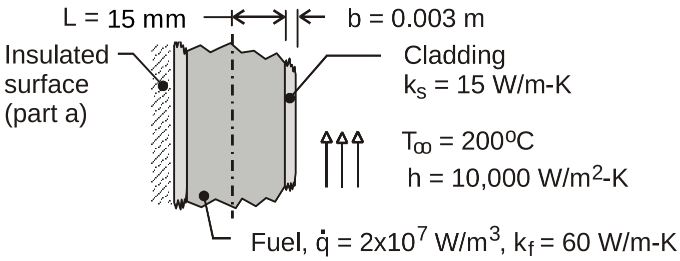
PROBLEM 3.91

KNOWN: Thermal conductivity, heat generation and thickness of fuel element. Thickness and thermal conductivity of cladding. Surface convection conditions.

FIND: (a) Temperature distribution in fuel element with one surface insulated and the other cooled by convection. Largest and smallest temperatures and corresponding locations. (b) Same as part (a) but with equivalent convection conditions at both surfaces, (c) Plot of temperature distributions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance.

ANALYSIS: (a) From Eq. C.1,

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{1}$$

With an insulated surface at x = -L, Eq. C.10 yields

$$T_{s,1} - T_{s,2} = \frac{2\dot{q}L^2}{k_f} \tag{2}$$

and with convection at x = L + b, Eq. C.13 yields

$$U\left(T_{s,2} - T_\infty\right) = \dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \tag{3}$$

where $U^{-1} = h^{-1} + b/k_s$. Subtracting Eq. (2) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{4\dot{q}L^2}{k_f}$$

$$T_{s,2} = T_\infty + \frac{2\dot{q}L}{U} \tag{4}$$

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PROBLEM 3.91 (Cont.)

Alternatively, this result could have been found from an energy balance on the wall which equates the generated heat to the heat leaving at L+b,

$$2\dot{q}L = U(T_{s,2} - T_\infty)$$

Substituting Eq. (4) into Eq. (2)

$$T_{s,1} = T_\infty + 2\dot{q}L\left(\frac{L}{k_f} + \frac{1}{U}\right) \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (1),

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2}{U} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty$$

Or,

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty \tag{6}$$ <

The maximum temperature occurs at x = - L and is

$$T(-L) = 2\dot{q}L\left(\frac{b}{k_s} + \frac{1}{h} + \frac{L}{k_f}\right) + T_\infty$$

$$T(-L) = 2 \times 2 \times 10^7\ \mathrm{W/m^3} \times 0.015\,\mathrm{m}\left(\frac{0.003\mathrm{m}}{15\,\mathrm{W/m\cdot K}} + \frac{1}{10,000\,\mathrm{W/m^2\cdot K}} + \frac{0.015\,\mathrm{m}}{60\,\mathrm{W/m\cdot K}}\right) + 200°\mathrm{C} = 530°\mathrm{C}$$ <

The lowest temperature is at x = + L and is

$$T(+L) = -\frac{3}{2}\frac{\dot{q}L^2}{k_f} + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty = 380°\mathrm{C}$$ <

(b) If a convection condition is maintained at x = - L, Eq. C.12 reduces to

$$U(T_\infty - T_{s,1}) = -\dot{q}L - \frac{k_f}{2L}(T_{s,2} - T_{s,1})$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}(T_{s,1} - T_\infty) - \frac{2\dot{q}L^2}{k_f} \tag{7}$$

Subtracting Eq. (7) from Eq. (3),

$$0 = \frac{2LU}{k_f}(T_{s,2} - T_\infty - T_{s,1} + T_\infty) \qquad \text{or} \qquad T_{s,1} = T_{s,2}$$

Hence, from Eq. (7)

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$$T_{s,1} = T_{s,2} = \frac{\dot{q}L}{U} + T_\infty = \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \qquad (8)$$

Substituting into Eq. (1), the temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \qquad (9) \quad <$$

The maximum temperature is at x = 0 and is

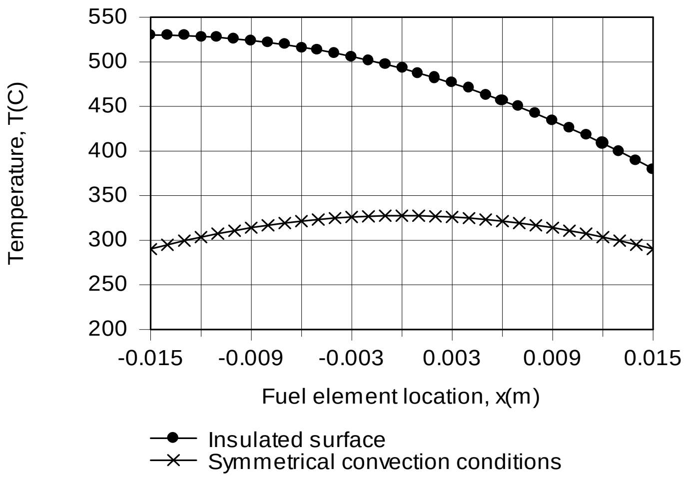
$$T(0) = \frac{2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})^2}{2\times60\ \mathrm{W/m\cdot K}} + 2\times10^7\ \mathrm{W/m^3}\times0.015\,\mathrm{m}\left(\frac{1}{10,000\ \mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\ \mathrm{W/m\cdot K}}\right) + 200°\mathrm{C}$$

$$T(0) = 37.5°\mathrm{C} + 90°\mathrm{C} + 200°\mathrm{C} = 327.5°\mathrm{C} \quad <$$

The minimum temperature at x = ± L is

$$T_{s,1} = T_{s,2} = 2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})\left(\frac{1}{10,000\ \mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\ \mathrm{W/m\cdot K}}\right) + 200°\mathrm{C} = 290°\mathrm{C} \quad <$$

(c) The temperature distributions are as shown.



The amount of heat generation is the same for both cases, but the ability to transfer heat from both surfaces for case (b) results in lower temperatures throughout the fuel element.

COMMENTS: Note that for case (a), the temperature in the insulated cladding is constant and equivalent to $T_{s,1}$ = 530°C.