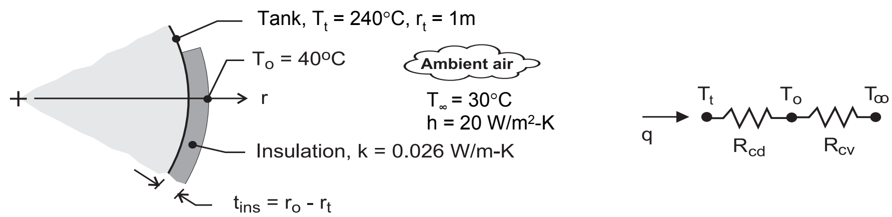
PROBLEM 3.71

KNOWN: Spherical tank of 2-m diameter containing an exothermic reaction and is at 240°C when the ambient air is at 30°C. Convection coefficient on outer surface is 20 W/m^2·K.

FIND: Determine the thickness of urethane foam required to reduce the exterior temperature to 40°C. Determine the percentage reduction in the heat rate achieved using the insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, (3) Convection coefficient is the same for bare and insulated exterior surface, and (4) Negligible radiation exchange between the insulation outer surface and the ambient surroundings, (5) Negligible contact resistances.

PROPERTIES: *Table A-3*, urethane, rigid foam (300 K): k = 0.026 W/m·K.

ANALYSIS: (a) The heat transfer situation for the heat rate from the tank can be represented by the thermal circuit shown above. The heat rate from the tank is

$$q = \frac{T_t - T_\infty}{R_{cd} + R_{cv}}$$

where the thermal resistances associated with conduction within the insulation (Eq. 3.41) and convection for the exterior surface, respectively, are

$$R_{cd} = \frac{(1/r_t - 1/r_o)}{4\pi k} = \frac{(1/1.0 - 1/r_o)}{4\pi \times 0.026\ \text{W/m}\cdot\text{K}} = \frac{(1/1.0 - 1/r_o)}{0.3267}\ \text{K/W}$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{4\pi h r_o^2} = \frac{1}{4\pi \times 20\ \text{W/m}^2\cdot\text{K} \times r_o^2} = 3.979\times10^{-3} r_o^{-2}\ \text{K/W}$$

To determine the required insulation thickness so that T_o = 40°C, perform an energy balance on the o-node.

$$\frac{T_t - T_o}{R_{cd}} + \frac{T_\infty - T_o}{R_{cv}} = 0$$

$$\frac{(240-40)\text{K}}{(1/1.0 - 1/r_o)/0.3267\ \text{K/W}} + \frac{(30-40)\text{K}}{3.979\times10^{-3} r_o^2\ \text{K/W}} = 0$$

$$r_o = 1.025\ \text{m} \qquad t = r_o - r_i = (1.025 - 1.0)\ \text{m} = 25.4\ \text{mm}$$ <

From the rate equation, for the bare and insulated surfaces, respectively,

$$q_o = \frac{T_t - T_\infty}{1/4\pi h r_t^2} = \frac{(240-30)\text{K}}{0.01592\ \text{K/W}} = 52.78\ \text{kW}$$

$$q_{ins} = \frac{T_t - T_\infty}{R_{cd} + R_{cv}} = \frac{(240-30)}{(0.0757 + 0.003785)\text{K/W}} = 2.642\ \text{kW}$$

Hence, the percentage reduction in heat loss achieved with the insulation is,

$$\frac{q_{ins} - q_o}{q_o} \times 100 = \frac{52.78 - 2.64}{52.78} \times 100 = 95\%$$ <

COMMENTS: (1) Contact resistances will reduce the required insulation thickness. (2) Radiation may be shown to be negligible by considering the case of $\varepsilon = 1$.