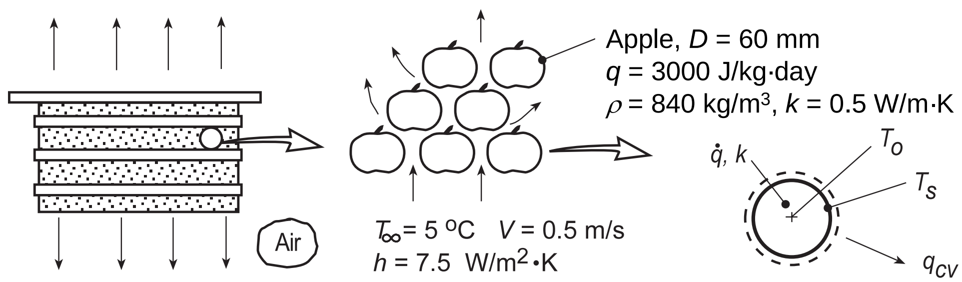
PROBLEM 3.105

KNOWN: Carton of apples, modeled as 60-mm diameter spheres, ventilated with air at 5°C and experiencing internal volumetric heat generation at a rate of 3000 J/kg·day.

FIND: (a) The apple center and surface temperatures when the convection coefficient is 7.5 $W/m^2 \cdot K$, and (b) Compute and plot the apple temperatures as a function of air velocity, V, for the range $0.1 \le V \le 1$ m/s, when the convection coefficient has the form $h = C_1 V^{0.425}$, where $C_1 = 10.1\ W/m^2 \cdot K \cdot (m/s)^{0.425}$.

SCHEMATIC:



ASSUMPTIONS: (1) Apples can be modeled as spheres, (2) Each apple experiences flow of ventilation air at $T_\infty = 5°C$, (3) One-dimensional radial conduction, (4) Constant properties and (5) Uniform heat generation.

ANALYSIS: (a) From Eq. C.24, the temperature distribution in a solid sphere (apple) with uniform generation is

$$T(r) = \frac{\dot{q} r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \qquad (1)$$

To determine T_s, perform an energy balance on the apple as shown in the sketch above, with volume $V = 4/3\pi r_o^3$,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad -q_{cv} + \dot{q}\forall = 0$$

$$-h\left(4\pi r_o^2\right)\left(T_s - T_\infty\right) + \dot{q}\left(4\pi r_o^3/3\right) = 0 \qquad (2)$$

$$-7.5\,W/m^2 \cdot K\left(4\pi \times 0.030^2 m^2\right)\left(T_s - 5^\circ C\right) + 38.9\,W/m^3\left(4\pi \times 0.030^3 m^3/3\right) = 0$$

where the volumetric generation rate is

$$\dot{q} = 3000\,J/kg \cdot day$$

$$\dot{q} = 3000\,J/kg \cdot day \times 840\,kg/m^3 \times \left(1\,day/24\,hr\right) \times \left(1\,hr/3600\,s\right)$$

$$\dot{q} = 29.2\,W/m^3$$

and solving for T_s, find

$$T_s = 5.04^\circ C$$ <

From Eq. (1), at r = 0, with T_s, find

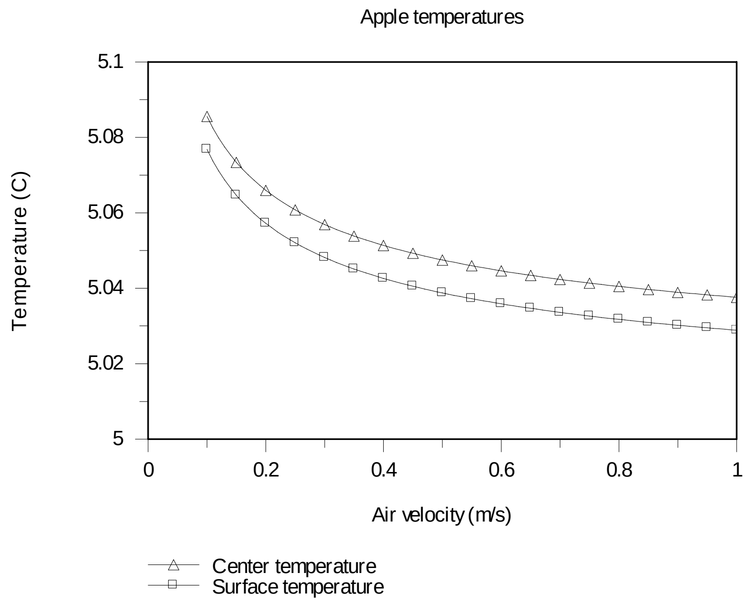
$$T(0) = \frac{29.2\,W/m^3 \times 0.030^2 m^2}{6 \times 0.5\,W/m \cdot K} + 5.04^\circ C = 0.009^\circ C + 5.04^\circ C = 5.05^\circ C$$ <

Continued...

(b) With the convection coefficient depending upon velocity,

$$h = C_1 V^{0.425}$$

with $C_1 = 10.1\ W/m^2 \cdot K \cdot (m/s)^{0.425}$, and using the energy balance of Eq. (2), calculate and plot T_s as a function of ventilation air velocity V.



COMMENTS: (1) While the temperature within the apple is nearly isothermal, the center temperature will track the ventilation air temperature which will increase as it passes through stacks of cartons.

(2) The *IHT* Workspace used to determine T_s for the base condition and generate the above plot is shown below.

```
// The temperature distribution, Eq (1),
T_r = qdot * ro^2 / (4 * k) * ( 1- r^2/ro^2 )  +  Ts

// Energy balance on the apple, Eq (2)
- qcv + qdot * Vol = 0
Vol = 4 / 3 * pi *  ro ^3

// Convection rate equation:
qcv = h* As * ( Ts - Tinf )
As = 4 * pi * ro^2

// Generation rate:
qdot = qdotm * (1/24) * (1/3600) * rho          // Generation rate, W/m^3; Conversions: days/h and h/sec

// Assigned variables:
ro = 0.030                // Radius of apple, m
k = 0.5                   // Thermal conductivity, W/m.K
qdotm = 3000              // Generation rate, J/kg.K
rho = 840                 // Specific heat, J/kg.K
r = 0                     // Center, m; location for T(0)
h =  7.5                  // Convection coefficient, W/m^2.K; base case, V = 0.5 m/s
//h = C1 * V^0.425        // Correlation
//C1 = 10.1
//V = 0.5                 // Air velocity, m/s; range 0.1 to 1 m/s
Tinf = 5                  // Air temperature, C
```