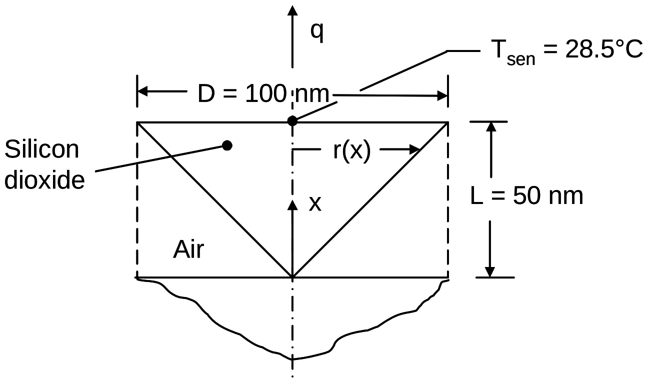
PROBLEM 3.44

KNOWN: Construction and dimensions of a device to measure the temperature of a surface. Ambient and sensing temperatures, and thermal resistance between the sensing element and the pivot point.

FIND: (a) Thermal resistance between the surface temperature and the sensing temperature, (b) Surface temperature for T_{sen} = 28.5°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible nanoscale effects, (4) Constant properties.

PROPERTIES: Table A.2, polycrystalline silicon dioxide (300 K): k = 1.38 W/m·K. Table A.4, air (300 K): k = 0.0263 W/m·K.

ANALYSIS:

(a) At any x location, heat transfer in the x-direction occurs by conduction in the air as well as conduction in the probe. Applying Fourier's law,

$$q_x = -k_a A_a \frac{dT}{dx} - k_p A_p \frac{dT}{dx} \qquad (1)$$

Since the probe radius is r = Dx/2L, the probe area is

$$A_p = \frac{\pi D^2}{4L^2} x^2 \quad \text{and} \quad A_a = \frac{\pi D^2}{4} - A_p = \frac{\pi D^2}{4}\left[1 - \frac{x^2}{L^2}\right] \qquad (2a, 2b)$$

Substituting Eqs. (2a) and (2b) into Eq. (1) yields

$$q_x = -\frac{\pi D^2}{4L^2}\left[k_a (L^2 - x^2) + k_p x^2\right]\frac{dT}{dx}$$

Separating variables and integrating,

Continued…

PROBLEM 3.44 (Cont.)

$$q_x \int_{x=0}^{L} \frac{dx}{k_a (L^2 - x^2) + k_p x^2} = -\frac{\pi D^2}{4L^2} \int_{T=T_{surf}}^{T_{sen}} dT = -\frac{\pi D^2}{4L^2}\left(T_{sen} - T_{surf}\right)$$

Therefore, the thermal resistance associated with the probe is

$$R_{sen} = \frac{(T_{surf} - T_{sen})}{q_x} = \frac{4L^2}{\pi D^2} \int_{x=0}^{L} \frac{dx}{k_a L^2 + (k_p - k_a)x^2}$$

Carrying out the integration yields

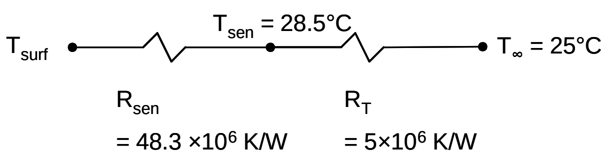
$$R_{sen} = \frac{4L^2}{\pi D^2} \frac{1}{\sqrt{k_a (k_p - k_a)}} \tan^{-1}\sqrt{\frac{k_p - k_a}{k_a}}$$

Substituting values gives

$$R_{sen} = \frac{4 \times 50 \times 10^{-9}\ \text{m}}{\pi \times (100 \times 10^{-9}\ \text{m})^2} \times \frac{1}{\sqrt{0.0263\ \text{W/m}\cdot\text{K} \times (1.38 - 0.0263)\ \text{W/m}\cdot\text{K}}}$$

$$\times \tan^{-1}\sqrt{\frac{(1.38 - 0.0263)\ \text{W/m}\cdot\text{K}}{0.0263\ \text{W/m}\cdot\text{K}}} = 48.3 \times 10^6\ \text{K/W}$$ <

(b) The thermal circuit is



Hence,

$$\frac{(T_{surf} - T_{sen})}{R_{sen}} = \frac{(T_{sen} - T_\infty)}{R_T}$$

$$T_{surf} = (T_{sen} - T_\infty)\frac{R_{sen}}{R_T} + T_{sen} = (28.5 - 25)°\text{C} \times \frac{48.3 \times 10^6\ \text{K/W}}{5 \times 10^6\ \text{K/W}} + 28.5°\text{C}$$

$$T_{surf} = 62.3°\text{C}$$ <

COMMENT: Heat transfer within the probe region will not be one-dimensional and modification of heat transfer due to nanoscale effects may be important. However, the probe may be calibrated by measuring the surface temperature of a large isothermal object.