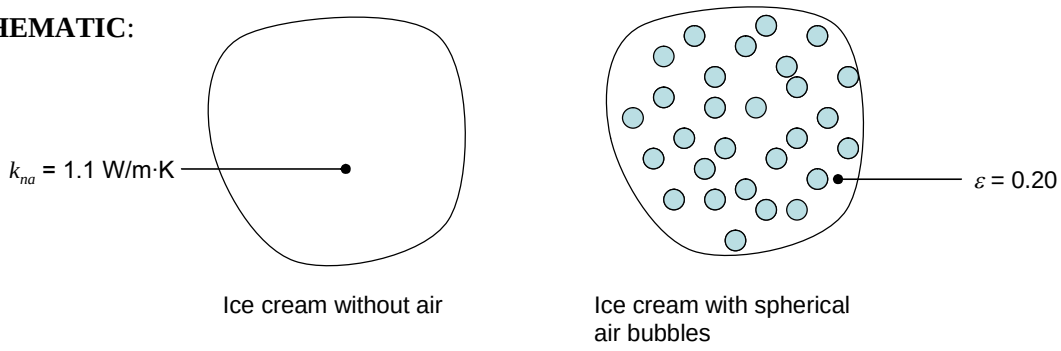
PROBLEM 3.36

KNOWN: Thermal conductivity of ice cream containing no air at T = -20°C. Shape and volume fraction of air bubbles.

FIND: The thermal conductivity of commercial ice cream characterized by $\varepsilon = 0.20$ at T = -20°C.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Spherical air bubbles.

PROPERTIES: *Table A.4*, Air (300 K): $k_{air} = 0.0225$ W/m·K.

ANALYSIS: Maxwell's expression for the effective thermal conductivity may be used, with $k_f = k_{air}$ and $k_s = k_{na}$. Hence,

$$k_{eff} = \left[\frac{k_f + 2k_s - 2\varepsilon\left(k_s - k_f\right)}{k_f + 2k_s + \varepsilon\left(k_s - k_f\right)}\right] k_s$$

$$= \left[\frac{0.0225\ \text{W/m}\cdot\text{K} + 2\times 1.1\ \text{W/m}\cdot\text{K} - 2\times 0.2\times\left(1.1\ \text{W/m}\cdot\text{K} - 0.0225\ \text{W/m}\cdot\text{K}\right)}{0.0225\ \text{W/m}\cdot\text{K} + 2\times 1.1\ \text{W/m}\cdot\text{K} + 0.2\times\left(1.1\ \text{W/m}\cdot\text{K} - 0.0225\ \text{W/m}\cdot\text{K}\right)}\right] \times 1.1\ \text{W/m}\cdot\text{K}$$

$$= 0.81\ \text{W/m}\cdot\text{K} \qquad <$$

COMMENTS: (1) The reduction in the effective thermal conductivity due to the presence of the air bubbles is 26%. (2) The predicted thermal conductivity is in good agreement with measured values. Furthermore, Maxwell's equation accurately predicts the measured thermal conductivity of ice cream over the range $0 \le \varepsilon \le 0.5$. The thermal conductivity as well as additional thermophysical properties of ice cream containing spherical air bubbles (called *overrun* ice cream) are available in Cogne, Andrieu, Laurent, Besson and Noequet, "Experimental Data and Modelling of Thermal Properties of Ice Cream," *Journal of Food Engineering*, Vol. 58, p, 331, 2003.