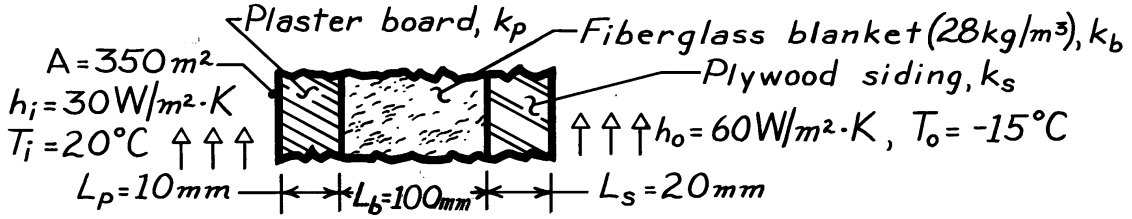
PROBLEM 3.13

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: (a) Expression for thermal resistance of house wall, R_{tot}; (b) Total heat loss, q(W); (c) Effect on heat loss due to increase in outside heat transfer convection coefficient, h_o; and (d) Controlling resistance for heat loss from house.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

PROPERTIES: *Table A-3*, $\left(\overline{T} = \left(T_i + T_o\right)/2 = \left(20-15\right)^\circ C/2 = 2.5^\circ C \approx 300K\right)$: Fiberglass blanket, 28 kg/m^3, k_b = 0.038 W/m·K; Plywood siding, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}. \quad <$$

(b) The total heat loss through the house wall is

$$q = \Delta T/R_{tot} = \left(T_i - T_o\right)/R_{tot}.$$

Substituting numerical values, find

$$R_{tot} = \frac{1}{30 W/m^2\cdot K \times 350 m^2} + \frac{0.01m}{0.17 W/m\cdot K \times 350 m^2} + \frac{0.10m}{0.038 W/m\cdot K \times 350 m^2} + \frac{0.02m}{0.12 W/m\cdot K \times 350 m^2} + \frac{1}{60 W/m^2\cdot K \times 350 m^2}$$

$$R_{tot} = \left[9.52 + 16.8 + 752 + 47.6 + 4.76\right]\times 10^{-5}\ ^\circ C/W = 831\times 10^{-5}\ ^\circ C/W$$

The heat loss is then,

$$q = \left[20-(-15)\right]^\circ C/831\times 10^{-5}\ ^\circ C/W = 4.21\ kW. \quad <$$

(c) If h_o changes from 60 to 300 W/m^2·K, $R_o = 1/h_o A$ changes from 4.76×10^{-5} °C/W to 0.95×10^{-5} °C/W. This reduces R_{tot} to 826×10^{-5} °C/W, which is a 0.6% decrease and hence a 0.6% increase in q.

(d) From the expression for R_{tot} in part (b), note that the insulation resistance, $L_b/k_b A$, is 752/830 ≈ 90% of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.