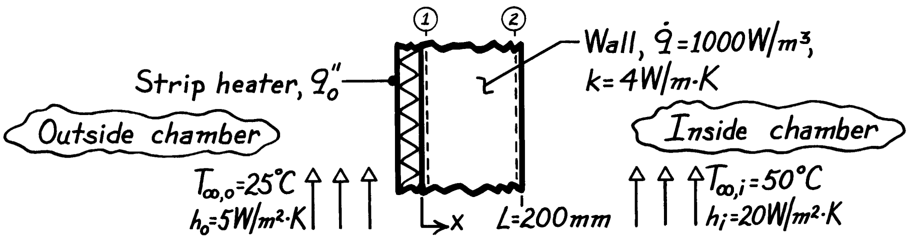
PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

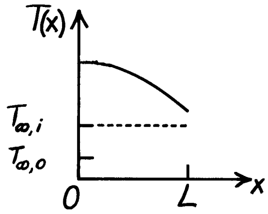
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}=0$ but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \tag{1}$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \qquad \rightarrow \qquad C_1 = 0. \tag{2}$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \tag{3}$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \tag{4}$$

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and from Eq. (3) with x = L and T(L) = T_2,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \qquad (5,6)$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C}+1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2 \cdot \text{K}=50^\circ\text{C}+10^\circ\text{C}=60^\circ\text{C} \qquad <$$

$$T_1 = 60^\circ\text{C}+1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2 \times 4\ \text{W/m} \cdot \text{K}=65^\circ\text{C}. \qquad <$$

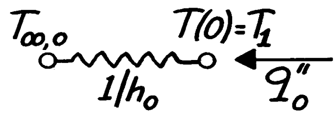
Approach No. 2: Using the boundary condition

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

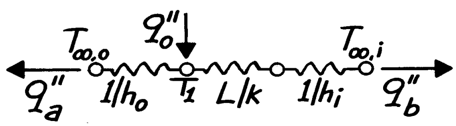
(c) The value of q''_o when T(0) = T_1 = 65°C follows from the circuit



$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2 \cdot \text{K}\,(65\text{-}25)^\circ\,\text{C}=200\ \text{W/m}^2. \qquad <$$

(d) With $\dot{q}=0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k+1/h_i}$$

which yields

$$T_1 = 55^\circ\text{C}. \qquad <$$