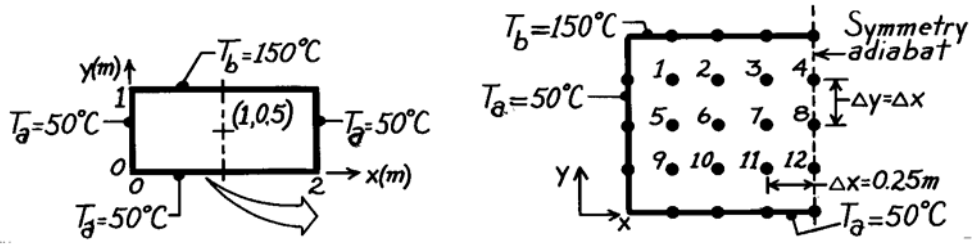
PROBLEM 4.75

KNOWN: Rectangular plate subjected to uniform temperature boundaries.

FIND: Temperature at the midpoint using a finite-difference method with space increment of 0.25m

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: For the nodal network above, 12 finite-difference equations must be written. It follows that node 8 represents the midpoint of the rectangle. Since all nodes are interior nodes, Eq. 4.29 is appropriate and is written in the form

$$4T_m - \sum T_{neighbors} = 0.$$

For nodes on the symmetry adiabat, the neighboring nodes include two symmetrical nodes. Hence, for Node 4, the neighbors are T_b, T_8 and $2T_3$. Because of the simplicity of the finite-difference equations, we may proceed directly to the matrices [A] and [C] – see Eq. 4.48 – and matrix inversion can be used to find the nodal temperatures T_m.

$$A = \begin{bmatrix} -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & -4 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & -4 \end{bmatrix} \quad C = \begin{bmatrix} -200 \\ -150 \\ -150 \\ -150 \\ -50 \\ 0 \\ 0 \\ 0 \\ -100 \\ -50 \\ -50 \\ -50 \end{bmatrix} \quad T = \begin{bmatrix} 96.5 \\ 112.9 \\ 118.9 \\ 120.4 \\ 73.2 \\ 86.2 \\ 92.3 \\ 94.0 \\ 59.9 \\ 65.5 \\ 69.9 \\ 71.0 \end{bmatrix}$$

The temperature at the midpoint (Node 8) is

$$T(1,0.5) = T_8 = 94.0^\circ C. \quad <$$

COMMENTS: Using the exact analytical, solution – see Eq. 4.19 and Problem 4.2 – the midpoint temperature is found to be 94.5°C. To improve the accuracy of the finite-difference method, it would be necessary to decrease the nodal mesh size.