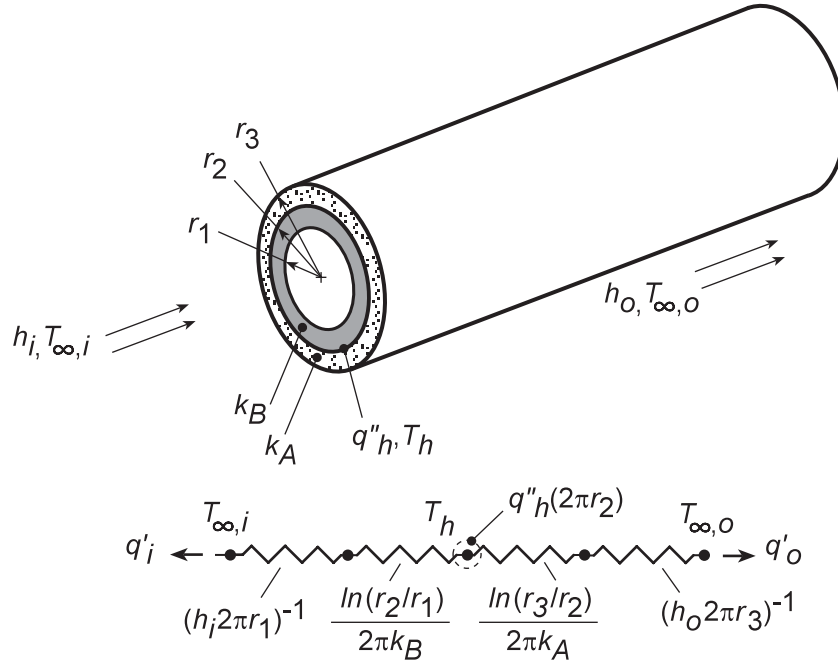# PROBLEM 3.57

KNOWN: Conditions associated with a composite wall and a thin electric heater.

FIND: (a) Equivalent thermal circuit, (b) Expression for heater temperature, (c) Ratio of outer and inner heat flows and conditions for which ratio is minimized.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties, (3) Isothermal heater, (4) Negligible contact resistance(s).

ANALYSIS: (a) On the basis of a unit axial length, the circuit, thermal resistances, and heat rates are as shown in the schematic.

(b) Performing an energy balance for the heater, $\dot{E}_{in} = \dot{E}_{out}$, it follows that

$$q_h''\left(2\pi r_2\right) = q_i' + q_o' = \frac{T_h - T_{\infty,i}}{\left(h_i\, 2\pi r_1\right)^{-1} + \dfrac{\ln\left(r_2/r_1\right)}{2\pi k_B}} + \frac{T_h - T_{\infty,o}}{\left(h_o\, 2\pi r_3\right)^{-1} + \dfrac{\ln\left(r_3/r_2\right)}{2\pi k_A}} \qquad <$$

(c) From the circuit,

$$\frac{q_o'}{q_i'} = \frac{\left(T_h - T_{\infty,o}\right)}{\left(T_h - T_{\infty,i}\right)} \times \frac{\left(h_i\, 2\pi r_1\right)^{-1} + \dfrac{\ln\left(r_2/r_1\right)}{2\pi k_B}}{\left(h_o\, 2\pi r_3\right)^{-1} + \dfrac{\ln\left(r_3/r_2\right)}{2\pi k_A}} \qquad <$$

To reduce q_o'/q_i', one could increase k_B, h_i, and r_3/r_2, while reducing k_A, h_o and r_2/r_1.

COMMENTS: Contact resistances between the heater and materials A and B could be important.