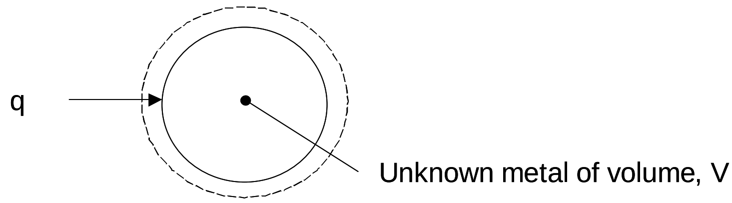
PROBLEM 2.20

KNOWN: Volume of unknown metal of high thermal conductivity. Known heating rate.

FIND: (a) Differential equation that may be used to determine the temperature response of the metal to heating. (b) If the model can be used to identify the metal, based upon matching the predicted and measured thermal responses.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible spatial temperature gradients, (2) Constant properties.

PROPERTIES: Table A.1 (T = 300 K): Aluminum; ρ = 2702 kg/m^3, c_p = 903 J/kg·K, Gold; ρ = 19300 kg/m^3, c_p = 129 J/kg·K, Silver; ρ = 10500 kg/m^3, c_p = 235 J/kg·K.

ANALYSIS: (a) An energy balance on the control volume yields $\dot{E}_{st} = \dot{E}_{in}$ which may be written

$$\rho V c_p \frac{dT}{dt} = q \quad \text{or} \quad \frac{dT}{dt} = \frac{q}{\rho V c_p} = \frac{q}{V}\frac{1}{\rho c_p}$$

The thermal response, dT/dt, may be measured. Alternatively, the expression may be integrated to find $T(t)$ given the initial temperature, volume, heat rate, and product of the density and specific heat.

(b) For a known metal volume, V, the thermal response to constant heating is determined by the product of the density and specific heat, ρc_p. This product is listed below for each of the three candidate materials.

Material	ρ (kg/m^3)	c_p (J/kg·K)	ρc_p (J/m^3·K)
Aluminum	2702	903	2.44×10^6
Gold	19300	129	2.49×10^6
Silver	10500	235	2.47×10^6

Because the product of the densities and specific heats are so similar for these four candidate materials, in general this approach cannot be used to distinguish which material is being heated. $<$

COMMENTS: (1) By neglecting spatial temperature gradients, the proposed approach is based only on thermodynamics principles. Therefore, it is limited in its usefulness relative to alternative schemes discussed in the text such as in Problem 2.23. (2) For many metals, the product of the density and specific heat lies within a relatively narrow band.