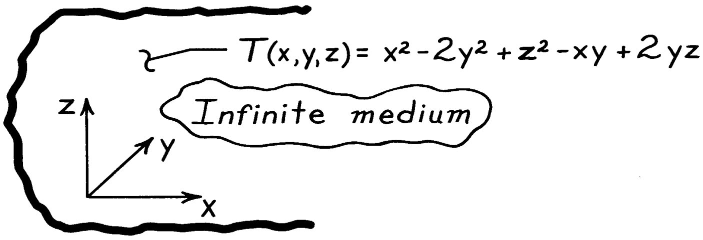
PROBLEM 2.26

KNOWN: Temperature distribution, T(x,y,z), within an infinite, homogeneous body at a given instant of time.

FIND: Regions where the temperature changes with time.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties of infinite medium and (2) No internal heat generation.

ANALYSIS: The temperature distribution throughout the medium, at any instant of time, must satisfy the heat equation. For the three-dimensional cartesian coordinate system, with constant properties and no internal heat generation, the heat equation, Eq. 2.21, has the form

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}. \qquad (1)$$

If T(x,y,z) satisfies this relation, conservation of energy is satisfied at every point in the medium. Substituting T(x,y,z) into the Eq. (1), first find the gradients, $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial z$.

$$\frac{\partial}{\partial x}(2x\text{-}y)+\frac{\partial}{\partial y}(-4y\text{-}x+2z)+\frac{\partial}{\partial z}(2z+2y)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Performing the differentiations,

$$2-4+2=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Hence,

$$\frac{\partial T}{\partial t}=0$$

which implies that, at the prescribed instant, the temperature is everywhere independent of time. <

COMMENTS: Since we do not know the initial and boundary conditions, we cannot determine the temperature distribution, T(x,y,z), at any future time. We can only determine that, for this special instant of time, the temperature will not change.