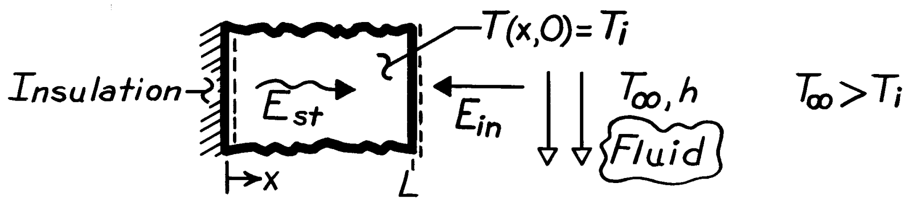
PROBLEM 2.57

KNOWN: Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

FIND: (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, T(x,t); (b) Sketch T(x,t) for these conditions: initial ($t \le 0$), steady-state, $t \to \infty$, and two intermediate times; (c) Sketch heat fluxes as a function of time for surface locations; (d) Expression for total energy transferred to wall per unit volume (J/m^3).

SCHEMATIC:



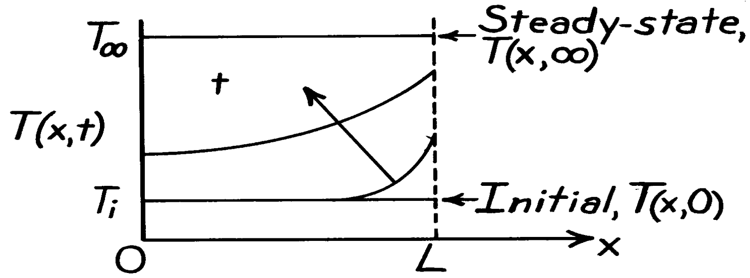
ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: (a) For one-dimensional conduction with constant properties, the heat equation has the form,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

and the conditions are:
$$\begin{cases} \text{Initial, } t \le 0: & T(x,0) = T_i & \text{uniform} \\ \text{Boundaries:} \quad x=0 & \partial T/\partial x)_0 = 0 & \text{adiabatic} \\ \qquad\qquad x=L & -k\partial T/\partial x)_L = h\left[T(L,t) - T_\infty\right] & \text{convection} \end{cases}$$

(b) The temperature distributions are shown on the sketch.



Note that the gradient at x = 0 is always zero, since this boundary is adiabatic. Note also that the gradient at x = L decreases with time.

(c) The heat flux, $q''_x(x,t)$, as a function of time, is shown on the sketch for the surfaces x = 0 and x = L.

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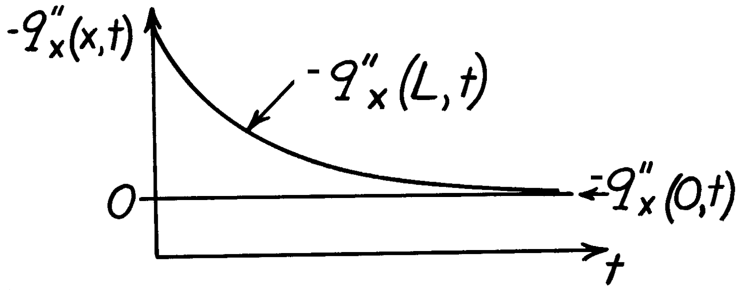

For the surface at $x = 0$, $q''_x(0,t) = 0$ since it is adiabatic. At x = L and t = 0, $q''_x(L,0)$ is a maximum (in magnitude)

$$\left|q''_x(L,0)\right| = h\left|T(L,0) - T_\infty\right|$$

where T(L,0) = T_i. The temperature difference, and hence the flux, decreases with time.

(d) The total energy transferred to the wall may be expressed as

$$E_{in} = \int_0^\infty q''_{conv} A_s dt$$
$$E_{in} = hA_s \int_0^\infty \left(T_\infty - T(L,t)\right) dt$$

Dividing both sides by $A_s L$, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L}\int_0^\infty \left[T_\infty - T(L,t)\right] dt \qquad \left[J/m^3\right]$$

COMMENTS: Note that the heat flux at x = L is into the wall and is hence in the negative x direction.