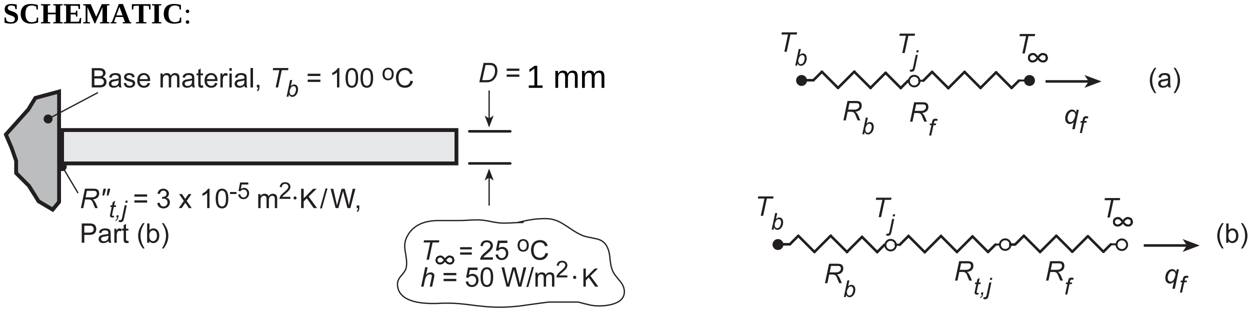
PROBLEM 4.32

KNOWN: Long fin of aluminum alloy with prescribed convection coefficient attached to different base materials (aluminum alloy or stainless steel) with and without thermal contact resistance $R''_{t,j}$ at the junction.

FIND: (a) Heat rate q_f and junction temperature T_j for base materials of aluminum and stainless steel, (b) Repeat calculations considering thermal contact resistance, $R''_{t,j}$, and (c) Plot as a function of h for the range $10 \le h \le 1000$ W/m^2·K for each base material.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Infinite fin.

PROPERTIES: (Given) Aluminum alloy, k = 240 W/m·K, Stainless steel, k = 15 W/m·K.

ANALYSIS: (a,b) From the thermal circuits, the heat rate and junction temperature are

$$q_f = \frac{T_b - T_\infty}{R_{tot}} = \frac{T_b - T_\infty}{R_b + R_{t,j} + R_f} \tag{1}$$

$$T_j = T_\infty + q_f R_f \tag{2}$$

and, with P = πD and $A_c = \pi D^2/4$, from Tables 4.1 and 3.4 find

$$R_b = 1/Sk_b = 1/(2Dk_b) = (2 \times 0.005\,m \times k_b)^{-1}$$

$$R_{t,j} = R''_{t,j}/A_c = 3\times10^{-5} m^2 \cdot K/W / \left[\pi(0.005m)^2/4\right] = 1.528 K/W$$

$$R_f = (hPkA_c)^{-1/2} = \left[50\,W/m^2\cdot K\,\pi^2 (0.005\,m)^3\, 240\,W/m\cdot K/4\right]^{-1/2} = 16.4\,K/W$$

		Without $R''_{t,j}$		With $R''_{t,j}$	
Base	R_b (K/W)	q_f (W)	T_j (°C)	q_f (W)	T_j (°C)
Al alloy	0.417	4.46	98.2	4.09	92.1
St. steel	6.667	3.26	78.4	3.05	75.1

(c) We used the *IHT Model for Extended Surfaces, Performance Calculations, Rectangular Pin Fin* to calculate q_f for $10 \le h \le 100$ W/m^2·K by replacing R''_{tc} (thermal resistance at fin base) by the sum of the contact and spreading resistances, $R''_{t,j} + R''_b$.

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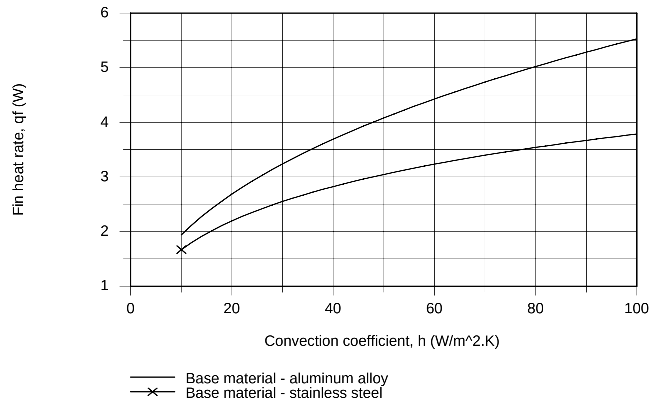
PROBLEM 4.32 (Cont.)



COMMENTS: (1) From part (a), the aluminum alloy base material has negligible effect on the fin heat rate and depresses the base temperature by only 2°C. The effect of the stainless steel base material is substantial, reducing the heat rate by 27% and depressing the junction temperature by 25°C.

(2) The contact resistance reduces the heat rate and increases the temperature depression relatively more with the aluminum alloy base.

(3) From the plot of q_f vs. h, note that at low values of h, the heat rates are nearly the same for both materials since the fin is the dominant resistance. As h increases, the effect of R''_b becomes more important.