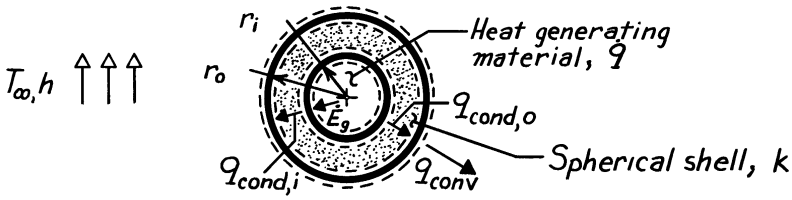
PROBLEM 3.75

KNOWN: Volumetric heat generation occurring within the cavity of a spherical shell of prescribed dimensions. Convection conditions at outer surface.

FIND: Expression for steady-state temperature distribution in shell.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Steady-state conditions, (3) Constant properties, (4) Uniform generation within the shell cavity, (5) Negligible radiation.

ANALYSIS: For the prescribed conditions, the appropriate form of the heat equation is

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=0$$

Integrate twice to obtain,

$$r^2\frac{dT}{dr}=C_1 \qquad \text{and} \qquad T=-\frac{C_1}{r}+C_2. \qquad (1,2)$$

The boundary conditions may be obtained from energy balances at the inner and outer surfaces. At the inner surface (r_i),

$$\dot{E}_g=\dot{q}\left(4/3\pi\, r_i^3\right)=q_{cond,i}=-k\left(4\pi\, r_i^2\right)dT/dr)_{r_i} \qquad dT/dr|_{r_i}=-\dot{q}r_i/3k. \qquad (3)$$

At the outer surface (r_o),

$$q_{cond,o}=-k4\pi\, r_o^2\; dT/dr)_{r_o}=q_{conv}=h4\pi\, r_o^2\left[T(r_o)-T_\infty\right]$$

$$dT/dr|_{r_o}=-(h/k)\;\left[T(r_o)-T_\infty\right]. \qquad (4)$$

From Eqs. (1) and (3), $C_1=-\dot{q}r_i^3/3k$. From Eqs. (1), (2) and (4)

$$-\frac{\dot{q}r_i^3}{3kr_o^2}=-\left[\frac{h}{k}\right]\left[\frac{\dot{q}r_i^3}{3r_o k}+C_2-T_\infty\right]$$

$$C_2=\frac{\dot{q}r_i^3}{3hr_o^2}-\frac{\dot{q}r_i^3}{3r_o k}+T_\infty.$$

Hence, the temperature distribution is

$$T=\frac{\dot{q}r_i^3}{3k}\left[\frac{1}{r}-\frac{1}{r_o}\right]+\frac{\dot{q}r_i^3}{3hr_o^2}+T_\infty. \qquad <$$

COMMENTS: Note that $\dot{E}_g=q_{cond,i}=q_{cond,o}=q_{conv}$.