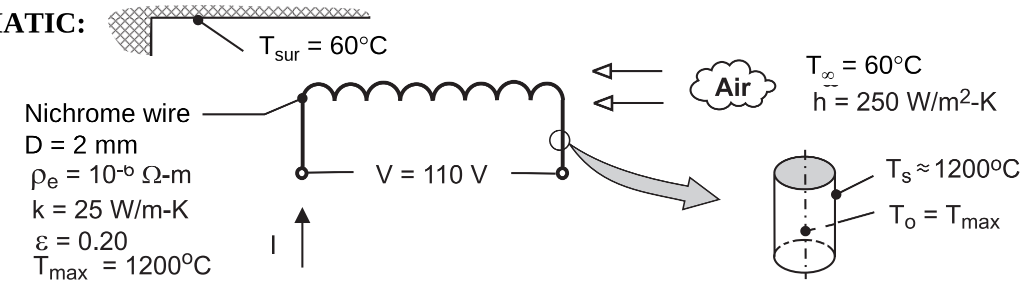
PROBLEM 3.86

KNOWN: Diameter, resistivity, thermal conductivity, emissivity, voltage, and maximum temperature of heater wire. Convection coefficient and air exit temperature. Temperature of surroundings.

FIND: Maximum operating current, heater length and power rating.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform wire temperature, (3) Constant properties, (4) Radiation exchange with large surroundings.

ANALYSIS: Assuming a uniform wire temperature, $T_{max} = T(r = 0) \equiv T_o \approx T_s$, the maximum volumetric heat generation may be obtained from Eq. (3.60), but with the total heat transfer coefficient, $h_t = h + h_r$, used in lieu of the convection coefficient h. With

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.20 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\left(1473 + 333\right)\mathrm{K}\left(1473^2 + 333^2\right)\mathrm{K}^2 = 46.7\,\mathrm{W/m^2 \cdot K}$$

$$h_t = \left(250 + 46.7\right)\mathrm{W/m^2 \cdot K} = 296.7\,\mathrm{W/m^2 \cdot K}$$

$$\dot{q}_{max} = \frac{2\,h_t}{r_o}\left(T_s - T_\infty\right) = \frac{2\left(296.7\,\mathrm{W/m^2 \cdot K}\right)}{0.001\mathrm{m}}\left(1140°\mathrm{C}\right) = 6.76 \times 10^8\,\mathrm{W/m^3}$$

Hence, with $\dot{q} = \dfrac{I^2 R_e}{\forall} = \dfrac{I^2\left(\rho_e L / A_c\right)}{L A_c} = \dfrac{I^2 \rho_e}{A_c^2} = \dfrac{I^2 \rho_e}{\left(\pi D^2/4\right)^2}$

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2}\frac{\pi D^2}{4} = \left(\frac{6.76 \times 10^8\,\mathrm{W/m^3}}{10^{-6}\,\Omega \cdot \mathrm{m}}\right)^{1/2}\frac{\pi\left(0.002\,\mathrm{m}\right)^2}{4} = 81.7\,\mathrm{A} \quad <$$

Also, with $\Delta E = I\,R_e = I\,(\rho_e L/A_c)$,

$$L = \frac{\Delta E \cdot A_c}{I_{max}\,\rho_e} = \frac{110\,\mathrm{V}\left[\pi\left(0.002\mathrm{m}\right)^2/4\right]}{81.7\,\mathrm{A}\left(10^{-6}\,\Omega \cdot \mathrm{m}\right)} = 4.23\mathrm{m} \quad <$$

and the power rating is

$$P_{elec} = \Delta E \cdot I_{max} = 110\,\mathrm{V}\left(81.7\,\mathrm{A}\right) = 8990\,\mathrm{W} = 8.99\,\mathrm{kW} \quad <$$

COMMENTS: To assess the validity of assuming a uniform wire temperature, Eq. (3.58) may be used to compute the centerline temperature corresponding to $\dot{q}_{max}$ and a surface temperature of 1200°C. It follows that $T_o = \dfrac{\dot{q}\,r_o^2}{4\,k} + T_s = \dfrac{6.76 \times 10^8\,\mathrm{W/m^3}\left(0.001\mathrm{m}\right)^2}{4\left(25\,\mathrm{W/m \cdot K}\right)} + 1200°\mathrm{C} = 1207°\mathrm{C}$. With only a 7°C temperature difference between the centerline and surface of the wire, the assumption is excellent.