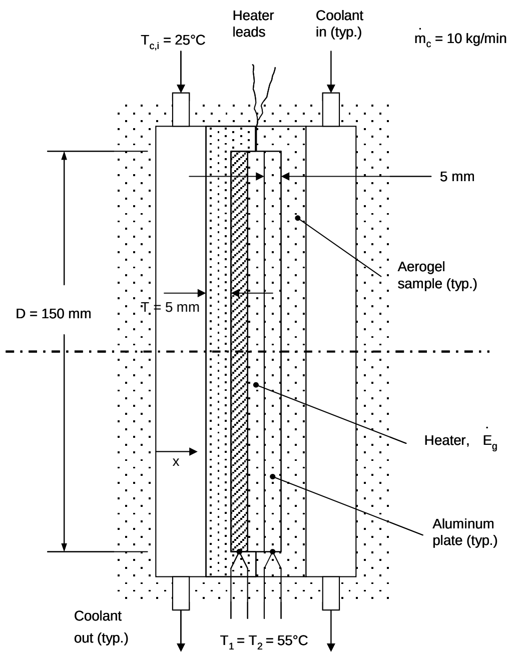
PROBLEM 2.18

KNOWN: Geometry and steady-state conditions used to measure the thermal conductivity of an aerogel sheet.

FIND: (a) Reason the apparatus of Problem 2.17 cannot be used, (b) Thermal conductivity of the aerogel, (c) Temperature difference across the aluminum sheets, and (d) Outlet temperature of the coolant.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer.

PROPERTIES: Table A.1, pure aluminum [$T = (T_1 + T_{c,i})/2 = 40°C = 313$ K]: $k_{al} = 239$ W/m·K. Table A.6, liquid water (25°C = 298 K): $c_p = 4180$ J/kg·K.

ANALYSIS:

(a) The apparatus of Problem 2.17 cannot be used because it operates under the assumption that the heat transfer is one-dimensional in the axial direction. Since the aerogel is expected to have an extremely small thermal conductivity, the insulation used in Problem 2.17 will likely have a higher thermal conductivity than aerogel. Radial heat losses would be significant, invalidating any measured results.

(b) The electrical power is

$$\dot{E}_g = V(I) = 10\text{V} \times 0.125\text{ A} = 1.25\text{ W}$$

Continued…

PROBLEM 2.18 (Cont.)

The conduction heat rate through each aerogel plate is

$$q = \frac{\dot{E}_g}{2} = -k_a A \frac{dT}{dx} = -k_a \left(\frac{\pi D^2}{4}\right)\left(\frac{T_c - T_l}{t}\right)$$

or

$$k_a = \frac{2\dot{E}_g t}{\pi D^2 (T_l - T_c)} = \frac{2 \times 1.25 \text{ W} \times 0.005 \text{ m}}{\pi \times (0.15 \text{ m})^2 \times (55 - 25)°\text{C}} = 5.9 \times 10^{-3} \frac{\text{W}}{\text{m} \cdot \text{K}} \qquad <$$

(c) The conduction heat flux through each aluminum plate is the same as through the aerogel. Hence,

$$-k_a \frac{(T_c - T_l)}{t} = -k_{al} \frac{\Delta T_{al}}{t}$$

or

$$\Delta T_{al} = \frac{k_a}{k_{al}}(T_l - T_c) = \frac{5.9 \times 10^{-3} \text{ W/m} \cdot \text{K}}{239 \text{ W/m} \cdot \text{K}} \times 30°\text{C} = 0.74 \times 10^{-3} °\text{C} \qquad <$$

The temperature difference across the aluminum plate is negligible. Therefore it is not important to know the location where the thermocouples are attached.

(d) An energy balance on the water yields

$$\dot{E}_g = \dot{m} c_p (T_{c,o} - T_{c,i})$$

or

$$T_{c,o} = T_{c,i} + \frac{\dot{E}_g}{\dot{m} c_p}$$

$$= 25°\text{C} + \frac{1.25 \text{ W}}{1 \text{ kg/min} \times \frac{1}{60} \text{ min/s} \times 4180 \text{ J/kg} \cdot \text{K}} = 25.02°\text{C} \qquad <$$

COMMENTS: (1) For all practical purposes the aluminum plates may be considered to be isothermal. (2) The coolant may be considered to be isothermal.