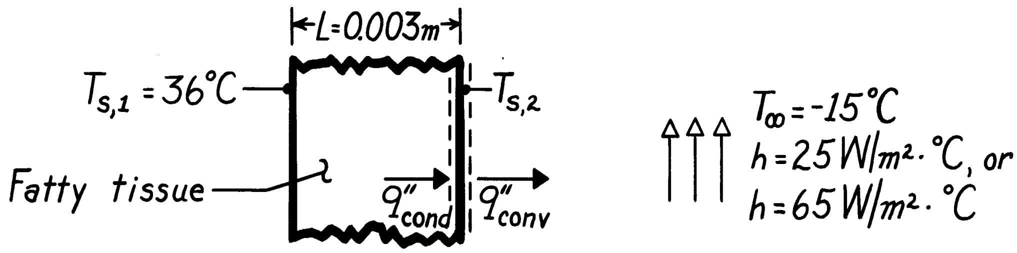
PROBLEM 3.10

KNOWN: A layer of fatty tissue with fixed inside temperature can experience different outside convection conditions.

FIND: (a) Ratio of heat loss for different convection conditions, (b) Outer surface temperature for different convection conditions, and (c) Temperature of still air which achieves same cooling as moving air (*wind chill* effect).

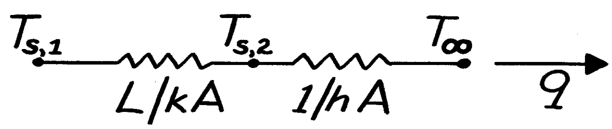
SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) Homogeneous medium with constant properties, (4) No internal heat generation (metabolic effects are negligible), (5) Negligible radiation effects.

PROPERTIES: *Table A-3*, Tissue, fat layer: k = 0.2 W/m·K.

ANALYSIS: The thermal circuit for this situation is



Hence, the heat rate is

$$q = \frac{T_{s,1} - T_\infty}{R_{tot}} = \frac{T_{s,1} - T_\infty}{L/kA + 1/hA}.$$

Therefore,

$$\frac{q''_{calm}}{q''_{windy}} = \frac{\left[\frac{L}{k} + \frac{1}{h}\right]_{windy}}{\left[\frac{L}{k} + \frac{1}{h}\right]_{calm}}.$$

Applying a surface energy balance to the outer surface, it also follows that

$$q''_{cond} = q''_{conv}.$$

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Hence,

$$\frac{k}{L}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_{\infty}\right)$$

$$T_{s,2}=\frac{T_{\infty}+\dfrac{k}{hL}T_{s,1}}{1+\dfrac{k}{hL}}.$$

To determine the wind chill effect, we must determine the heat loss for the windy day and use it to evaluate the hypothetical ambient air temperature, T'_{∞}, which would provide the same heat loss on a calm day, Hence,

$$q''=\frac{T_{s,1}-T_{\infty}}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{\text{windy}}}=\frac{T_{s,1}-T'_{\infty}}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{\text{calm}}}$$

From these relations, we can now find the results sought:

(a)
$$\frac{q''_{\text{calm}}}{q''_{\text{windy}}}=\frac{\dfrac{0.003\ \text{m}}{0.2\ \text{W/m}\cdot\text{K}}+\dfrac{1}{65\ \text{W/m}^2\cdot\text{K}}}{\dfrac{0.003\ \text{m}}{0.2\ \text{W/m}\cdot\text{K}}+\dfrac{1}{25\ \text{W/m}^2\cdot\text{K}}}=\frac{0.015+0.0154}{0.015+0.04}$$

$$\frac{q''_{\text{calm}}}{q''_{\text{windy}}}=0.553$$ <

(b)
$$T_{s,2}\big]_{\text{calm}}=\frac{-15^\circ\text{C}+\dfrac{0.2\ \text{W/m}\cdot\text{K}}{\left(25\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\ \text{m}\right)}36^\circ\text{C}}{1+\dfrac{0.2\ \text{W/m}\cdot\text{K}}{\left(25\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\ \text{m}\right)}}=22.1^\circ\text{C}$$ <

$$T_{s,2}\big]_{\text{windy}}=\frac{-15^\circ\text{C}+\dfrac{0.2\ \text{W/m}\cdot\text{K}}{\left(65\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}36^\circ\text{C}}{1+\dfrac{0.2\ \text{W/m}\cdot\text{K}}{\left(65\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}}=10.8^\circ\text{C}$$ <

(c)
$$T'_{\infty}=36^\circ\text{C}-\left(36+15\right)^\circ\text{C}\frac{\left(0.003/0.2+1/25\right)}{\left(0.003/0.2+1/65\right)}=-56.3^\circ\text{C}$$ <

COMMENTS: The wind chill effect is equivalent to a decrease of $T_{s,2}$ by 11.3°C and increase in the heat loss by a factor of $(0.553)^{-1}$ = 1.81.