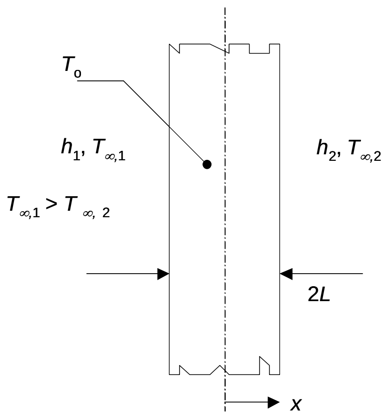
PROBLEM 2.55

KNOWN: Dimensions of one-dimensional plane wall, initial and boundary conditions.

FIND: (a) Differential equation, boundary and initial conditions used to determine $T(x,t)$, (b) Sketch of the temperature distributions for the initial condition, the steady-state condition, and for two intermediate times, (c) Sketch of the heat flux $q_x''(x,t)$ at the planes $x = 0$, $-L$, and $+L$, (d) Sketch of the temperature distributions for the initial condition, the steady-state condition, and for two intermediate times for h_1 twice the previous value, (e) Sketch of the heat flux $q_x''(x,t)$ at the planes $x = 0$, $-L$, and $+L$ for h_1 twice the previous value.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Constant properties, (3) No internal generation.

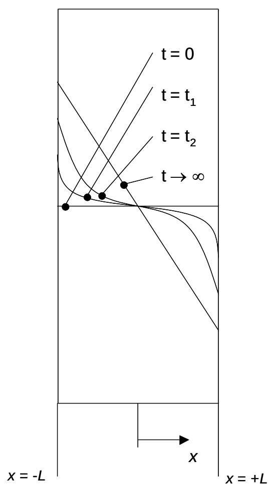
ANALYSIS: The differential equation may be found by simplifying the heat equation, Equation 2.21. The simplification yields

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The initial and boundary conditions are:

$$T(x,t=0) = T_o\,;\; -k\left.\frac{\partial T}{\partial x}\right|_{x=-L} = h_1\left[T_{\infty,1} - T(x=-L,t)\right];\, -k\left.\frac{\partial T}{\partial x}\right|_{x=+L} = h_2\left[T(x=+L,t) - T_{\infty,2}\right]$$

(b) The temperature distributions are shown in the sketch below.



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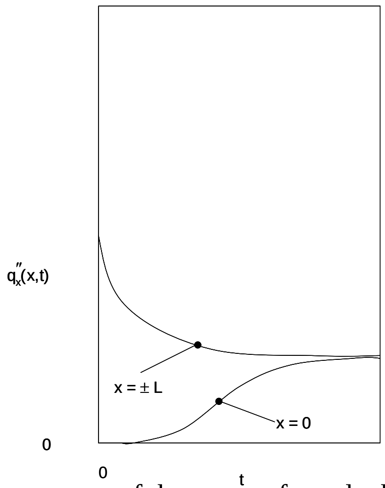
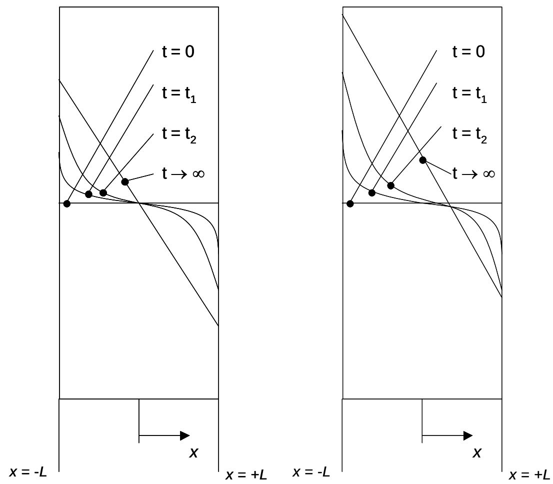
The temperature is uniform at the initial time, as required by the initial condition listed in part (a). Note the temperature gradients at the exposed surfaces are large at early times and decrease in magnitude as the convective heat flux is reduced, as required by the boundary conditions listed in part (a). The steady-state temperature distribution is linear. At the steady state,

$$-k\frac{dT}{dx} = h_1\left[T_{\infty,1} - T(x=-L)\right] = h_2\left[T(x=+L) - T_{\infty,2}\right]$$

(c) At any time, the heat fluxes at $x = \pm L$ are identical. The initial heat flux value is $q''_x(x=\pm L) =$ $h_1[T_{\infty,1} - T_o] = h_2[T_o - T_{\infty,2}]$. As time progresses, thermal effects propagate to $x = 0$, resulting in a uniform heat flux distribution throughout the wall thickness.



(d) A comparison of the transient response of the system for a doubled value of h_1 is shown in the RHS sketch below. Note that for all but the initial time, temperatures throughout the wall are higher relative to the case associated with the original value of h_1 (LHS). At intermediate times, temperature gradients at $x = -L$ are larger than temperature gradients at $x = +L$ due to the larger convection heat transfer coefficient at the left surface.



h_1 $\qquad$ $2h_1$

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(e) The heat flux histories are shown in the plot below as dashed lines. The results for part (c) are replicated as solid lines. Note that at the initial time, the heat flux at the left face is doubled relative to part (c) because the heat transfer coefficient is doubled. The heat flux at the initial time for the right face is the same as in part (c). The heat fluxes at the three planes asymptotically approach the steady-state value given by

$$-k\frac{dT}{dx} = 2h_1\left[T_{\infty,1} - T(x=-L)\right] = h_2\left[T(x=+L) - T_{\infty,2}\right]$$

Note that the overall heat flux is *not* doubled at the steady state since the temperature at the right face ($T(x = \pm L)$) is greater for the case of the doubled LHS heat transfer coefficient, h_1.