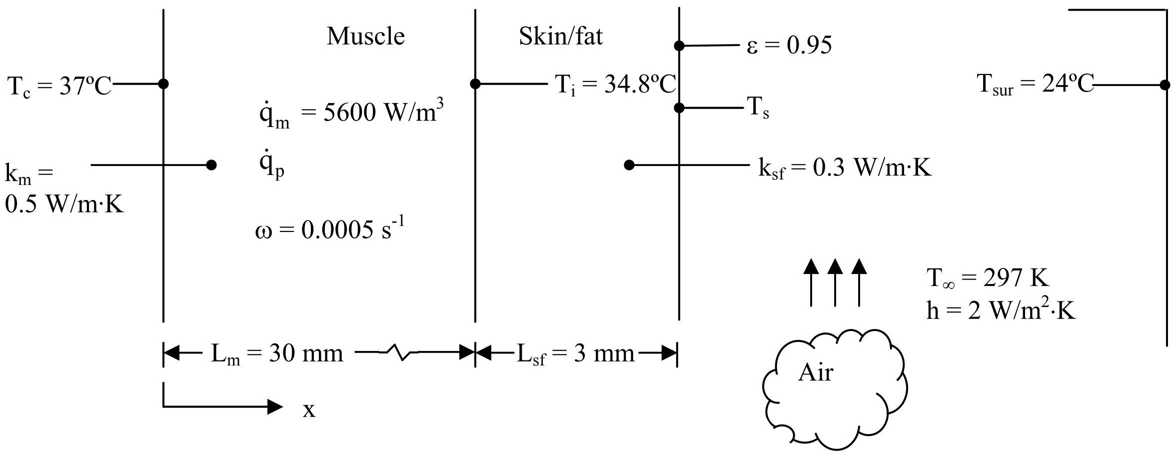
PROBLEM 3.165

KNOWN: Dimensions and thermal conductivities of a muscle layer and a skin/fat layer. Skin emissivity and surface area. Metabolic heat generation rate and perfusion rate within the muscle layer. Core body and arterial temperatures. Blood density and specific heat. Ambient conditions.

FIND: Perspiration rate to maintain same skin temperature as in Example 3.12.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer through the muscle and skin/fat layers, (3) Metabolic heat generation rate, perfusion rate, arterial temperature, blood properties, and thermal conductivities are all uniform, (4) Radiation heat transfer coefficient is known from Example 1.7, (5) Solar radiation is negligible, (6) Conditions are the same everywhere on the torso, limbs, etc., (7) Perspiration on skin has a negligible effect on heat transfer from the skin to the environment, that is, it adds a negligible thermal resistance and doesn't change the emissivity.

ANALYSIS: First we need to find the skin temperature, T_s, for the conditions of Example 3.12, in the air environment. Both q and T_i, the interface temperature between the muscle and the skin/fat layer, are known. The rate of heat transfer across the skin/fat layer is given by

$$q = \frac{k_{sf}A(T_i - T_s)}{L_{sf}} \tag{1}$$

Thus, the skin temperature is

$$T_s = T_i - \frac{qL_{sf}}{k_{sf}A} = 34.8°C - \frac{142\ W \times 0.003\ m}{0.3\ W/m\cdot K \times 1.8\ m^2} = 34.0°C$$

Now the heat transfer rate will change because of the increased metabolic heat generation rate. Heat transfer in the muscle layer is governed by Equation 3.114. In Example 3.12, this equation was solved subject to specified temperature boundary conditions, and the rate at which heat leaves the muscle and enters the skin/fat layer was found to be

$$q\big|_{x=L_m} = -k_m A\tilde{m}\theta_c \frac{(\theta_i/\theta_c)\cosh\tilde{m}L_m - 1}{\sinh\tilde{m}L_m} \tag{2}$$

Continued…

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This must equal the rate at which heat is transferred across the skin/fat layer, given by Equation (1). Equating Equations 1 and 2 and solving for T_i, recalling that T_i also appears in θ_i, yields

$$T_i = \frac{T_s \sinh\tilde{m}L_m + k_m\tilde{m}\dfrac{L_{sf}}{k_{sf}}\left[\theta_c + \left(T_a + \dfrac{\dot{q}_m}{\omega\rho_b c_b}\right)\cosh\tilde{m}L_m\right]}{\sinh\tilde{m}L_m + k_m\tilde{m}\dfrac{L_{sf}}{k_{sf}}\cosh\tilde{m}L_m}$$

where

$$\tilde{m} = \sqrt{\omega\rho_b c_b/k_m} = \left[0.0005\ s^{-1} \times 1000\ kg/m^3 \times 3600\ J/kg\cdot K/0.5\ W/m\cdot K\right]^{1/2} = 60\ m^{-1}$$

$$\sinh(\tilde{m}L_m) = \sinh(60\ m^{-1} \times 0.03\ m) = 2.94\ ; \quad \cosh(\tilde{m}L_m) = \cosh(60\ m^{-1} \times 0.03\ m) = 3.11$$

$$\theta_c = T_c - T_a - \frac{\dot{q}_m}{\omega\rho_b c_b} = -\frac{\dot{q}_m}{\omega\rho_b c_b} = -\frac{5600\ W/m^3}{0.0005\ s^{-1} \times 1000\ kg/m^3 \times 3600\ J/kg\cdot K} = -3.11\ K$$

The excess temperature can be expressed in kelvins or degrees Celsius, since it is a temperature difference. Thus

$$T_i = \frac{34.0°C \times 2.94 + 0.5\ W/m\cdot K \times 60\ m^{-1} \times \dfrac{0.003\ m}{0.3\ W/m\cdot K}\left[-3.11°C + \left(37°C + 3.11°C\right) \times 3.11\right]}{2.94 + 0.5\ W/m\cdot K \times 60\ m^{-1} \times \dfrac{0.003\ m}{0.3\ W/m\cdot K} \times 3.11}$$

$$T_i = 35.2°C$$

and again from Equation (1)

$$q = \frac{k_{sf}A\left(T_i - T_s\right)}{L_{sf}} = \frac{0.3\ W/m\cdot K \times 1.8\ m^2(35.2°C - 34.0°C)}{0.003\ m} = 222\ W$$

Since the skin temperature is unchanged from Example 3.12, the rate of heat transfer to the environment by convection and radiation will remain the same, and is therefore still 142 W. The difference of 80 W must be removed from the skin by perspiration, therefore

$$q_{per} = \dot{m}_{per}h_{fg} = 80\ W$$

Assuming the properties of perspiration are the same as that of water, evaluated at the skin temperature of 307 K, then from Table A.6 h_{fg} = 2421 kJ/kg and ρ = 994 kg/m^3. Thus the volume rate of perspiration is

$$\dot{\forall} = \frac{\dot{m}_{per}}{\rho} = \frac{q_{per}}{\rho h_{fg}} = \frac{80\ W}{994\ kg/m^3 \times 2421 \times 10^3\ J/kg} = 3.3 \times 10^{-8}\ m^3/s = 3.3 \times 10^{-5}\ \ell/s$$ <

COMMENTS: (1) This is a moderate rate of perspiration. In one hour, it would account for around 0.1 ℓ. (2) In reality, our bodies adjust in many ways to maintain core and skin temperatures. Exercise will likely cause an increase in perfusion rate near the skin surface, to locally elevate the temperature and increase the rate of heat transfer to the environment.