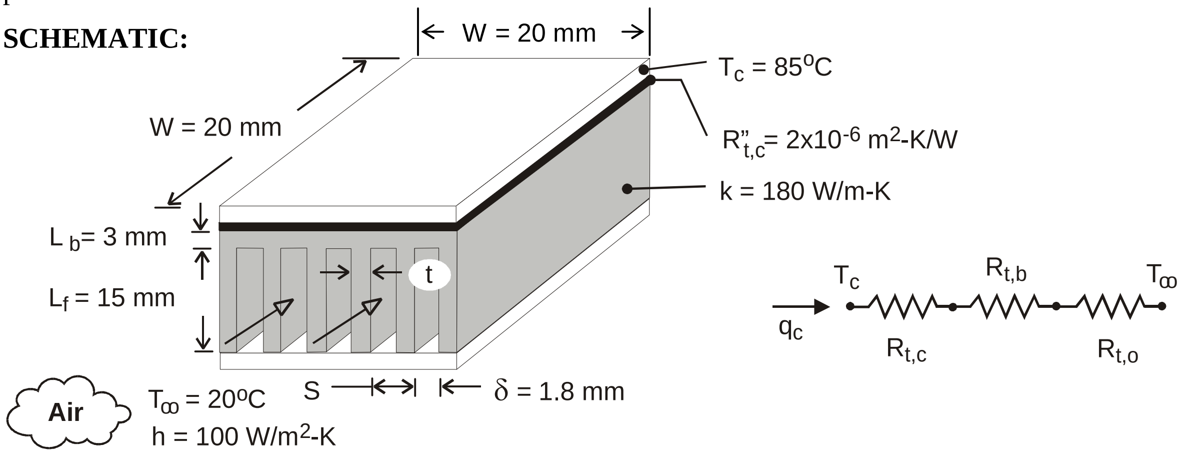
PROBLEM 3.144

KNOWN: Dimensions and maximum allowable temperature of an electronic chip. Thermal contact resistance between chip and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through the heat sink.

FIND: (a) Maximum allowable chip power for heat sink with prescribed number of fins, fin thickness, and fin pitch, and (b) Effect of fin thickness/number and convection coefficient on performance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surfaces of heat sink, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{tot}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

where $R_{t,c} = R''_{t,c}/W^2 = 2\times10^{-6}\,m^2\cdot K/W/(0.02m)^2 = 0.005\ K/W$ and $R_{t,b} = L_b/k\left(W^2\right)$

$= 0.003m/180\ W/m\cdot K\,(0.02m)^2 = 0.042\ K/W$. From Eqs. (3.108), (3.107), and (3.104)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \qquad \eta_o = 1 - \frac{N A_f}{A_t}(1-\eta_f), \qquad A_t = N A_f + A_b$$

where $A_f = 2WL_f = 2 \times 0.02m \times 0.015m = 6 \times 10^{-4}\ m^2$ and $A_b = W^2 - N(tW) = (0.02m)^2 - 11(0.182 \times 10^{-3}\ m \times 0.02m) = 3.6 \times 10^{-4}\ m^2$. With $mL_f = (2h/kt)^{1/2} L_f = (200\ W/m^2\cdot K/180\ W/m\cdot K \times 0.182 \times 10^{-3} m)^{1/2}\ (0.015m) = 1.17$, $\tanh mL_f = 0.824$ and Eq. (3.92) yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

It follows that $A_t = 6.96 \times 10^{-3}\ m^2$, $\eta_o = 0.719$, $R_{t,o} = 2.00$ K/W, and

$$q_c = \frac{(85-20)°C}{(0.005+0.042+2.00)K/W} = 31.8\ W \qquad <$$

(b) The following results are obtained from parametric calculations performed to explore the effect of decreasing the number of fins and increasing the fin thickness.

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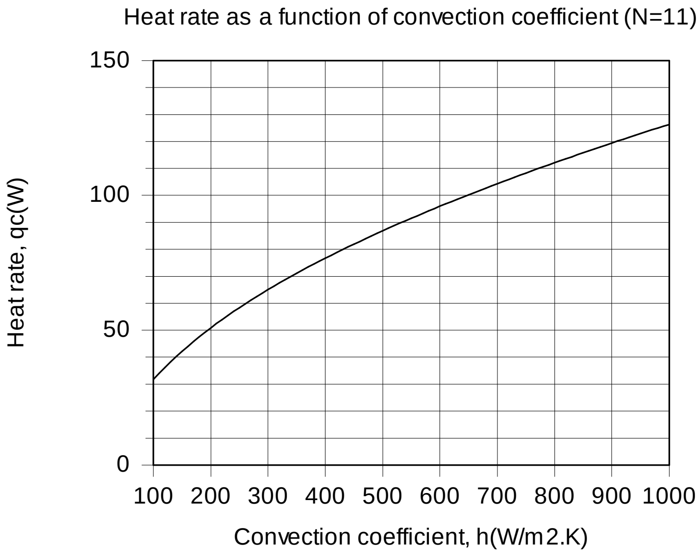
PROBLEM 3.144 (Cont.)

N	t(mm)	η_f	$R_{t,o}$ (K/W)	q_c (W)	A_t (m^2)
6	1.833	0.957	2.76	23.2	0.00378
7	1.314	0.941	2.40	26.6	0.00442
8	0.925	0.919	2.15	29.7	0.00505
9	0.622	0.885	1.97	32.2	0.00569
10	0.380	0.826	1.89	33.5	0.00632
11	0.182	0.704	2.00	31.8	0.00696

Although η_f (and η_o) increases with decreasing N (increasing t), there is a reduction in A_t which yields a minimum in $R_{t,o}$, and hence a maximum value of q_c, for N = 10. For N = 11, the effect of h on the performance of the heat sink is shown below.



With increasing h from 100 to 1000 W/m^2·K, $R_{t,o}$ decreases from 2.00 to 0.47 K/W, despite a decrease in η_f (and η_o) from 0.704 (0.719) to 0.269 (0.309). The corresponding increase in q_c is significant.

COMMENTS: (1) The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with h = 100 W/m^2·K, R_{tot} = 2.05 K/W from Part (a) would be replaced by $R_{cnv} = 1/hW^2$ = 25 K/W, yielding q_c = 2.60 W. (2) The air temperature will increase as it flows through the heat sink. Therefore the required air velocity will be greater than determined here. See Problem 11.89.