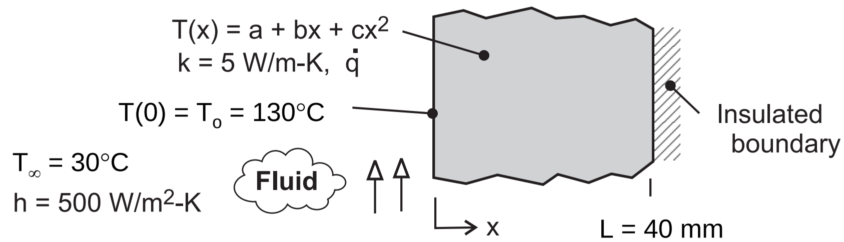
PROBLEM 2.34

KNOWN: Steady-state conduction with uniform internal energy generation in a plane wall; temperature distribution has quadratic form. Surface at x=0 is prescribed and boundary at x = L is insulated.

FIND: (a) Calculate the internal energy generation rate, $\dot{q}$, by applying an overall energy balance to the wall, (b) Determine the coefficients a, b, and c, by applying the boundary conditions to the prescribed form of the temperature distribution; plot the temperature distribution and label as Case 1, (c) Determine new values for a, b, and c for conditions when the convection coefficient is halved, and the generation rate remains unchanged; plot the temperature distribution and label as Case 2; (d) Determine new values for a, b, and c for conditions when the generation rate is doubled, and the convection coefficient remains unchanged (h = 500 $W/m^2 \cdot K$); plot the temperature distribution and label as Case 3.

SCHEMATIC:



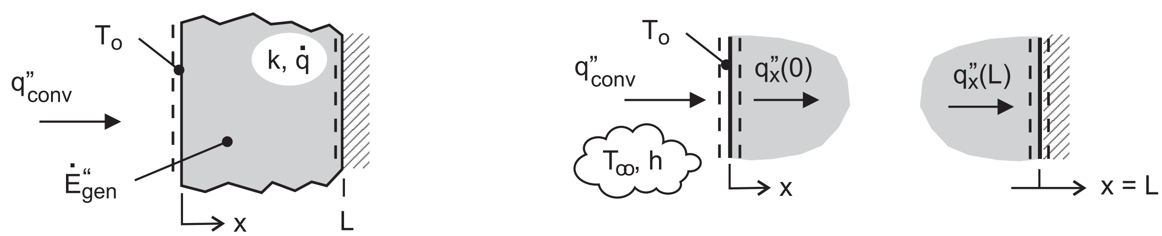
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties and uniform internal generation, and (3) Boundary at x = L is adiabatic.

ANALYSIS: (a) The internal energy generation rate can be calculated from an overall energy balance on the wall as shown in the schematic below.

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_{gen} = 0 \qquad \text{where} \qquad \dot{E}''_{in} = q''_{conv}$$

$$h\left(T_\infty - T_o\right) + \dot{q}\,L = 0 \tag{1}$$

$$\dot{q} = -h\left(T_\infty - T_o\right)/L = -500\,W/m^2 \cdot K\left(30-130\right)°C/0.040\ m = 1.25\times10^6\ W/m^3 \quad <$$



(a) Overall energy balance **(b) Surface energy balances**

(b) The coefficients of the temperature distribution, $T(x) = a + bx + cx^2$, can be evaluated by applying the boundary conditions at x = 0 and x = L. See Table 2.2 for representation of the boundary conditions, and the schematic above for the relevant surface energy balances.

Boundary condition at x = 0, convection surface condition

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conv} - q''_x(0) = 0 \qquad \text{where} \qquad q''_x(0) = -k\left.\frac{dT}{dx}\right|_{x=0}$$

$$h\left(T_\infty - T_o\right) - \left[-k\left(0 + b + 2cx\right)_{x=0}\right] = 0$$

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PROBLEM 2.34 (Cont.)

$$b = -h\left(T_\infty - T_o\right)/k = -500\,W/m^2\cdot K\left(30-130\right)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \qquad (2) <$$

Boundary condition at x = L, adiabatic or insulated surface

$$\dot{E}_{in} - \dot{E}_{out} = -q''_x\left(L\right) = 0 \text{ where} \qquad q''_x\left(L\right) = -k\left.\frac{dT}{dx}\right|_{x=L}$$

$$k\left[0+b+2cx\right]_{x=L} = 0 \qquad (3)$$

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.040m\right) = -1.25\times10^5\,K/m^2 \qquad <$$

Since the surface temperature at x = 0 is known, $T(0) = T_o = 130°C$, find

$$T\left(0\right) = 130°C = a + b\cdot0 + c\cdot0 \qquad \text{or} \qquad a = 130°C \qquad (4) <$$

Using the foregoing coefficients with the expression for T(x) in the Workspace of IHT, the temperature distribution can be determined and is plotted as Case 1 in the graph below.

(c) Consider Case 2 when the convection coefficient is halved, $h_2 = h/2 = 250\ W/m^2\cdot K$, $\dot{q} = 1.25\times10^6\ W/m^3$ and other parameters remain unchanged except that $T_o \neq 130°C$. We can determine a, b, and c for the temperature distribution expression by repeating the analyses of parts (a) and (b).

Overall energy balance on the wall, see Eqs. (1,4)

$$a = T_o = \dot{q}\,L/h + T_\infty = 1.25\times10^6\,W/m^3\times0.040m/250\,W/m^2\cdot K + 30°C = 230°C \qquad <$$

Surface energy balance at x = 0, see Eq. (2)

$$b = -h\left(T_\infty - T_o\right)/k = -250\,W/m^2\cdot K\left(30-230\right)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \qquad <$$

Surface energy balance at x = L, see Eq. (3)

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.040m\right) = -1.25\times10^5\,K/m^2 \qquad <$$

The new temperature distribution, T_2 (x), is plotted as Case 2 below.

(d) Consider Case 3 when the internal energy volumetric generation rate is doubled, $\dot{q}_3 = 2\dot{q} = 2.5\times10^6\,W/m^3$, $h = 500\ W/m^2\cdot K$, and other parameters remain unchanged except that $T_o \neq 130°C$. Following the same analysis as part (c), the coefficients for the new temperature distribution, T (x), are

$$a = 230°C \qquad b = 2\times10^4\,K/m \quad c = -2.50\times10^5\,K/m^2 \qquad <$$

and the distribution is plotted as Case 3 below.

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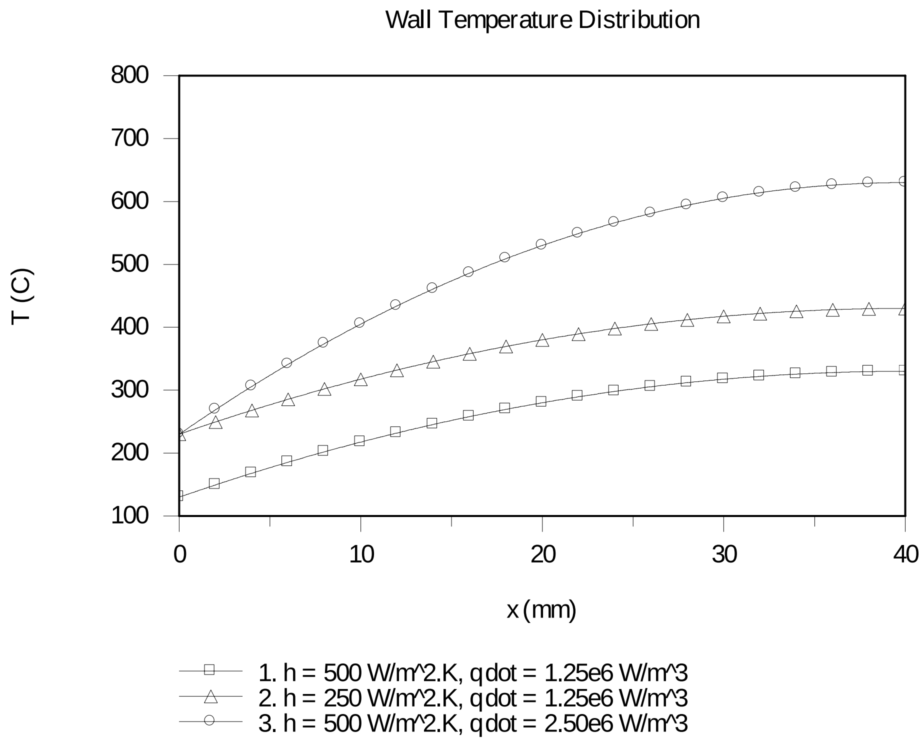

COMMENTS: Note the following features in the family of temperature distributions plotted above. The temperature gradients at x = L are zero since the boundary is insulated (adiabatic) for all cases. The shapes of the distributions are all quadratic, with the maximum temperatures at the insulated boundary.

By halving the convection coefficient for Case 2, we expect the surface temperature T_o to increase relative to the Case 1 value, since the same heat flux is removed from the wall $(\dot{q}L)$ but the convection resistance has increased.

By doubling the generation rate for Case 3, we expect the surface temperature T_o to increase relative to the Case 1 value, since double the amount of heat flux is removed from the wall $(2\dot{q}L)$.

Can you explain why T_o is the same for Cases 2 and 3, yet the insulated boundary temperatures are quite different? Can you explain the relative magnitudes of T(L) for the three cases?