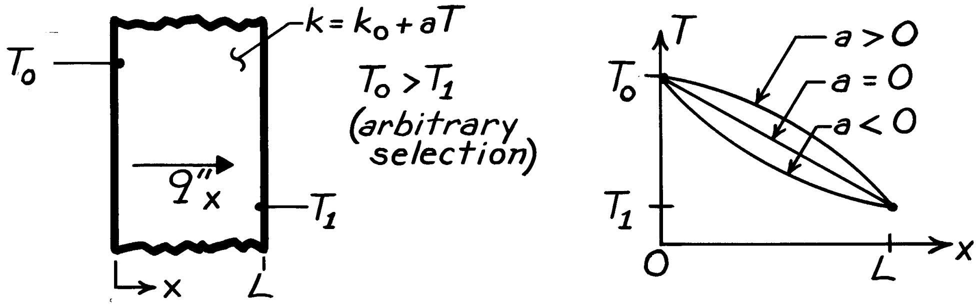
PROBLEM 3.41

KNOWN: Temperature dependence of the thermal conductivity, k.

FIND: Heat flux and form of temperature distribution for a plane wall.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: For the assumed conditions, q_x and A(x) are constant and Eq. 3.26 gives

$$q''_x \int_0^L dx = -\int_{T_o}^{T_1} (k_o + aT) dT$$

$$q''_x = \frac{1}{L}\left[k_o (T_o - T_1) + \frac{a}{2}\left(T_o^2 - T_1^2\right)\right].$$

From Fourier's law,

$$q''_x = -(k_o + aT)\ dT/dx.$$

Hence, since the product of (k_o+aT) and dT/dx) is constant, decreasing T with increasing x implies,

a > 0: decreasing (k_o+aT) and increasing |dT/dx| with increasing x

a = 0: k = k_o => constant (dT/dx)

a < 0: increasing (k_o+aT) and decreasing |dT/dx| with increasing x.

The temperature distributions appear as shown in the above sketch.