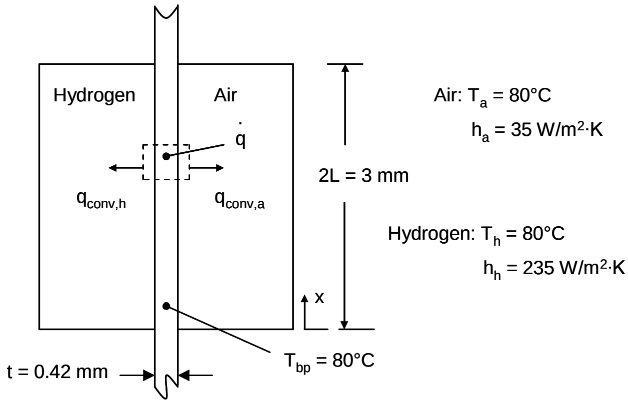
PROBLEM 3.118

KNOWN: Dimensions, convective conditions, bipolar plate, hydrogen and air temperatures within a fuel cell.

FIND: (a) The differential equation governing the membrane temperature distribution, T(x), (b) Solution of the equation of part (a), (c) Temperature distributions associated with carbon nanotube loadings of 0 and 10 volume percent.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Uniform volumetric energy generation, (4) Negligible contact resistance.

ANALYSIS:

(a) Performing an energy balance on the differential control volume,

$$q'_x + dq'_g = q'_{x+dx} + dq'_{conv,a} + dq'_{conv,h} \qquad (1)$$

$$q'_{x+dx} = q'_x + (dq'_x/dx)dx$$

where $dq'_g = \dot{q} \cdot t \cdot dx$, $dq'_{conv,a} = h_a(T - T_a)dx$, $dq'_{conv,h} = h_h(T - T_h)dx$

Noting that $T_a = T_h$, Eq, (1) becomes

$$\dot{q} \cdot t \cdot dx = (dq'_x / dx)dx + h_a(T - T_a)dx + h_h(T - T_h)dx = (dq'_x/dx)dx + \left[(h_a + h_h)(T - T_h)\right]dx$$

From Fourier's law,

$$q'_x = - kt\, dT/dx$$

and $$\frac{d^2T}{dx^2} - \frac{1}{k_{eff,x}t}\left[(h_a + h_h)(T - T_a)\right] + \frac{\dot{q}}{k_{eff,x}} = 0$$ <

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(b) Defining $\theta = T - T_a$ and $\dfrac{d^2T}{dx^2} = \dfrac{d^2\theta}{dx^2}$, the differential equation becomes

$$\frac{d^2\theta}{dx^2} - \frac{(h_a + h_h)}{k_{eff,x}t}\theta + \frac{\dot{q}}{k_{eff,x}} = 0$$

This is the second-order, differential equation, and its general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2$$

where $\lambda = \left(\dfrac{h_a + h_h}{k_{eff,x}t}\right)^{1/2}$, $S = \dfrac{\dot{q}}{k_{eff,x}}$

Appropriate boundary conditions are:

$$\theta(0) = T_0 - T_\infty = \theta_0, \quad d\theta/dx)_{x=L} = 0.$$

Hence, $\theta_0 = C_1 + C_2 + S/\lambda^2$

$$d\theta/dx)_{x=L} = C_1\lambda e^{+\lambda L} - C_2\lambda e^{-\lambda L} = 0 \qquad C_2 = C_1 e^{2\lambda L}$$

Hence, $C_1 = \left(\theta_0 - S/\lambda^2\right)\Big/\left(1 + e^{2\lambda L}\right) \qquad C_2 = \left(\theta_0 - S/\lambda^2\right)\Big/\left(1 + e^{-2\lambda L}\right)$

$$\theta = (\theta_0 - S/\lambda^2)\left[\frac{e^{\lambda x}}{1 + e^{2\lambda L}} + \frac{e^{-\lambda x}}{1 + e^{-2\lambda L}}\right] + S/\lambda^2. \qquad <$$

(c) For $h_a = 35\ W/m^2 \cdot K$, $h_h = 235\ W/m^2 \cdot K$ and $k_{eff,x} = 0.79\ W/m \cdot K$

$$\lambda = \left[\frac{35\ W/m^2 \cdot K + 235\ W/m^2 \cdot K}{0.79\ W/m \cdot K \times 0.42 \times 10^{-3}\ m}\right]^{1/2} = 902\ m^{-1}$$

For $\dot{q} = 10 \times 10^6\ W/m^3$, $S = \dfrac{10 \times 10^6\ W/m^3}{0.79\ W/m \cdot K} = 12.7 \times 10^6\ K/m^2$

The temperature distribution without, and with carbon nanotube loading, is shown below. <

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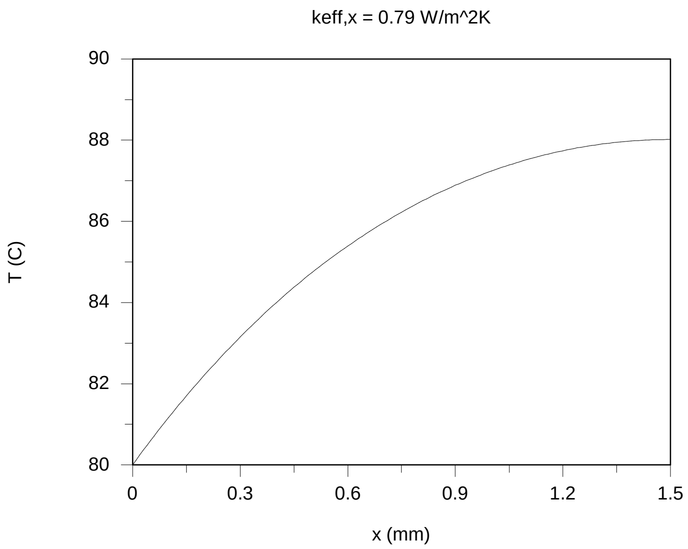
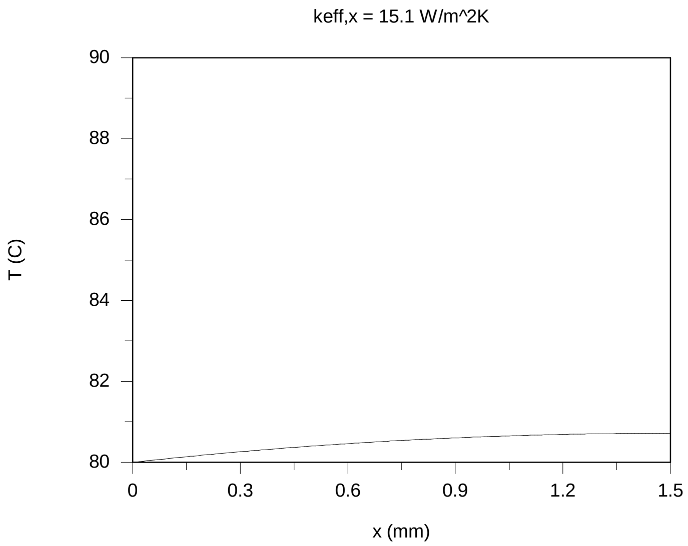
PROBLEM 3.118 (Cont.)



Without carbon nanotube loading.



With carbon nanotube loading.

COMMENTS: (1) The carbon nanotubes appear to be effective in reducing the maximum temperature of the membrane. (2) Contact resistances between the bipolar plates and the membrane can be large. Hence, the actual membrane temperature will be higher than indicated with this analysis.