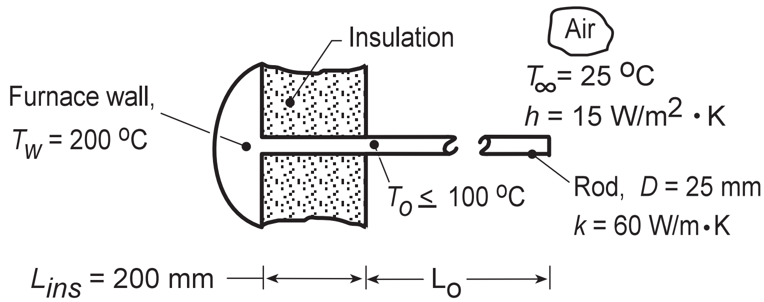
PROBLEM 3.122

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \leq 100°C$? If not, what design parameters would you change?

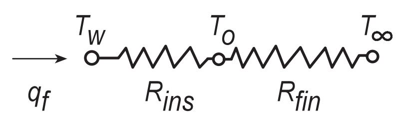
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.81,

$$R_{fin} = \theta_b/q_f = \frac{1}{\left(hPkA_c\right)^{1/2} \tanh\left(mL_o\right)} \qquad (2)$$

$$m = \left(hP/kA_c\right)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}\left(T_w - T_\infty\right) \qquad (6)$$ <

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25°C + \frac{6.298}{6.790 + 6.298}\left(200 - 25\right)° C = 109° C$$ <

$$R_{ins} = \frac{0.200\,\text{m}}{60\,\text{W/m}\cdot\text{K} \times 4.909 \times 10^{-4}\,\text{m}^2} = 6.790\,\text{K/W} \qquad A_c = \pi\left(0.025\,\text{m}\right)^2/4 = 4.909 \times 10^{-4}\,\text{m}^2$$

$$R_{fin} = 1\Big/\left(0.0347\,\text{W}^2/\text{K}^2\right)^{1/2} \tanh\left(6.324 \times 0.200\right) = 6.298\,\text{K/W}$$

$$\left(hPkA_c\right) = \left(15\,\text{W/m}^2\cdot\text{K} \times \pi\left(0.025\,\text{m}\right) \times 60\,\text{W/m}\cdot\text{K} \times 4.909 \times 10^{-4}\,\text{m}^2\right) = 0.0347\,\text{W}^2/\text{K}^2$$

Continued...

PROBLEM 3.122 (Cont.)

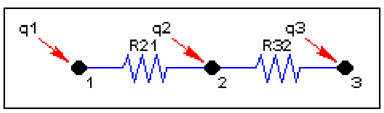
$$m = \left(hP/kA_c\right)^{1/2} = \left(15\,W/m^2 \cdot K \times \pi\left(0.025\,m\right)/60\,W/m \cdot K \times 4.909 \times 10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

Consider the following design changes aimed at reducing $T_o \le 100°C$. (1) Increasing length of the fin portions: with L_o = 400 and 600 mm, T_o is 102.8°C and 102.3°C, respectively. Hence, increasing L_o will reduce T_o only modestly. (2) Decreasing the thermal conductivity: backsolving the above equation set with T_0 = 100°C, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for T_o = 100°C, the required insulation thickness would be L_{ins} = 211 mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by "tack welding" (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit. (6) Using a tube rather than a rod will decrease A_c. For a 3 mm tube wall and 25 mm outside diameter, $A_c = 2.07 \times 10^{-4}\,m^2$, R_{ins} = 16.103 K/W and R_{fin} = 8.61 K/W, yielding T_o = 86°C. (conduction within the air inside the tube is neglected).

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:

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// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw             // Furnace wall temperature, C
//q1 =              // Heat rate, W
T2 = To             // To, beginning of rod exposed length
q2 =  0             // Heat rate, W; node 2; no external heat source
T3 =  Tinf          // Ambient air temperature, C
//q3 =              // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)          // Conduction resistance, K/W
Ac = pi * D^2 / 4               // Cross sectional area of rod, m^2
// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                     //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )          // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                      // Perimeter, m
// Other Assigned Variables:
Tw = 200            // Furnace wall temperature, C
k = 60              // Rod thermal conductivity, W/m.K
Lins = 0.200        // Insulated length, m
D = 0.025           // Rod diameter, m
h = 15              // Convection coefficient, W/m^2.K
Tinf = 25           // Ambient air temperature,C
Lo = 0.200          // Exposed length, m
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