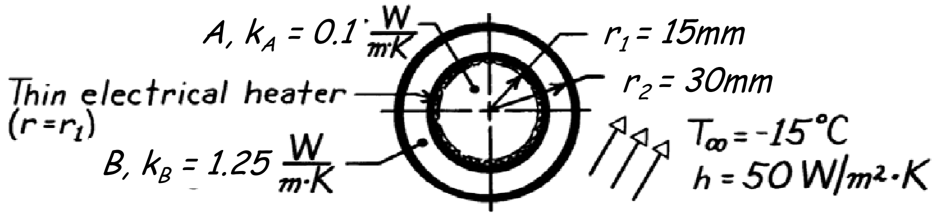
PROBLEM 3.52

KNOWN: Thin electrical heater fitted between two concentric cylinders, the outer surface of which experiences convection.

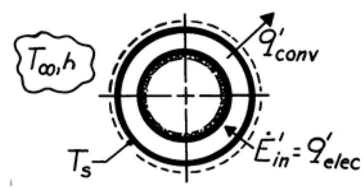
FIND: (a) Electrical power required to maintain outer surface at a specified temperature, (b) Temperature at the center.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Heater element has negligible thickness, (4) Negligible contact resistance between cylinders and heater, (5) Constant properties, (6) No generation.

ANALYSIS: (a) Perform an energy balance on the composite system to determine the power required to maintain $T(r_2) = T_s = 5°C$.



$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$+q'_{elec} - q'_{conv} = 0.$$

Using Newton's law of cooling,

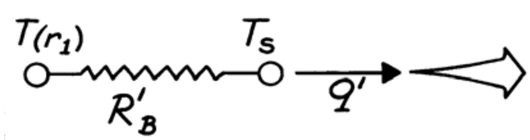
$$q'_{elec} = q'_{conv} = h \cdot 2\pi\ r_2 \left(T_s - T_\infty\right)$$

$$q'_{elec} = 50 \frac{W}{m^2 \cdot K} \times 2\pi\left(0.030m\right)\ \left[5-\left(-15\right)\right]^\circ C = 188\ W/m.$$ <

(b) From a control volume about Cylinder A, we recognize that the cylinder must be isothermal, that is,

$$T(0) = T(r_1).$$

Represent Cylinder B by a thermal circuit:



$$q' = \frac{T\left(r_1\right) - T_s}{R'_B}$$

For the cylinder, from Eq. 3.28,

$$R'_B = \ln r_2 / r_1 / 2\pi\ k_B$$

giving

$$T\left(r_1\right) = T_s + q'R'_B = 5^\circ C + 188 \frac{W}{m} \frac{\ln 30/15}{2\pi \times 1.25\ W/m \cdot K} = 21.6^\circ C$$

Hence, $T(0) = T(r_1) = 21.6°C$. <

Note that k_A has no influence on the temperature T(0).