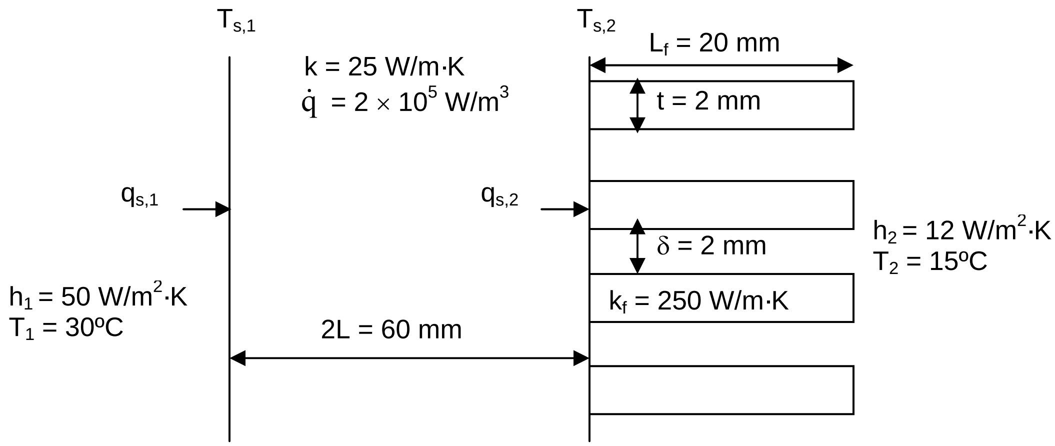
PROBLEM 3.156

KNOWN: Wall with known heat generation rate, thermal conductivity, and thickness. Dimensions and thermal conductivity of fins. Heat transfer coefficients and environment temperatures.

FIND: Maximum temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Wall surface temperatures are uniform. (3) No contact resistance between fins and wall, (4) Heat transfer from the fin tips can be neglected.

ANALYSIS: The temperature distribution in a wall with uniform volumetric heat generation and specified temperature boundary conditions is, from Equation 3.46

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{1}$$

The heat transfer rates at the two surfaces, for a wall section of area A, can be found from Fourier's law:

$$q_{s,1} = -kA\left.\frac{dT}{dx}\right|_{x=-L} = -\dot{q}LA - kA\frac{T_{s,2} - T_{s,1}}{2L} \tag{2}$$

$$q_{s,2} = -kA\left.\frac{dT}{dx}\right|_{x=L} = \dot{q}LA - kA\frac{T_{s,2} - T_{s,1}}{2L} \tag{3}$$

We can express these same heat transfer rates alternatively, as follows:

$$q_{s,1} = h_1A(T_1 - T_{s,1}) \tag{4}$$

$$q_{s,2} = h_2A_t(T_{s,2} - T_2)\eta_o \tag{5}$$

where η_o is given by Equation 3.107. Equating the two expressions for $q_{s,1}$, Equations (2) and (4), and equating the expressions for $q_{s,2}$, Equations (3) and (5), and solving for $T_{s,1}$ and $T_{s,2}$ yields

Continued…

$$T_{s,1} = \frac{\left(\frac{k}{2L} + h_2\tilde{A}\right)h_1T_1 + \frac{k}{2L}h_2\tilde{A}T_2 + \left(\frac{k}{L} + h_2\tilde{A}\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\tilde{A} + \frac{kh_2\tilde{A}}{2L}}$$

$$T_{s,2} = \frac{\frac{k}{2L}h_1T_1 + \left(\frac{k}{2L} + h_1\right)h_2\tilde{A}T_2 + \left(\frac{k}{L} + h_1\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\tilde{A} + \frac{kh_2\tilde{A}}{2L}}$$

where

$$\tilde{A} = \frac{A_t\eta_o}{A} = \frac{A_t}{A} - \frac{NA_f}{A}(1 - \eta_f)$$

Performing the calculations:

$$m = \sqrt{\frac{h_2P}{k_fA_c}} = \sqrt{\frac{2h_2}{k_ft}} = \sqrt{\frac{2 \times 12\ \mathrm{W/m^2 \cdot K}}{250\ \mathrm{W/m \cdot K} \times 0.002\ \mathrm{m}}} = 6.9\ \mathrm{m^{-1}}$$

$$\eta_f = \frac{\tanh(mL_f)}{mL_f} = \frac{\tanh\left(6.9\ \mathrm{m^{-1}} \times 0.02\ \mathrm{m}\right)}{6.9\ \mathrm{m^{-1}} \times 0.02\ \mathrm{m}} = 0.994$$

$$\frac{NA_f}{A} = \frac{N2wL_f}{(\delta + t)Nw} = \frac{2L_f}{\delta + t} = \frac{2 \times 0.02\ \mathrm{m}}{0.004\ \mathrm{m}} = 10.0$$

$$\frac{A_t}{A} = \frac{NA_f}{A} + \frac{A_b}{A} = \frac{NA_f}{A} + \frac{\delta Nw}{(\delta + t)Nw} = \frac{NA_f}{A} + \frac{\delta}{\delta + t} = 10. + \frac{0.002\ \mathrm{m}}{0.004\ \mathrm{m}} = 10.5$$

$$\tilde{A} = 10.5 - 10.(1 - 0.994) = 10.4$$

$$h_2\tilde{A} = 12\ \mathrm{W/m^2 \cdot K} \times 10.4 = 125\ \mathrm{W/m^2 \cdot K}$$

$$\frac{k}{2L} = \frac{25\ \mathrm{W/m \cdot K}}{0.06\ \mathrm{m}} = 417\ \mathrm{W/m^2 \cdot K}$$

Thus

$$T_{s,1} = \left(\begin{array}{l} (417 + 125)\ \mathrm{W/m^2 \cdot K} \times 50\ \mathrm{W/m^2 \cdot K} \times 30°\mathrm{C} \\ + 417\ \mathrm{W/m^2 \cdot K} \times 125\ \mathrm{W/m^2 \cdot K} \times 15°\mathrm{C} \\ + (2 \times 417 + 125)\ \mathrm{W/m^2 \cdot K} \times 2 \times 10^5\ \mathrm{W/m^3} \times 0.03\ \mathrm{m} \end{array}\right) \Bigg/ \left(\begin{array}{l} (417 \times 50 \\ + 50 \times 125 \\ + 417 \times 125)\ \left(\mathrm{W/m^2 \cdot K}\right)^2 \end{array}\right)$$

Continued…

$$T_{s,1} = 92.7°C$$

Similarly,

$$T_{s,2} = 85.8°C$$

The location of the maximum temperature in the wall can be found by setting the gradient of the temperature (from Equation (1)) to zero:

$$\frac{dT}{dx} = -\frac{\dot{q}x}{k} + \frac{T_{s,2} - T_{s,1}}{2L} = 0$$

Thus, $x_{max} = k\frac{T_{s,2} - T_{s,1}}{2L\dot{q}}$. Substituting this back into the temperature distribution,

$$T_{max} = \frac{\dot{q}L^2}{2k} + \frac{k\left(T_{s,2} - T_{s,1}\right)^2}{8L^2\dot{q}} + \frac{T_{s,1} + T_{s,2}}{2}$$

$$= \frac{2 \times 10^5 \text{ W/m}^3 \times (0.03 \text{ m})^2}{2 \times 25 \text{ W/m} \cdot \text{K}} + \frac{25 \text{ W/m} \cdot \text{K}\left(85.8°C - 92.7°C\right)^2}{8 \times (0.03 \text{ m})^2 \times 2 \times 10^5 \text{ W/m}^3}$$

$$+ \frac{92.7°C + 85.8°C}{2} = 93.7°C$$ <