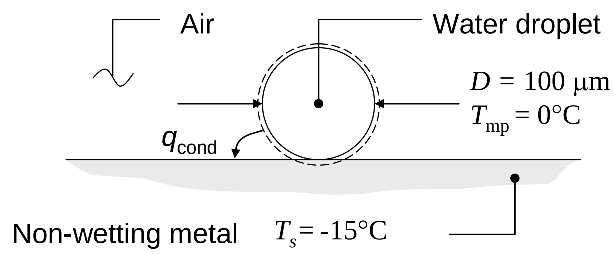
PROBLEM 4.15

KNOWN: Dimensions and temperature of water droplet.

FIND: Time for droplet to freeze completely.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible convection and radiation, (3) Isothermal water particle, (4) Semi-infinite medium.

PROPERTIES: Table A.4, Air (265 K): k_a = 0.0235 W/m·K. Table A.6, Liquid water (273 K): ρ_w = 1000 kg/m^3.

ANALYSIS: An energy balance on the droplet yields

$$t = \frac{\Delta E}{q_{\text{cond}}} = \frac{V\rho_w h_{sf}}{Sk(T_{\text{mp}} - T_s)} \tag{1}$$

The shape factor S is that of Case 1 of Table 4.1 with $z = D/2$

$$S = \frac{2\pi D}{1 - D/4z} = 4\pi D \tag{2}$$

Combining Equations (1) and (2) with the expression for the droplet volume $V = \pi D^3/6$ yields

$$t = \frac{D^2 \rho_w h_{sf}}{24 k_a (T_{mp} - T_s)} = \frac{(100\times10^{-6}\text{ m})^2 \times 1000\text{ kg/m}^3 \times 334{,}000\text{ J/kg}}{24\times 0.0235\text{ W/m}\cdot\text{K}\times 15\text{ K}} = 0.39\text{ s}$$ <

COMMENTS: (1) Solidification might initiate in the lower region of the droplet. The ice that forms would pose an additional conduction resistance between the cold metal surface and the liquid water. This would increase the time needed for the droplet to solidify completely. (2) The air thermal conductivity in the vicinity of the contact point would be reduced by the nanoscale effects described in Chapter 2. In applying this shape factor for the $z = D/2$ case we have implicitly assumed that nanoscale effects are negligible. See B. Gebhart, *Heat Conduction and Mass Diffusion*, McGraw-Hill, 1993.