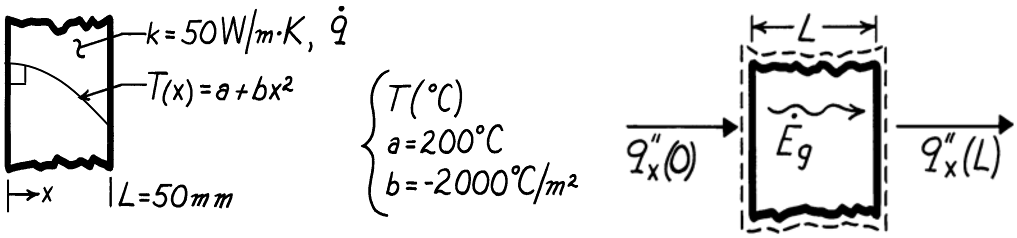
PROBLEM 2.30

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.21 re-written as

$$\dot{q}=-k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q}=-k\frac{d}{dx}\left[\frac{d}{dx}\left(a+bx^2\right)\right]=-k\frac{d}{dx}\left[2bx\right]=-2bk$$

$$\dot{q}=-2\left(-2000^\circ\text{C/m}^2\right)\times 50\ \text{W/m}\cdot\text{K}=2.0\times10^5\ \text{W/m}^3.$$ <

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x)=-k\left.\frac{dT}{dx}\right]_x .$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x)=-k\frac{d}{dx}\left[a+bx^2\right]=-2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0)=0$$ <

$$q''_x(L)=-2kbL=-2\times 50\text{W/m}\cdot\text{K}\left(-2000^\circ\text{C/m}^2\right)\times 0.050\text{m}$$

$$q''_x(L)=10,000\ \text{W/m}^2.$$ <

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=0 \qquad q''_x(0)-q''_x(L)+\dot{q}L=0$$

$$\dot{q}=\frac{q''_x(L)-q''_x(0)}{L}=\frac{10,000\ \text{W/m}^2-0}{0.050\text{m}}=2.0\times10^5\,\text{W/m}^3.$$