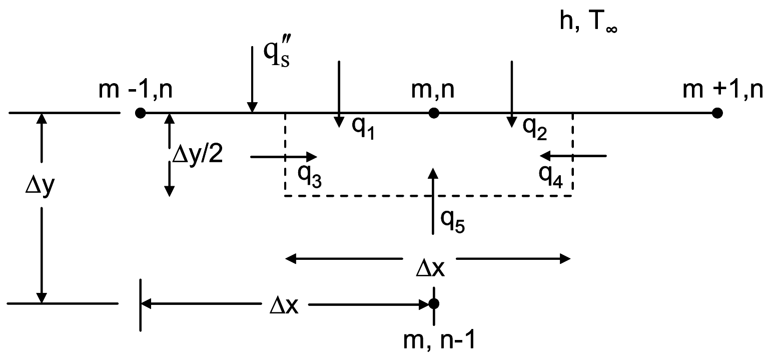
PROBLEM 4.41

KNOWN: Boundary conditions that change from specified heat flux to convection.

FIND: The finite difference equation for the node at the point where the boundary condition changes.

SCHEMATIC:



ASSUMPTIONS: (1) Two dimensional, steady-state conduction with no generation, (2) Constant properties.

ANALYSIS: Performing an energy balance on the control volume Δx • Δy/2,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad\qquad q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

Expressing q_1 in terms of the specified heat flux, q_2 in terms of the known heat transfer coefficient and environment temperature, and the remaining heat rates using the conduction rate equation,

$$q_1 = q_s'' \frac{\Delta x}{2} \cdot 1$$

$$q_2 = h(T_\infty - T_{m,n}) \frac{\Delta x}{2} \cdot 1$$

$$q_3 = \frac{k(T_{m-1,n} - T_{m,n})}{\Delta x} \frac{\Delta y}{2} \cdot 1$$

$$q_4 = \frac{k(T_{m+1,n} - T_{m,n})}{\Delta x} \frac{\Delta y}{2} \cdot 1$$

$$q_5 = \frac{k(T_{m,n-1} - T_{m,n})}{\Delta y} \Delta x \cdot 1$$

Letting Δx = Δy, substituting these expressions into the energy balance, and rearranging yields

$$T_{m-1,n} + T_{m+1,n} + 2T_{m,n-1} - \left[4 + \frac{h\Delta x}{k}\right] T_{m,n} + \frac{h\Delta x}{k} T_\infty + \frac{q_s''\Delta x}{k} = 0$$ <