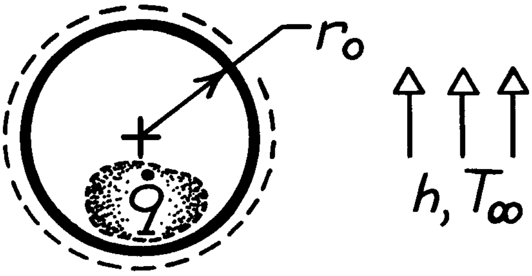
PROBLEM 3.102

KNOWN: Radius, thermal conductivity, heat generation and convection conditions associated with a solid sphere.

FIND: Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation.

ANALYSIS: Integrating the appropriate form of the heat diffusion equation,

$$\frac{1}{r^2}\frac{d}{dr}\left[kr^2\frac{dT}{dr}\right]+\dot{q}=0 \qquad \text{or} \qquad \frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=-\frac{\dot{q}r^2}{k}$$

$$r^2\frac{dT}{dr}=-\frac{\dot{q}r^3}{3k}+C_1 \qquad \frac{dT}{dr}=-\frac{\dot{q}r}{3k}+\frac{C_1}{r^2}$$

$$T(r)=-\frac{\dot{q}r^2}{6k}-\frac{C_1}{r}+C_2.$$

The boundary conditions are: $\left.\frac{dT}{dr}\right]_{r=0}=0$ hence $C_1=0,$ and

$$-k\left.\frac{dT}{dr}\right]_{r_o}=h\left[T(r_o)-T_\infty\right].$$

Substituting into the second boundary condition ($r = r_o$), find

$$\frac{\dot{q}r_o}{3}=h\left[-\frac{\dot{q}r_o^2}{6k}+C_2-T_\infty\right] \qquad C_2=\frac{\dot{q}r_o}{3h}+\frac{\dot{q}r_o^2}{6k}+T_\infty.$$

The temperature distribution has the form

$$T(r)=\frac{\dot{q}}{6k}\left(r_o^2-r^2\right)+\frac{\dot{q}r_o}{3h}+T_\infty. \qquad <$$

COMMENTS: To verify the above result, obtain $T(r_o) = T_s$,

$$T_s=\frac{\dot{q}r_o}{3h}+T_\infty$$

Applying energy balance to the control volume about the sphere,

$$\dot{q}\left[\frac{4}{3}\pi\, r_o^3\right]=h4\pi\, r_o^2\left(T_s-T_\infty\right) \qquad \text{find} \qquad T_s=\frac{\dot{q}r_o}{3h}+T_\infty.$$