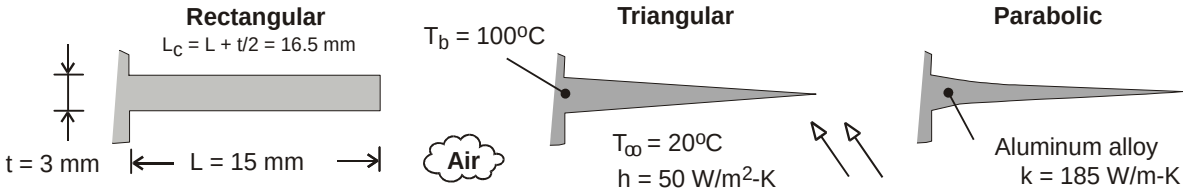
PROBLEM 3.134

KNOWN: Length, thickness and temperature of straight fins of rectangular, triangular and parabolic profiles. Ambient air temperature and convection coefficient.

FIND: Heat rate per unit width, efficiency and volume of each fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

ANALYSIS: For each fin,

$$q'_f = q'_{max} = \eta_f \, hA'_f \, \theta_b, \qquad V' = A_p$$

where η_f depends on the value of $m = (2h/kt)^{1/2} = (100\ W/m^2\cdot K/185\ W/m\cdot K \times 0.003m)^{1/2} = 13.4m^{-1}$ and the product $mL = 13.4m^{-1} \times 0.015m = 0.201$ or $mL_c = 0.222$. Expressions for η_f, A'_f and A_p are obtained from Table 3-5.

Rectangular Fin:

$$\eta_f = \frac{\tanh\, mL_c}{mL_c} = \frac{0.218}{0.222} = 0.982, \; A'_f = 2L_c = 0.033m \qquad <$$

$$q' = 0.982\left(50\,W/m^2\cdot K\right)0.033m\left(80°C\right) = 129.6\,W/m, \; V' = tL = 4.5\times10^{-5}\ m^2 \qquad <$$

Triangular Fin:

$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} = \frac{0.205}{(0.201)1.042} = 0.978, \; A'_f = 2\left[L^2 + (t/2)^2\right]^{1/2} = 0.030m \qquad <$$

$$q' = 0.978\left(50\,W/m^2\cdot K\right)0.030m\left(80°C\right) = 117.3\,W/m, \; V' = (t/2)L = 2.25\times10^{-5}\,m^2 \qquad <$$

Parabolic Fin:

$$\eta_f = \frac{2}{\left[4(mL)^2+1\right]^{1/2}+1} = 0.963, \; A'_f = \left[C_1L + \left(L^2/t\right)\ln\left(t/L + C_1\right)\right] = 0.030m \qquad <$$

$$q'_f = 0.963\left(50\,W/m^2\cdot K\right)0.030m\left(80°C\right) = 115.6\,W/m, \; V' = (t/3)L = 1.5\times10^{-5}\,m^2 \qquad <$$

COMMENTS: Although the heat rate is slightly larger (~10%) for the rectangular fin than for the triangular or parabolic fins, the heat rate per unit volume (or mass) is larger and largest for the triangular and parabolic fins, respectively.