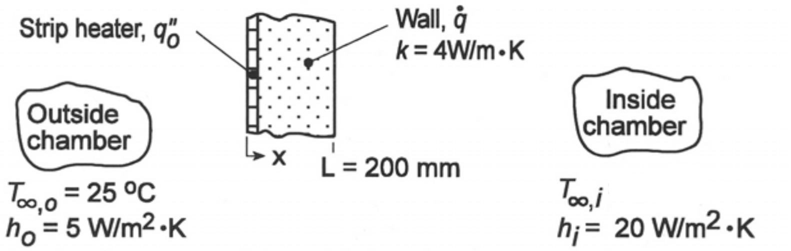
PROBLEM 3.92

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation and strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions ($T_{\infty,i}$, h_i, $T_{\infty,o}$, h_o). Strip heater acts to guard against heat losses from the wall to the outside.

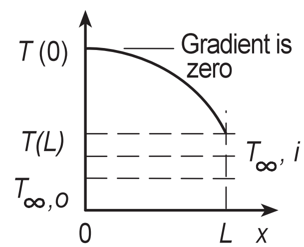
FIND: Compute and plot q''_o and T(0) as a function of $\dot{q}$ for $200 \le \dot{q} \le 2000\ \mathrm{W/m^3}$ and $T_{\infty,i}$ = 30, 50 and 70°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: If no heat generated within the wall will be lost to the outside of the chamber, the gradient at the position x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution must be parabolic as shown in the sketch.



To determine the required heater flux q''_o as a function of the operation conditions $\dot{q}$ and $T_{\infty,i}$, the analysis begins by considering the temperature distribution in the wall and then surface energy balances at the two wall surfaces. The analysis is organized for easy treatment with equation-solving software.

Temperature distribution in the wall, T(x): The general solution for the temperature distribution in the wall is, Eq. 3.45,

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$$

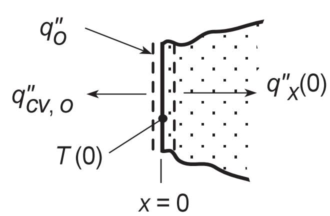
and the guard condition at the outer wall, x = 0, requires that the conduction heat flux be zero. Using Fourier's law,

$$q''_x(0) = -k\left.\frac{dT}{dx}\right|_{x=0} = -kC_1 = 0 \qquad (C_1 = 0) \tag{1}$$

At the outer wall, x = 0,

$$T(0) = C_2 \tag{2}$$

Heater energy balance, x = 0:



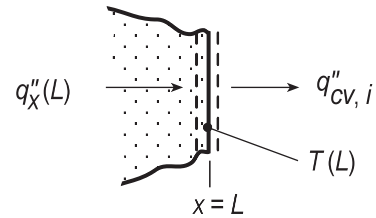
$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = 0$$

$$0 + q''_o - q''_{cv,o} - q''_x(0) = 0 \tag{3}$$

$$q''_{cv,o} = h_o\left(T(0) - T_{\infty,o}\right), q''_x(0) = 0 \tag{4a,b}$$

Continued...

Surface energy balance, x = L:



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

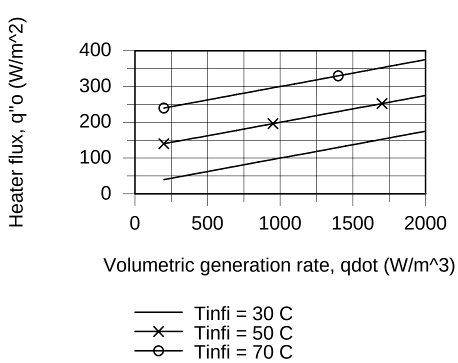
$$q''_x(L) - q''_{cv,i} = 0 \qquad (5)$$

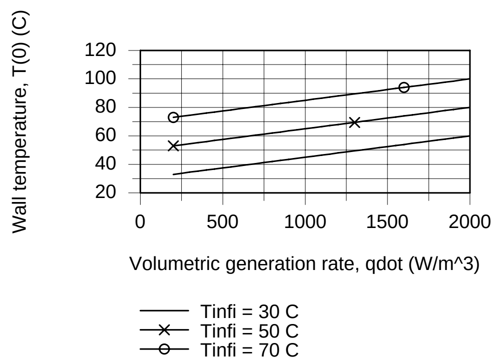
$$q''_x(L) = -k\left.\frac{dT}{dx}\right|_{x=L} = +\dot{q}L \qquad (6)$$

$$q''_{cv,i} = h_i\left[T(L) - T_{\infty,i}\right]$$

$$q''_{cv,i} = h_i\left[-\frac{\dot{q}}{2k}L^2 + T(0) - T_{\infty,i}\right] \qquad (7)$$

Solving Eqs. (3) through (7) simultaneously with appropriate numerical values and performing the parametric analysis, the results are plotted below.





From the first plot, the heater flux q''_o is a linear function of the volumetric generation rate $\dot{q}$. As expected, the higher $\dot{q}$ and $T_{\infty,i}$, the higher the heat flux required to maintain the guard condition ($q''_x(0)$ = 0). Notice that for any $\dot{q}$ condition, equal changes in $T_{\infty,i}$ result in equal changes in the required q''_o. The outer wall temperature T(0) is also linearly dependent upon $\dot{q}$. From our knowledge of the temperature distribution, it follows that for any $\dot{q}$ condition, the outer wall temperature T(0) will track changes in $T_{\infty,i}$.