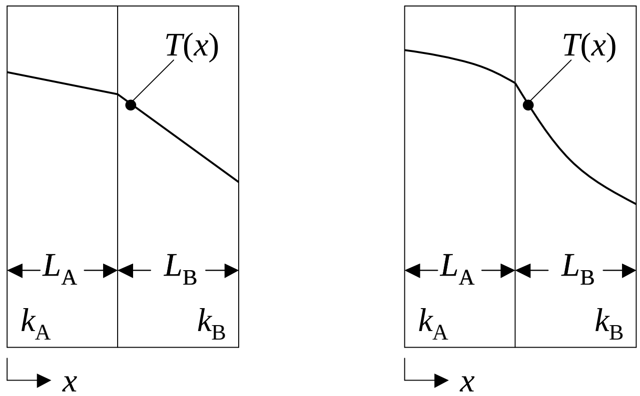
PROBLEM 2.58

KNOWN: Qualitative temperature distributions in two cases.

FIND: For each of two cases, determine which material (A or B) has the higher thermal conductivity, how the thermal conductivity varies with temperature, description of the heat flux distribution through the composite wall, effect of simultaneously doubling the wall thickness and thermal conductivity.

SCHEMATIC:



Case 1. Case 2.

ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible contact resistances, (3) No internal energy generation.

ANALYSIS: Under steady-state conditions with no internal generation, the conservation of energy requirement dictates that the heat flux through the wall must be constant. $<$

For Materials A and B, Fourier's law is written $q''_{\mathrm{A}} = -k_{\mathrm{A}} \dfrac{dT_{\mathrm{A}}}{dx} = q''_{\mathrm{B}} = -k_{\mathrm{B}} \dfrac{dT_{\mathrm{B}}}{dx}$. Therefore,

$$\frac{k_{\mathrm{A}}}{k_{\mathrm{B}}} = \frac{dT_{\mathrm{B}}/dx}{dT_{\mathrm{A}}/dx} > 1 \quad \text{and} \quad k_{\mathrm{B}} < k_{\mathrm{A}} \text{ for both cases.}$$ $<$

Since the heat flux through the wall is constant, Fourier's law dictates that lower thermal conductivity material must exist where temperature gradients are larger. For Case 1, the temperature distributions are linear. Therefore, the temperature gradient is constant in each material, and the thermal conductivity of each material must not vary significantly with temperature. For Case 2, Material A, the temperature gradient is larger at lower temperatures. Hence, for Material A the thermal conductivity increases with increasing material temperature. For Case 2, Material B, the temperature gradient is smaller at lower temperatures. Hence, for Material B the thermal conductivity decreases with increases in material temperature. $<$

COMMENTS: If you were given information regarding the relative values of the thermal conductivities and how the thermal conductivities vary with temperature in each material, you should be able to sketch the temperature distributions provided in the problem statement.