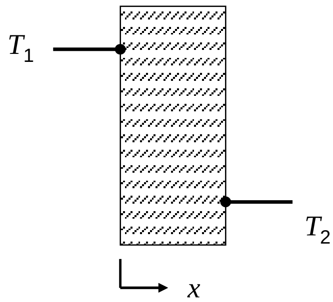
PROBLEM 3.35

KNOWN: Density of glass fiber insulation.

FIND: Maximum and minimum possible values of the effective thermal conductivity of the insulation at T = 300 K, and comparison with the value listed in Table A.3.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation, (3) Atmospheric pressure.

PROPERTIES: *Table A.3,* Glass (plate): k_{gl} = 1.4 W/m·K, ρ_{gl} = 2500 kg/m^3.Glass fiber batt (paper faced, ρ_{ins} = 28 kg/m^3) k_{ins} = 0.038 W/m·K. *Table A.4,* Air (300 K): k_{air} = 0.0263 W/m·K, ρ_{air} = 1.1614 kg/m^3. Given: ρ_{ins} = 28 kg/m^3.

ANALYSIS: The density of the glass fiber insulation may be related to the density of the air and glass phases, and the volume fraction, ε, as follows.

$$\rho_{\text{ins}} = \varepsilon\rho_{\text{air}} + (1-\varepsilon)\rho_{\text{gl}}$$

Therefore,

$$\varepsilon = \frac{\rho_{\text{gl}} - \rho_{\text{ins}}}{\rho_{\text{gl}} - \rho_{\text{air}}} = \frac{2500-28}{2500-1.164} = 0.989$$

The minimum effective thermal conductivity is

$$k_{\text{eff,min}} = \frac{1}{(1-\varepsilon)/k_{\text{glass}} + \varepsilon/k_{\text{air}}} = \frac{1}{(1-0.989)/1.4\text{ W/m}\cdot\text{K} + 0.989/0.0263\text{ W/m}\cdot\text{K}} \quad <$$
$$= 0.0265\text{ W/m}\cdot\text{K}$$

The maximum effective thermal conductivity is

$$k_{\text{eff,max}} = \varepsilon k_f + (1-\varepsilon)k_s = 0.989\times0.0263\text{ W/m}\cdot\text{K} + (1-0.989)\times1.4\text{ W/m}\cdot\text{K} \quad <$$
$$= 0.0414\text{ W/m}\cdot\text{K}$$

As expected, the predicted minimum and maximum effective thermal conductivities bracket the actual effective thermal conductivity of k_{ins} = 0.038 W/m·K. $<$

COMMENT: Radiation internal to the glass fiber batting may be significant. If so, this will reduce the insulating capability of the matt.