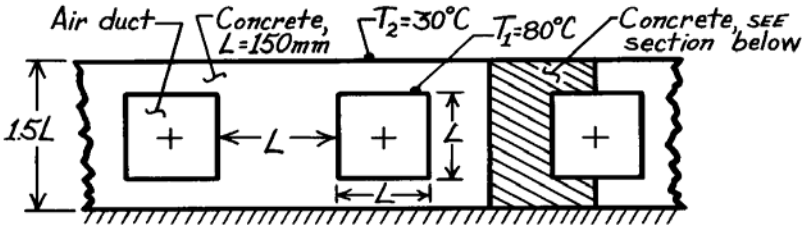
PROBLEM 4.67

KNOWN: Rectangular air ducts having surfaces at 80°C in a concrete slab with an insulated bottom and upper surface maintained at 30°C.

FIND: Heat rate from each duct per unit length of duct, q'.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) No internal volumetric generation, (4) Constant properties.

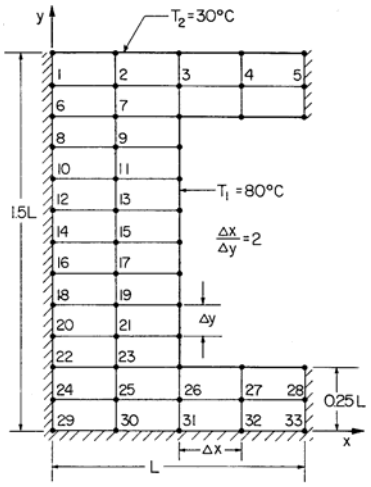
PROPERTIES: Concrete (given): k = 1.4 W/m·K.

ANALYSIS: Taking advantage of symmetry, the nodal network, using the suggested grid spacing

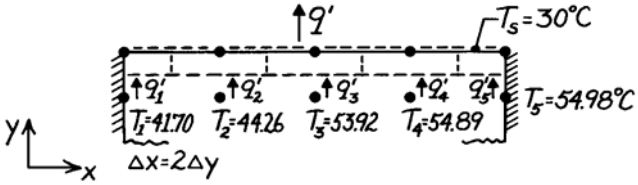
$$\Delta x = 2\Delta y = 37.50 \text{ mm}$$

$$\Delta y = 0.125L = 18.75 \text{ mm}$$

where L = 150 mm, is shown in the sketch. To evaluate the heat rate, we need the temperatures T_1, T_2, T_3, T_4, and T_5. All the nodes may be treated as interior nodes (considering symmetry for those nodes on insulated boundaries), Eq. 4.29. Use matrix notation, Eq. 4.48, [A][T] = [C], and perform the inversion.



The heat rate per unit length from the prescribed section of the duct follows from an energy balance on the nodes at the top isothermal surface.



$$q' = q'_1 + q'_2 + q'_3 + q'_4 + q'_5$$

$$q' = k(\Delta x/2)\frac{T_1 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_2 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_3 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_4 - T_s}{\Delta y} + k(\Delta x/2)\frac{T_5 - T_s}{\Delta y}$$

$$q' = k\left[(T_1 - T_s) + 2(T_2 - T_s) + 2(T_3 - T_s) + 2(T_4 - T_s) + (T_5 - T_s)\right]$$

$$q' = 1.4 \text{ W/m}\cdot\text{K}\left[(41.70 - 30) + 2(44.26 - 30) + 2(53.92 - 30) + 2(54.89 - 30) + (54.98 - 30)\right]$$

$$q' = 228 \text{ W/m}.$$

Since the section analyzed represents one-half of the region about an air duct, the heat loss per unit length for each duct is,

$$q'_{duct} = 2xq' = 456 \text{ W/m}. \quad <$$

Continued …

PROBLEM 4.67 (Cont.)

Coefficient matrix [A]

-1.0	.2	0	0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.1	-1.0	.1	0	0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	.1	-1.0	.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	.1	-1.0	.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	.2	-1.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.4	0	0	0	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	.4	0	0	0	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	.4	.1	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	.4	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	.1	0	0	0	.4	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0	0	.4	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0	0	.4	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0	0	.4	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0	0	.4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.2	-1.0

RHS Vector	*Solution Vector*
-12.0	41.70
-12	44.26
-44.0	53.92
-44.0	54.89
44	54.98
0	52.13
-80.0	56.75
0	60.24
-80.0	64.58
0	66.19
-80.0	69.64
0	70.41
-80.0	72.98
0	73.35
-80.0	75.20
0	75.37
-80.0	76.68
0	76.73
-80.0	77.66
0	77.62
-80.0	78.30
0	78.16
-80.0	78.68
0	78.45
0	78.85
-32.0	79.75
-32.0	79.94
-32.0	79.97
0	78.54
0	78.91
0	79.68
0	79.92
0	79.96