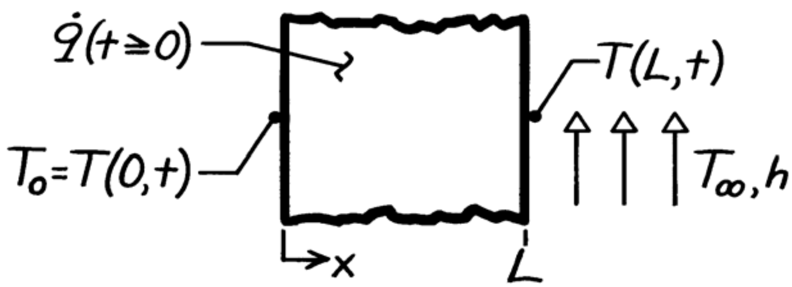
PROBLEM 2.60

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty > T_o$,h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. x) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; also show distribution when there is no heat flow at the x = L boundary, (b) Sketch the heat flux $\left(q''_x \text{ vs. } t\right)$ at the boundaries x = 0 and L.

SCHEMATIC:



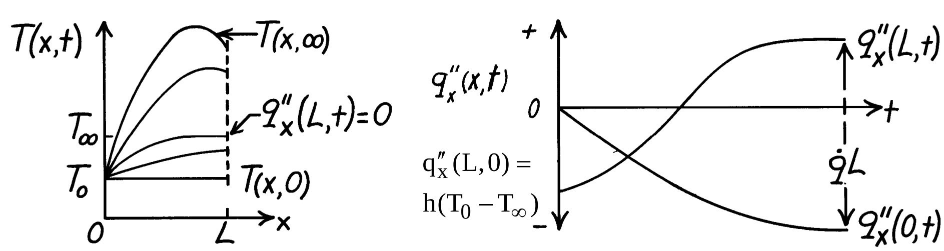
ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_o < T_\infty$ and $\dot{q}$ large enough that $T(x,\infty) > T_\infty$ for some x.

ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

Initial ($t \le 0$):	$T(x,0) = T_o$	Uniform temperature
Boundary:	$x = 0 \quad T(0,t) = T_o$	Constant temperature
	$x = L \qquad -k\left.\dfrac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right]$	Convection process.

The temperature distributions are shown on the T-x coordinates below. Note the special condition when the heat flux at (x = L) is zero.

(b) The heat flux as a function of time at the boundaries, $q''_x(0,t)$ and $q''_x(L,t)$, can be inferred from the temperature distributions using Fourier's law.



COMMENTS: Since $T(x,\infty) > T_\infty$ for some x and $T_\infty > T_o$, heat transfer at both boundaries must be out of the wall at steady state. From an overall energy balance at steady state, $+q''_x(L,\infty) - q''_x(0,\infty) = \dot{q}L.$