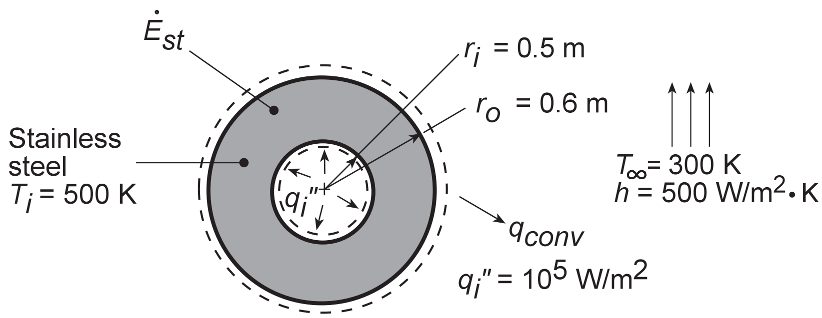
PROBLEM 1.64

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: *Table A.1*, Stainless Steel, AISI 302: ρ = 8055 kg/m^3, c_p = 535 J/kg·K.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$. Identifying relevant processes and solving for dT/dt,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho \frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q_i'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right].$$

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right)_i = \frac{3\left[10^5 \frac{W}{m^2}(0.5m)^2 - 500\frac{W}{m^2 \cdot K}(0.6m)^2(500-300)K\right]}{8055\frac{kg}{m^3}\,510\frac{J}{kg \cdot K}\left[(0.6)^3 - (0.5)^3\right]m^3}$$

$$\left.\frac{dT}{dt}\right)_i = -0.084\,K/s\,.$$ <

(b) Under steady-state conditions with $\dot{E}_{st}$ = 0, it follows that

$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300K + \frac{10^5\,W/m^2}{500W/m^2 \cdot K}\left(\frac{0.5m}{0.6m}\right)^2 = 439K$$ <

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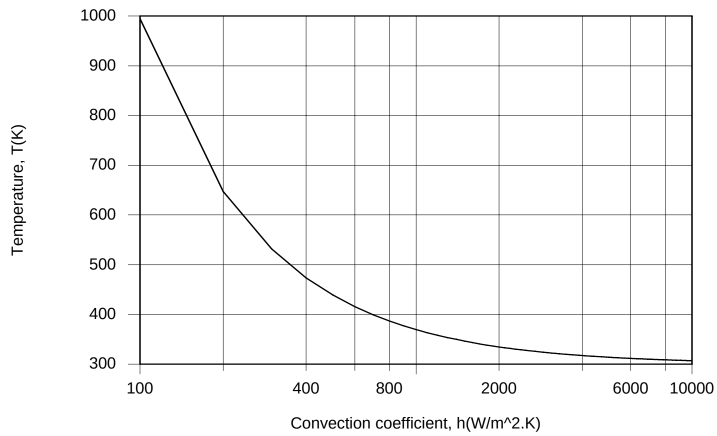
PROBLEM 1.64 (Cont.)

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Hollow Sphere*. As shown below, there is a sharp increase in temperature with decreasing values of h < 1000 $W/m^2 \cdot K$. For T > 380 K, boiling will occur at the canister surface, and for T > 410 K a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in h and increase in T.



Although the canister remains well below the melting point of stainless steel for h = 100 $W/m^2 \cdot K$, boiling should be avoided, in which case the convection coefficient should be maintained at h > 1000 $W/m^2 \cdot K$.

COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is $\theta = (S/R)\left(1 - e^{-Rt}\right) + \theta_i e^{-Rt}$, where $\theta \equiv T - T_\infty$,

$S \equiv 3q_i'' r_i^2 / \rho c_p \left(r_o^3 - r_i^3\right)$, $R = 3hr_o^2/\rho c_p \left(r_o^3 - r_i^3\right)$. Note results for $t \rightarrow \infty$ and for S = 0.