.

$$E_{in}'' - E_{out}'' + E_{gen}'' = \Delta E_{st}'' = E_f'' - E_i'' \tag{1}$$

Note that $E_{in}'' - E_{out}'' = 0$, and the dissipated electrical energy is

$$E_{gen}'' = q_o''\Delta t_o = 4000\,\mathrm{W/m^2}(15\times 60)\mathrm{s} = 3.600\times 10^6\ \mathrm{J/m^2} \tag{2}$$

For the final condition,

$$E_f'' = \rho c(2L)[T_f - T_\infty] = 2500\,\mathrm{kg/m^3}\times 700\,\mathrm{J/kg\cdot K}(2\times 0.025\mathrm{m})[T_f - 20]°\mathrm{C}$$
$$E_f'' = 8.75\times 10^4[T_f - 20]\,\mathrm{J/m^2} \tag{3}$$

where $T_f = T_3$, the final uniform temperature, Case 3. For the initial condition,

$$E_i'' = \rho c\int_{-L}^{+L}[T_2(x) - T_\infty]dx = \rho c\left\{\int_{-L}^{0}[T_2(x) - T_\infty]dx + \int_0^{+L}[T_2(0) - T_\infty]dx\right\} \tag{4}$$

where $T_2(x)$ is linear for $-L \le x \le 0$ and constant at $T_2(0)$ for $0 \le x \le +L$.

$$T_2(x) = T_2(0) + [T_2(0) - T_2(L)]x/L \qquad -L \le x \le 0$$

$$T_2(x) = 50°\mathrm{C} + [50 - 30]°\mathrm{C}\,x/0.025\mathrm{m}$$

$$T_2(x) = 50°\mathrm{C} + 800x \tag{5}$$

Substituting for $T_2(x)$, Eq. (5), into Eq. (4)

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$$E_i'' = \rho c\left\{\int_{-L}^{0}\left[50+800x-T_\infty\right]dx+\left[T_2(0)-T_\infty\right]L\right\}$$

$$E_i'' = \rho c\left\{\left[50x+400x^2-T_\infty x\right]_{-L}^{0}+\left[T_2(0)-T_\infty\right]L\right\}$$

$$E_i'' = \rho c\left\{-\left[-50L+400L^2+T_\infty L\right]+\left[T_2(0)-T_\infty\right]L\right\}$$

$$E_i'' = \rho cL\left\{+50-400L-T_\infty+T_2(0)-T_\infty\right\}$$

$$E_i'' = 2500\,\mathrm{kg/m^3}\times 700\,\mathrm{J/kg\cdot K}\times 0.025\ \mathrm{m}\left\{+50-400\times 0.025-20+50-20\right\}\mathrm{K}$$

$$E_i'' = 2.188\times 10^6\ \mathrm{J/m^2} \tag{6}$$

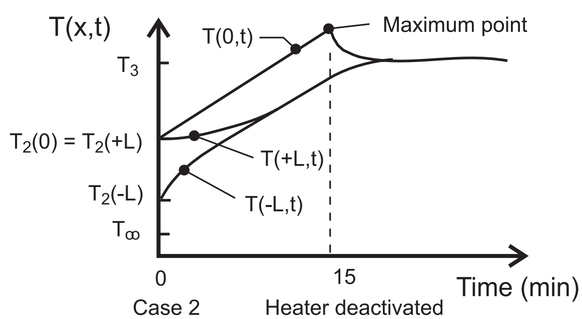
Returning to the energy balance, Eq. (1), and substituting Eqs. (2), (3) and (6), find $T_f = T_3$.

$$3.600\times 10^6\ \mathrm{J/m^2} = 8.75\times 10^4\left[T_3-20\right]-2.188\times 10^6\ \mathrm{J/m^2}$$

$$T_3 = (66.1+20)\,^\circ\mathrm{C} = 86.1\,^\circ\mathrm{C}$$ <

The temperature distribution is shown on the T-x coordinates above and labeled Case 3. The distribution is uniform, and considerably higher than the maximum value for Case 2.

(d) The temperature-time history at the plate locations x = 0, ± L during the transient period between the distributions for Case 2 and Case 3 are shown on the T-t coordinates below.



Note the temperatures for the locations at time t = 0 corresponding to the instant when the surface x = - L becomes adiabatic. These temperatures correspond to the distribution for Case 2. The heater remains energized for yet another 15 minutes and then is deactivated. The midpoint temperature, T(0,t), is always the hottest location and the maximum value slightly exceeds the final temperature T_3.