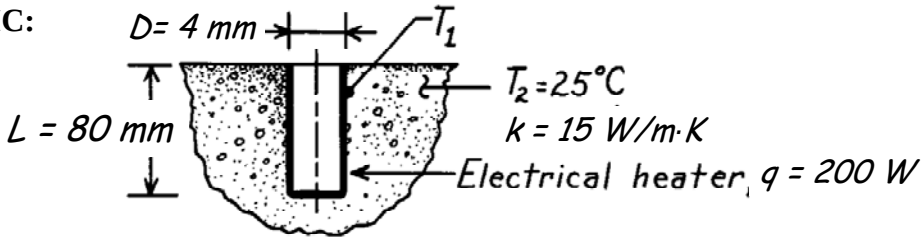
PROBLEM 4.13

KNOWN: Electrical heater of cylindrical shape inserted into a hole drilled normal to the surface of a large block of material with prescribed thermal conductivity.

FIND: Temperature reached when heater dissipates 200 W with the block at 25°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Block approximates semi-infinite medium with constant properties, (3) Negligible heat loss to surroundings above block surface, (4) Heater can be approximated as isothermal at T_1.

ANALYSIS: The temperature of the heater surface follows from the rate equation written as

$$T_1 = T_2 + q/kS$$

where S can be estimated from the conduction shape factor given in Table 4.1 for a "vertical cylinder in a semi-infinite medium,"

$$S = 2\pi L/\ell n\left(4L/D\right).$$

Substituting numerical values, find

$$S = 2\pi \times 0.08\text{m}/\ell n\left[\frac{4\times 0.08\text{m}}{0.004\text{m}}\right] = 0.115\text{m}.$$

The temperature of the heater is then

$$T_1 = 25°\text{C} + 200\ \text{W}/(15\ \text{W/m}\cdot\text{K} \times 0.114\text{m}) = 141°\text{C}. \quad <$$

COMMENTS: (1) Note that the heater has L >> D, which is a requirement of the shape factor expression.

(2) Our calculation presumes there is negligible thermal contact resistance between the heater and the medium. In practice, this would not be the case unless a conducting paste were used.

(3) Since L >> D, assumption (3) is reasonable.

(4) This configuration has been used to determine the thermal conductivity of materials from measurement of q and T_1.