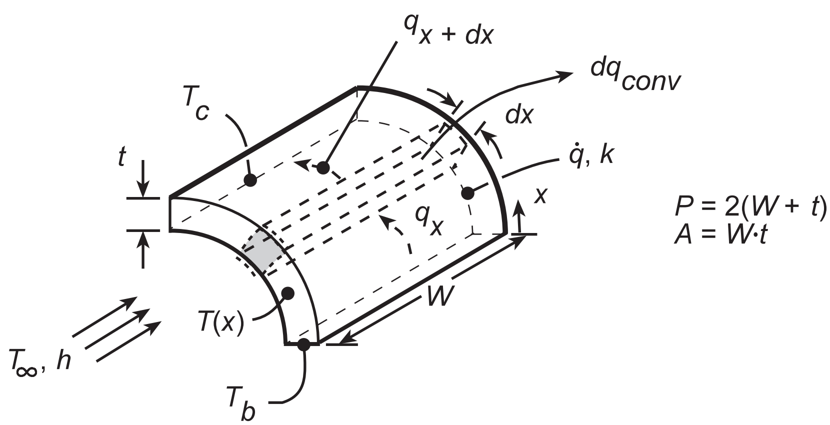
PROBLEM 3.125

KNOWN: Dimensions, end temperatures and volumetric heating of wire leads. Convection coefficient and ambient temperature.

FIND: (a) Equation governing temperature distribution in the leads, (b) Form of the temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x, (3) Uniform volumetric heating, (4) Uniform h (both sides), (5) Negligible radiation, (6) Constant properties.

ANALYSIS: (a) Performing an energy balance for the differential control volume,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q_x - q_{x+dx} - dq_{conv} + \dot{q}dV = 0$$

$$-kA_c \frac{dT}{dx} - \left[-kA_c \frac{dT}{dx} - \frac{d}{dx}\left(kA_c \frac{dT}{dx}\right)dx\right] - hPdx\left(T - T_\infty\right) + \dot{q}A_c dx = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}\left(T - T_\infty\right) + \frac{\dot{q}}{k} = 0 \qquad <$$

(b) With a *reduced temperature* defined as $\Theta \equiv T - T_\infty - \left(\dot{q}A_c/hP\right)$ and $m^2 \equiv hP/kA_c$, the differential equation may be rendered homogeneous, with a general solution and boundary conditions as shown

$$\frac{d^2\Theta}{dx^2} - m^2\Theta = 0 \qquad \Theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\Theta_b = C_1 + C_2 \qquad \Theta_c = C_1 e^{mL} + C_2 e^{-mL}$$

it follows that

$$C_1 = \frac{\Theta_b e^{-mL} - \Theta_c}{e^{-mL} - e^{mL}} \qquad C_2 = \frac{\Theta_c - \Theta_b e^{mL}}{e^{-mL} - e^{mL}}$$

$$\Theta(x) = \frac{\left(\Theta_b e^{-mL} - \Theta_c\right)e^{mx} + \left(\Theta_c - \Theta_b e^{mL}\right)e^{-mx}}{e^{-mL} - e^{mL}} \qquad <$$

COMMENTS: If $\dot{q}$ is large and h is small, temperatures within the lead may readily exceed the prescribed boundary temperatures.