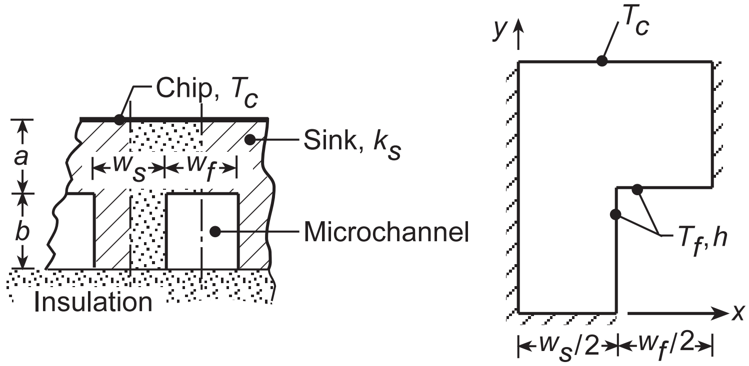
PROBLEM 4.81

KNOWN: Heat sink for cooling computer chips fabricated from copper with microchannels passing fluid with prescribed temperature and convection coefficient.

FIND: (a) Using a square nodal network with 100 μm spatial increment, determine the temperature distribution and the heat rate to the coolant per unit channel length for maximum allowable chip temperature $T_{c,max}$ = 75°C; estimate the thermal resistance betweeen the chip surface and the fluid, $R'_{t,c-f}$ (m·K/W); maximum allowable heat dissipation for a chip that measures 10 x 10 mm on a side; (b) The effect of grid spacing by considering spatial increments of 50 and 25 μm; and (c) Consistent with the requirement that a + b = 400 μm, explore altering the sink dimensions to decrease the thermal resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, and (3) Convection coefficient is uniform over the microchannel surface and independent of the channel dimensions and shape.

ANALYSIS: (a) The square nodal network with Δx = Δy = 100 μm is shown below. Considering symmetry, the nodes 1, 2, 3, 4, 7, and 9 can be treated as interior nodes and their finite-difference equations representing nodal energy balances can be written by inspection. Using the, *IHT Finite-Difference Equations Tool*, appropriate FDEs for the nodes experiencing surface convection can be obtained. The IHT code along with results is included in the Comments. Having the temperature distribution, the heat rate to the coolant per unit channel length for two symmetrical elements can be obtained by applying Newton's law of cooling to the surface nodes,

$$q'_{cv} = 2\left[h\left(\Delta y/2+\Delta x/2\right)\left(T_5-T_\infty\right)+h\left(\Delta x/2\right)\left(T_6-T_\infty\right)+h\left(\Delta y\right)\left(T_8-T_\infty\right)+h\left(\Delta y/2\right)\left(T_{10}-T_\infty\right)\right]$$

$$q'_{cv} = 2\times 30,000\,\mathrm{W/m^2\cdot K}\times 100\times 10^{-6}\,\mathrm{m}\left[(74.02-25)+(74.09-25)/2+(73.60-25)+(73.37-25)/2\right]\mathrm{K}$$

$$q'_{cv} = 878\,\mathrm{W/m} \qquad <$$

The thermal resistance between the chip and fluid per unit length for each microchannel is

$$R'_{t,c-f} = \frac{T_c-T_\infty}{q'_{cv}} = \frac{(75-25)^\circ\mathrm{C}}{878\,\mathrm{W/m}} = 5.69\times 10^{-2}\,\mathrm{m\cdot K/W} \qquad <$$

The maximum allowable heat dissipation for a 10 mm × 10 mm chip is

$$P_{chip,max} = q''_c\times A_{chip} = 2.20\times 10^6\,\mathrm{W/m^2}\times(0.01\times 0.01)\,\mathrm{m^2} = 220\,\mathrm{W} \qquad <$$

where A_{chip} = 10 mm × 10 mm and the heat flux on the chip surface ($w_f + w_s$) is

$$q''_c = q'_{cv}/\left(w_f+w_s\right) = 878\,\mathrm{W/m}/\left(200+200\right)\times 10^{-6}\,\mathrm{m} = 2.20\times 10^6\,\mathrm{W/m^2}$$

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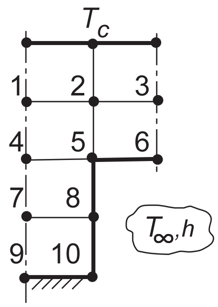
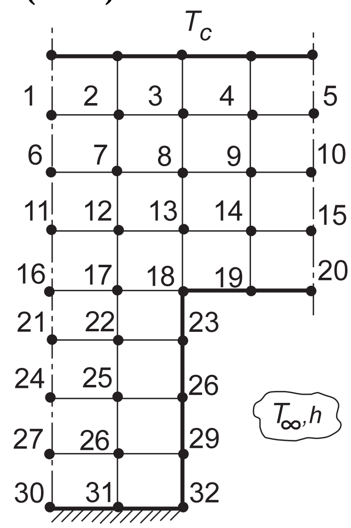




(b) To investigate the effect of grid spacing, the analysis was repreated with a spatial increment of 50 μm (32 nodes as shown above) with the following results

$$q'_{cv} = 881\,\mathrm{W/m} \qquad R'_{t,c-f} = 5.67 \times 10^{-2}\ \mathrm{m \cdot K/W}$$ <

Using a finite-element package with a mesh around 25 μm, we found $R'_{t,c-f} = 5.70 \times 10^{-2}\ \mathrm{m \cdot K/W}$ which suggests the grid spacing effect is not very significant.

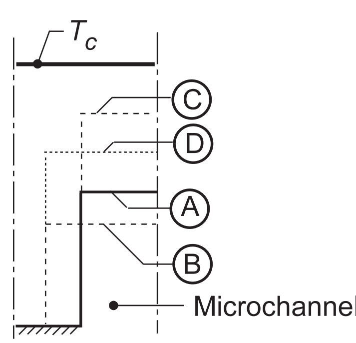
(c) Requring that the overall dimensions of the symmetrical element remain unchanged, we explored what effect changes in the microchannel cross-section would have on the overall thermal resistance, $R'_{t,c-f}$. It is important to recognize that the sink conduction path represents the dominant resistance, since for the convection process

$$R'_{t,cv} = 1/A'_s = 1\Big/\left(30,000\,\mathrm{W/m^2 \cdot K} \times 600 \times 10^{-6}\,\mathrm{m}\right) = 5.55 \times 10^{-2}\,\mathrm{m \cdot K/W}$$

where $A'_s = (w_f + 2b) = 600$ μm.

Using a finite-element package, the thermal resistances per unit length for three additional channel cross-sections were determined and results summarized below.

	Microchannel (μm)		$R'_{t,c-s} \times 10^2$
Case	Height	Half-width	(m·K/W)
A	200	100	5.70
B	133	150	6.12
C	300	100	4.29
D	250	150	4.25



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PROBLEM 4.81 (Cont.)

COMMENTS: (1) The IHT Workspace for the 5x5 coarse node analysis with results follows.

```
// Finite-difference equations - energy balances
// First row - treating as interior nodes considering symmetry
T1 = 0.25 * ( Tc + T2 + T4 + T2 )
T2 = 0.25 * ( Tc + T3 + T5 + T1 )
T3 = 0.25 * ( Tc + T2 + T6 + T2 )

/* Second row - Node 4 treat as interior node; for others, use Tools: Finite-Difference Equations,
Two-Dimensional, Steady-State; be sure to delimit replicated q''a = 0 equations.  */
T4 = 0.25 * ( T1 + T5+ T7 + T5 )
/* Node 5: internal corner node, e-s orientation; e, w, n, s labeled 6, 4, 2, 8. */
0.0 = fd_2d_ic_es(T5,T6,T4,T2,T8,k,qdot,deltax,deltay,Tinf,h,q''a)
q''a = 0                        // Applied heat flux, W/m^2; zero flux shown
/* Node 6: plane surface node, s-orientation; e, w, n labeled 5, 5, 3 . */
0.0 = fd_2d_psur_s(T6,T5,T5,T3,k,qdot,deltax,deltay,Tinf,h,q''a)
//q''a = 0                      // Applied heat flux, W/m^2; zero flux shown

/* Third row - Node 7 treat as interior node; for others, use Tools: Finite-Difference Equations,
Two-Dimensional, Steady-State; be sure to delimit replicated q''a = 0 equations.  */
T7 = 0.25 * (T4 + T8 + T9 + T8)
/* Node 8: plane surface node, e-orientation; w, n, s labeled 7, 5, 10. */
0.0 = fd_2d_psur_e(T8,T7,T5,T10,k,qdot,deltax,deltay,Tinf,h,q''a)
//q''a = 0                      // Applied heat flux, W/m^2; zero flux shown

/* Fourth row - Node 9 treat as interior node; for others, use Tools: Finite-Difference Equations,
Two-Dimensional, Steady-State; be sure to delimit replicated q''a = 0 equations.  */
T9 = 0.25 * (T7 + T10 +T7 + T10)
/* Node 10: plane surface node, e-orientation; w, n, s labeled 9, 8, 8. */
0.0 = fd_2d_psur_e(T10,T9,T8,T8,k,qdot,deltax,deltay,Tinf,h,q''a)
//q''a = 0                      // Applied heat flux, W/m^2; zero flux shown

// Assigned variables
// For the FDE functions,
qdot = 0                        // Volumetric generation, W/m^3
deltax = deltay                 // Spatial increments
deltay = 100e-6                 // Spatial increment, m
Tinf = 25                       // Microchannel fluid temperature, C
h = 30000                       // Convection coefficient, W/m^2.K
// Sink and chip parameters
k = 400                         // Sink thermal conductivity, W/m.K
Tc = 75                         // Maximum chip operating temperature, C
wf = 200e-6                     // Channel width, m
ws = 200e-6                     // Sink width, m

/* Heat rate per unit length, for two symmetrical elements about one microchannel, */
q'cv= 2 * (q'5 + q'6 + q'8 + q'10)
q'5 = h* (deltax / 2 + deltay / 2) * (T5 - Tinf)
q'6 = h * deltax / 2 * (T6 - Tinf)
q'8 = h * deltax  * (T8 - Tinf)
q'10 = h * deltax / 2 * (T10 - Tinf)

/* Thermal resistance between chip and fluid, per unit channel length, */
R'tcf = (Tc - Tinf) / q'cv      // Thermal resistance, m.K/W

// Total power for a chip of 10mm x 10mm, Pchip (W),
q''c = q'cv / (wf + ws)         // Heat flux on chip surface, W/m^2
Pchip = Achip * q''c            // Power, W
Achip = 0.01 * 0.01             // Chip area, m^2

/* Data Browser results: chip power, thermal resistance, heat rates and temperature distribution
Pchip    R'tcf     q''c      q'cv
219.5    0.05694   2.195E6   878.1

T1       T2       T3       T4       T5       T6       T7       T8       T9       T10
74.53    74.52    74.53    74.07    74.02    74.09    73.7     73.6     73.53    73.37 */
```