PROBLEM 4.6

KNOWN: Uniform media of prescribed geometry.

FIND: (a) Shape factor expressions from thermal resistance relations for the plane wall, cylindrical shell and spherical shell, (b) Shape factor expression for the isothermal sphere of diameter D buried in an infinite medium.

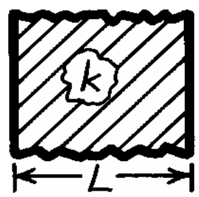
ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform properties.

ANALYSIS: (a) The relationship between the shape factor and thermal resistance of a shape follows from their definitions in terms of heat rates and overall temperature differences.

$$q = kS\Delta T \qquad (4.20), \qquad q = \frac{\Delta T}{R_t} \qquad (3.19), \qquad S = 1/kR_t \qquad (4.21)$$

Using the thermal resistance relations developed in Chapter 3, their corresponding shape factors are:

Plane wall:



$$R_t = \frac{L}{kA} \qquad S = \frac{A}{L}.$$ <

Cylindrical shell:

$$R_t = \frac{\ln(r_2/r_1)}{2\pi Lk} \qquad S = \frac{2\pi L}{\ln r_2/r_1.}$$ <

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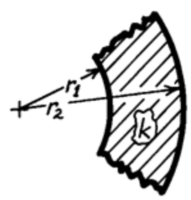
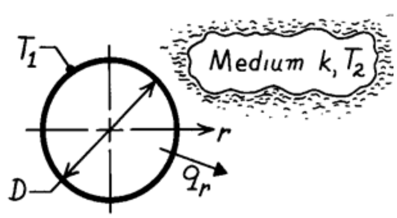


Spherical shell:

$$R_t = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad S = \frac{4\pi}{1/r_1 - 1/r_2}.$$ <

(b) The shape factor for the sphere of diameter D in an infinite medium can be derived using the alternative conduction analysis of Section 3.2. For this situation, q_r is a constant and Fourier's law has the form



$$q_r = -k\left(4\pi\, r^2\right)\frac{dT}{dr}.$$

Separate variables, identify limits and integrate.

$$-\frac{q_r}{4\pi k}\int_{D/2}^{\infty}\frac{dr}{r^2} = \int_{T_1}^{T_2} dT \qquad -\frac{q_r}{4\pi k}\left[-\frac{1}{r}\right]_{D/2}^{\infty} = -\frac{q_r}{4\pi k}\left[0 - \frac{2}{D}\right] = \left(T_2 - T_1\right)$$

$$q_r = 4\pi k\left[\frac{D}{2}\right]\left(T_1 - T_2\right) \qquad \text{or} \qquad S = 2\pi D.$$ <

COMMENTS: Note that the result for the buried sphere, $S = 2\pi D$, can be obtained from the expression for the spherical shell with $r_2 = \infty$. Also, the shape factor expression for the "isothermal sphere buried in a semi-infinite medium" presented in Table 4.1 provides the same result with $z \rightarrow \infty$.