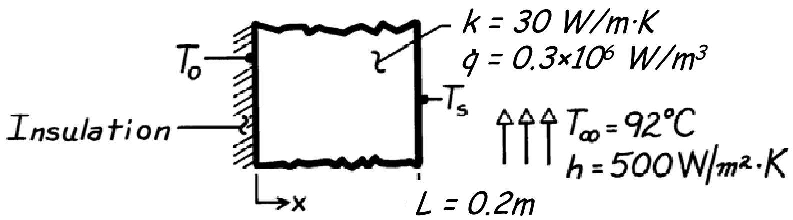
PROBLEM 3.81

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: The temperature at the inner surface is given by Eq. 3.48 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2 / 2k + T_s.$$

The outer surface temperature follows from Eq. 3.51,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92^\circ\text{C} + 0.4\times10^6 \frac{\text{W}}{\text{m}^3} \times 0.2\text{m}/500\text{W/m}^2\cdot\text{K} = 92^\circ\text{C} + 160^\circ\text{C} = 252^\circ\text{C}.$$

It follows that

$$T_o = 0.4\times10^6\text{W/m}^3 \times (0.2\text{m})^2 / 2\times30\text{W/m}\cdot\text{K} + 252^\circ\text{C}$$

$$T_o = 267^\circ\text{C} + 252^\circ\text{C} = 519^\circ\text{C}.$$ <

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 500\text{W/m}^2\cdot\text{K}(252-92)^\circ\text{C} = 80\text{kW/m}^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

$$q''_{conv} = \dot{q}L = 0.4\times10^6\text{W/m}^3 \times 0.2\text{m} = 80\text{kW/m}^2.$$