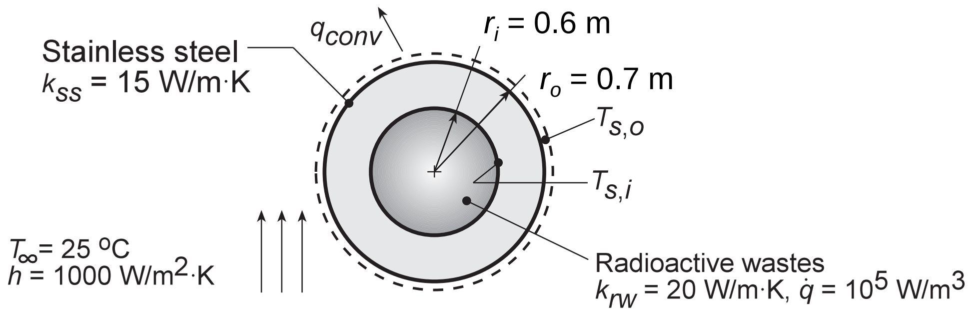
PROBLEM 3.104

KNOWN: Dimensions and thermal conductivity of a spherical container. Thermal conductivity and volumetric energy generation within the container. Outer convection conditions.

FIND: (a) Outer surface temperature, (b) Container inner surface temperature, (c) Temperature distribution within and center temperature of the wastes, (d) Feasibility of operating at twice the energy generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

ANALYSIS: (a) For a control volume which includes the container, conservation of energy yields $\dot{E}_g - \dot{E}_{out} = 0$, or $\dot{q}V - q_{conv} = 0$. Hence

$$\dot{q}(4/3)\left(\pi r_i^3\right) = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

and with $\dot{q} = 10^5$ W/m^3,

$$T_{s,o} = T_\infty + \frac{\dot{q}r_i^3}{3hr_o^2} = 25^\circ\text{C} + \frac{10^5\,\text{W/m}^2\,(0.6\,\text{m})^3}{3000\,\text{W/m}^2\cdot\text{K}\,(0.7\,\text{m})^2} = 39.7^\circ\text{C}. \qquad <$$

(b) Performing a surface energy balance at the outer surface, $\dot{E}_{in} - \dot{E}_{out} = 0$ or $q_{cond} - q_{conv} = 0$.
Hence

$$\frac{4\pi k_{ss}\left(T_{s,i} - T_{s,o}\right)}{(1/r_i) - (1/r_o)} = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

$$T_{s,i} = T_{s,o} + \frac{h}{k_{ss}}\left(\frac{r_o}{r_i} - 1\right) r_o\left(T_{s,o} - T_\infty\right) = 39.7^\circ\text{C} + \frac{1000\,\text{W/m}^2\cdot\text{K}}{15\,\text{W/m}\cdot\text{K}}(0.167)\,0.7\,\text{m}\left(14.7^\circ\text{C}\right) = 154^\circ\text{C}. <$$

(c) The heat equation in spherical coordinates is

$$k_{rw}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \dot{q}r^2 = 0.$$

Solving,

$$r^2\frac{dT}{dr} = -\frac{\dot{q}r^3}{3k_{rw}} + C_1 \qquad \text{and} \qquad T(r) = -\frac{\dot{q}r^2}{6k_{rw}} - \frac{C_1}{r} + C_2$$

Applying the boundary conditions,

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \qquad \text{and} \qquad T(r_i) = T_{s,i}$$

$$C_1 = 0 \qquad \text{and} \qquad C_2 = T_{s,i} + \dot{q}r_i^2/6k_{rw}.$$

Continued...

PROBLEM 3.104 (Cont.)

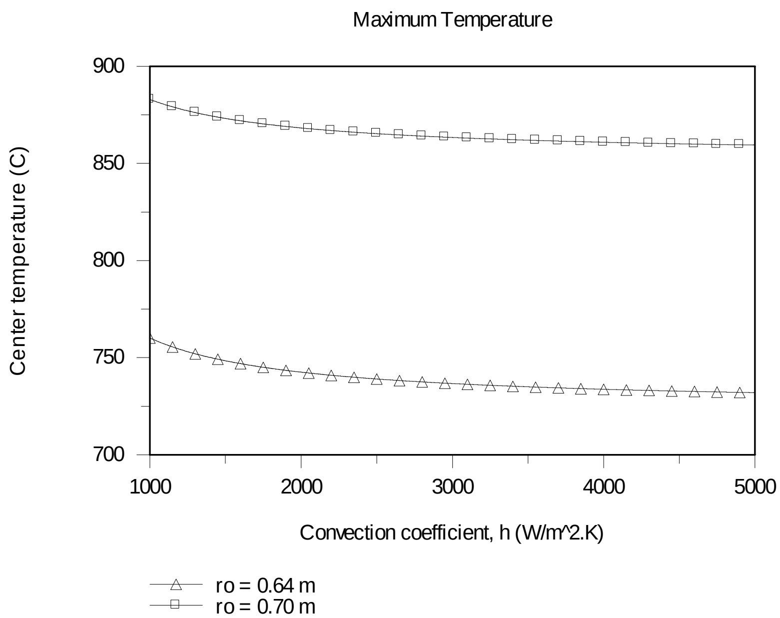
Hence

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k_{rw}}\left(r_i^2 - r^2\right) \qquad <$$

At r = 0,

$$T(0) = T_{s,i} + \frac{\dot{q}r_i^2}{6k_{rw}} = 154^\circ\text{C} + \frac{10^5\,\text{W/m}^3\,(0.6\,\text{m})^2}{6(20\,\text{W/m}\cdot\text{K})} = 454^\circ\text{C} \qquad <$$

(d) The feasibility assessment may be performed by using a commercial software package such as IHT. Results for the center temperature are shown below.



Clearly, even with r_o = 0.64 m = $r_{o,min}$ and h = 6,000 W/m^2·K (a practical upper limit), T(0) > 475°C and the desired condition can not be met. *The proposed extension is not feasible.*

COMMENTS: A value of $\dot{q}$ = 1.27 × 10^5 W/m^3 would allow for operation at T(0) = 475°C with r_o = 0.64 m and h = 5000 W/m^2·K.