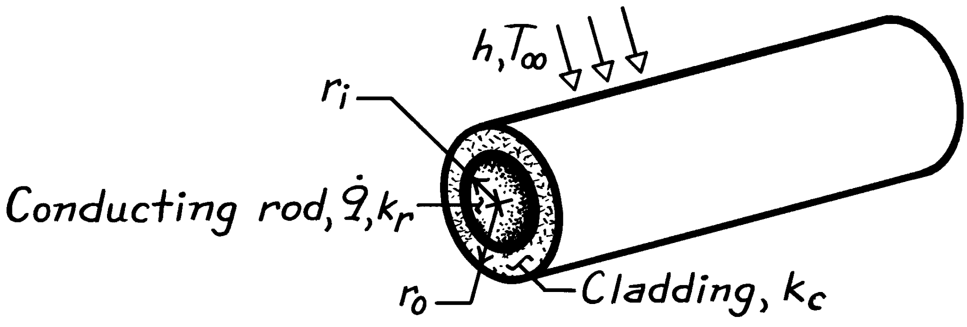
PROBLEM 2.48

KNOWN: Radii and thermal conductivity of conducting rod and cladding material. Volumetric rate of thermal energy generation in the rod. Convection conditions at outer surface.

FIND: Heat equations and boundary conditions for rod and cladding.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: From Equation 2.26, the appropriate forms of the heat equation are

Conducting Rod:

$$\frac{k_r}{r}\frac{d}{dr}\left(r\frac{dT_r}{dr}\right)+\dot{q}=0$$ <

Cladding:

$$\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0.$$ <

Appropriate boundary conditions are:

(a) $dT_r / dr|_{r=0} = 0$ <

(b) $T_r(r_i) = T_c(r_i)$ <

(c) $k_r \frac{dT_r}{dr}|_{r_i} = k_c \frac{dT_c}{dr}|_{r_i}$ <

(d) $- k_c \frac{dT_c}{dr}|_{r_o} = h\left[T_c(r_o) - T_\infty\right]$ <

COMMENTS: Condition (a) corresponds to symmetry at the centerline, while the interface conditions at $r = r_i$ (b,c) correspond to requirements of thermal equilibrium and conservation of energy. Condition (d) results from conservation of energy at the outer surface. Note that contact resistance at the interface between the rod and cladding has been neglected.