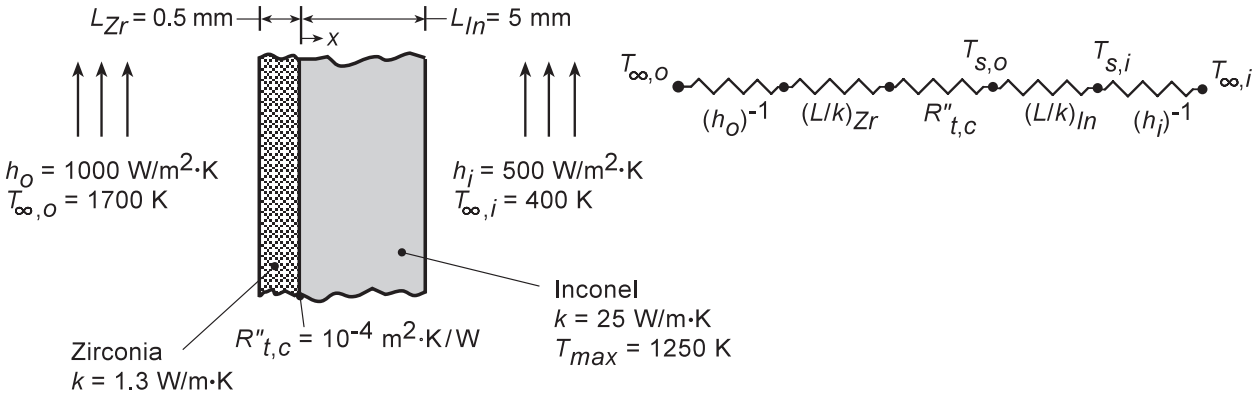
PROBLEM 3.30

KNOWN: Outer and inner surface convection conditions associated with zirconia-coated, Inconel turbine blade. Thicknesses, thermal conductivities, and interfacial resistance of the blade materials. Maximum allowable temperature of Inconel.

FIND: Whether blade operates below maximum temperature. Temperature distribution in blade, with and without the TBC.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in a composite plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$R''_{tot,w} = h_o^{-1} + (L/k)_{Zr} + R''_{t,c} + (L/k)_{In} + h_i^{-1}$$

$$R''_{tot,w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right) m^2\cdot K/W = 3.69\times10^{-3} m^2\cdot K/W$$

With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{tot,w}} = \frac{1300\,K}{3.69\times10^{-3}\,m^2\cdot K/W} = 3.52\times10^5\ W/m^2$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(w)} = T_{\infty,i} + (q''_w/h_i) = 400\,K + \left(3.52\times10^5\ W/m^2 / 500\ W/m^2\cdot K\right) = 1104\,K$$

$$T_{s,o(w)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right] q''_w = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right) m^2\cdot K/W \left(3.52\times10^5\ W/m^2\right) = 1174\,K$$

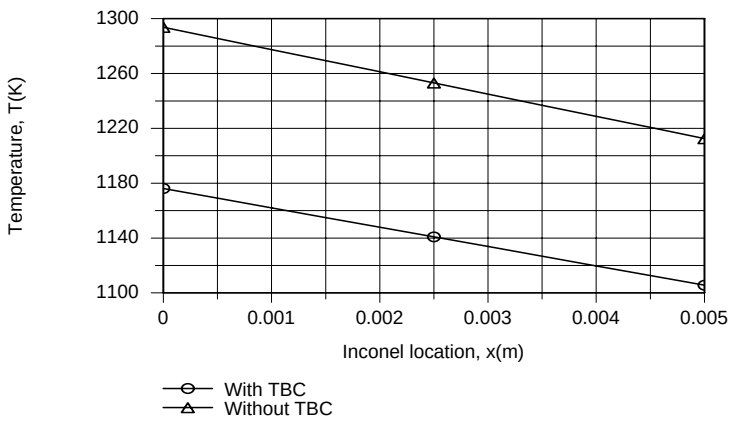
Without the TBC, $R''_{tot,wo} = h_o^{-1} + (L/k)_{In} + h_i^{-1} = 3.20\times10^{-3} m^2\cdot K/W$, and $q''_{wo} = (T_{\infty,o} - T_{\infty,i})/R''_{tot,wo}$ = (1300 K)/3.20×10^{-3} m^2·K/W = 4.06×10^5 W/m^2. The inner and outer surface temperatures of the Inconel are then

$$T_{s,i(wo)} = T_{\infty,i} + (q''_{wo}/h_i) = 400\,K + \left(4.06\times10^5\ W/m^2 / 500\ W/m^2\cdot K\right) = 1212\,K$$

$$T_{s,o(wo)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right] q''_{wo} = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right) m^2\cdot K/W \left(4.06\times10^5\ W/m^2\right) = 1293\,K$$

Continued...

PROBLEM 3.30 (Cont.)



Use of the TBC facilitates operation of the Inconel below T_{max} = 1250 K.

COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to the thickness are associated with reliability considerations.