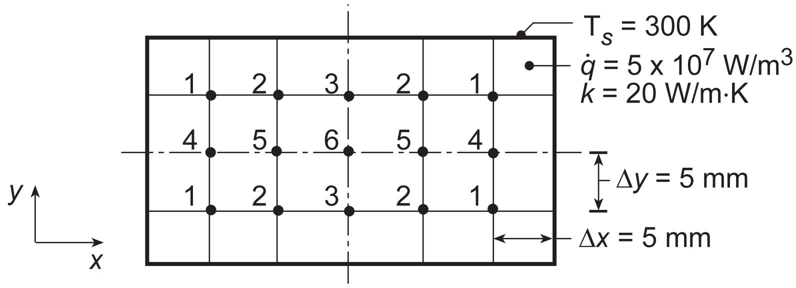
PROBLEM 4.53

KNOWN: Volumetric heat generation in a rectangular rod of uniform surface temperature.

FIND: (a) Temperature distribution in the rod, and (b) With boundary conditions unchanged, heat generation rate causing the midpoint temperature to reach 600 K.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) Uniform volumetric heat generation.

ANALYSIS: (a) From symmetry it follows that six unknown temperatures must be determined. Since all nodes are interior ones, the finite-difference equations may be obtained from Eq. 4.35 written in the form

$$T_i = 1/4 \sum T_{neighbors} + 1/4\left(\dot{q}\left(\Delta x \Delta y\right)/k\right).$$

With $\dot{q}\left(\Delta x \Delta y\right)/4k$ = 62.5 K, the system of finite-difference equations is

$$T_1 = 0.25\left(T_s + T_2 + T_4 + T_s\right) + 15.625 \qquad (1)$$

$$T_2 = 0.25\left(T_s + T_3 + T_5 + T_1\right) + 15.625 \qquad (2)$$

$$T_3 = 0.25\left(T_s + T_2 + T_6 + T_2\right) + 15.625 \qquad (3)$$

$$T_4 = 0.25\left(T_1 + T_5 + T_1 + T_s\right) + 15.625 \qquad (4)$$

$$T_5 = 0.25\left(T_2 + T_6 + T_2 + T_4\right) + 15.625 \qquad (5)$$

$$T_6 = 0.25\left(T_3 + T_5 + T_3 + T_5\right) + 15.625 \qquad (6)$$

With T_s = 300 K, the set of equations was written directly into the IHT workspace and solved for the nodal temperatures,

T_1	T_2	T_3	T_4	T_5	T_6 (K)
348.6	368.9	374.6	362.4	390.2	398.0

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(b) With the boundary conditions unchanged, the $\dot{q}$ required for T_6 = 600 K can be found using the same set of equations in the IHT workspace, but with these changes: (1) replace the last term on the RHS (15.625) of Eqs. (1-6) by $\dot{q}$ ($\Delta x \Delta y$)/4k = (0.005 m)^2 $\dot{q}$ /4×20 W/m·K = 3.125 × 10^{-7} $\dot{q}$ and (2) set T_6 = 600 K. The set of equations has 6 unknown, five nodal temperatures plus $\dot{q}$. Solving find

$$\dot{q} = 1.53 \times 10^8 \ W/m^3$$ <