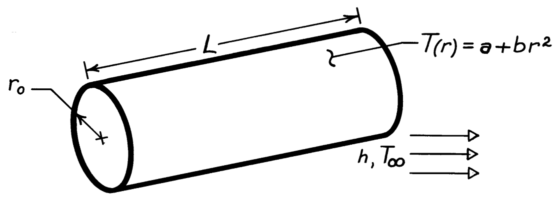
PROBLEM 2.13

KNOWN: Temperature distribution in solid cylinder and convection coefficient at cylinder surface.

FIND: Expressions for heat rate at cylinder surface and fluid temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The heat rate from Fourier's law for the radial (cylindrical) system has the form

$$q_r = -kA_r \frac{dT}{dr}.$$

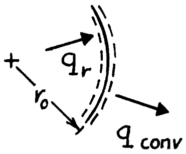
Substituting for the temperature distribution, $T(r) = a + br^2$,

$$q_r = -k(2\pi rL)\ 2br = -4\pi kbLr^2.$$

At the outer surface ($r = r_o$), the conduction heat rate is

$$q_{r=r_o} = -4\pi kbLr_o^2. \qquad <$$

From a surface energy balance at $r = r_o$,



$$q_{r=r_o} = q_{conv} = h(2\pi r_o L)\ \left[T(r_o) - T_\infty\right],$$

Substituting for $q_{r=r_o}$ and solving for T_∞,

$$T_\infty = T(r_o) + \frac{2kbr_o}{h}$$

$$T_\infty = a + br_o^2 + \frac{2kbr_o}{h}$$

$$T_\infty = a + br_o\left[r_o + \frac{2k}{h}\right]. \qquad <$$