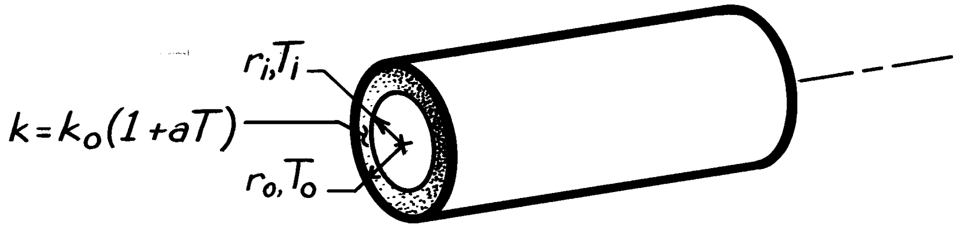
PROBLEM 3.42

KNOWN: Temperature dependence of tube wall thermal conductivity.

FIND: Expressions for heat transfer per unit length and tube wall thermal (conduction) resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal heat generation.

ANALYSIS: From Eq. 3.29, the appropriate form of Fourier's law is

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi\ rL)\frac{dT}{dr}$$

$$q_r' = -2\pi\ kr\frac{dT}{dr}$$

$$q_r' = -2\pi\ rk_o(1 + aT)\frac{dT}{dr}.$$

Separating variables,

$$-\frac{q_r'}{2\pi}\frac{dr}{r} = k_o(1 + aT)dT$$

and integrating across the wall, find

$$-\frac{q_r'}{2\pi}\int_{r_i}^{r_o}\frac{dr}{r} = k_o\int_{T_i}^{T_o}(1+aT)dT$$

$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[T + \frac{aT^2}{2}\right]\Bigg|_{T_i}^{T_o}$$

$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[(T_o - T_i) + \frac{a}{2}\left(T_o^2 - T_i^2\right)\right]$$

$$q_r' = -2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]\frac{(T_o - T_i)}{\ln(r_o/r_i)}. \qquad <$$

It follows that the overall thermal resistance per unit length is

$$R_t' = \frac{T_i - T_o}{q_r'} = \frac{\ln(r_o/r_i)}{2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]}. \qquad <$$

COMMENT: Note the necessity of the stated assumptions to treating q_r' as independent of r.