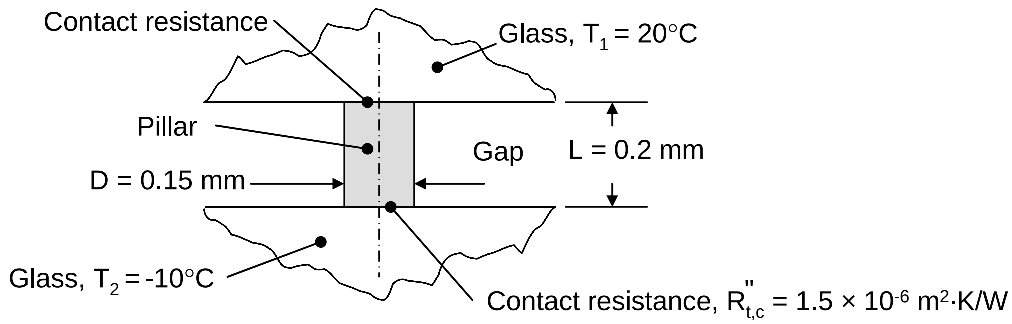
PROBLEM 4.22

KNOWN: Dimensions of stainless steel pillar and nominal glass temperatures. Contact resistance between pillar and glass.

FIND: Conduction rate through the pillar.

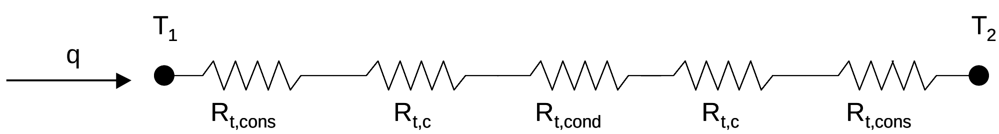
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible radiation, (4) Two-dimensional conduction, (5) Glass behaves as a semi-infinite medium.

PROPERTIES: Table A.1, AISI 302 stainless steel (300 K): k_p = 15.1 W/m·K. Table A.3, plate glass (300 K): k_g = 1.4 W/m·K.

ANALYSIS: Conduction through the pillar results in a depression of the glass temperature adjacent to the pillar. This is associated with a *constriction resistance* within each glass sheet. Therefore, the resistance network consists of two constriction resistances, two contact resistances, and a conduction resistance through the pillar as shown below.



Using the shape factor for Case 10 of Table 4.1(a) the resistances are:

$$R_{t,\text{cons}} = 1/(Sk_g) = 1/(2Dk_g) = 1/(2\times 0.15\times 10^{-3}\text{m}\times 1.4\text{ W/m}\cdot\text{K}) = 2381\text{ K/W}$$

$$R_{t,\text{c}} = R''_{t,c}/A_p = 1.5\times 10^{-6}\text{m}^2\cdot\text{K/W}/\left[\pi\times\left(0.15\times 10^{-3}\text{m}\right)^2/4\right] = 84.88\text{ K/W}$$

$$R_{t,\text{cond}} = L/k_pA_p = L/k_p\left(\pi D_p^2/4\right) = 0.2\times 10^{-3}\text{ m}/\left[15.1\text{ W/m}\cdot\text{K}\times\pi\times\left(0.15\times 10^{-3}\text{ m}\right)^2/4\right] = 749.5\text{ K/W}$$

Therefore, the total resistance is

$$R_{tot} = 2(R_{t,cons} + R_{t,c}) + R_{t,cond} = 2\times(2381\text{ K/W} + 84.88\text{ K/W}) + 749.5\text{ K/W} = 5681\text{ K/W}$$

and the conduction through an individual pillar is

$$q = (T_1 - T_2)/R_{tot} = [20 - (-10)°\text{C}]/[5681\text{ K/W}] = 5.28\times 10^{-3}\text{ W} = 5.28\text{ mW}$$ <

COMMENTS: (1) Constriction of the heat flow within the glass poses the largest resistance to heat transfer. (2) Radiation between the two glass sheets exists, and may be important in determining the overall heat transfer through the window. (3) Extremely high vacuum between the two glass sheets is required to eliminate conduction within the gap. (4) See Manz, Brunner and Wullschleger, "Triple Vacuum Glazing: Heat Transfer and Basic Design Constraints," *Solar Energy*, Vol. 80, pp. 1632-1642, 2006 for more information.