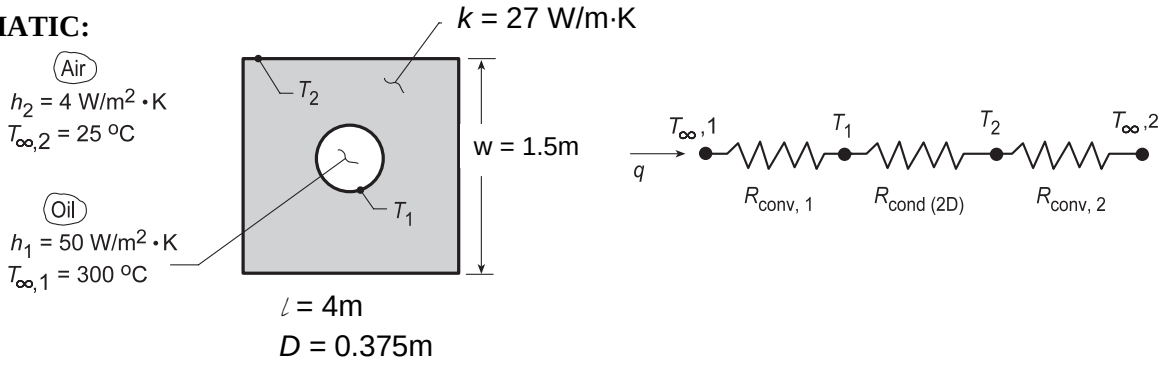
PROBLEM 4.31

KNOWN: Dimensions, shape factor, and thermal conductivity of square rod with drilled interior hole. Interior and exterior convection conditions.

FIND: Heat rate and surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) Uniform convection coefficients at inner and outer surfaces.

ANALYSIS: The heat loss can be expressed as

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{conv,1} + R_{cond(2D)} + R_{conv,2}}$$

where

$$R_{conv,1} = \left(h_1 \pi D L\right)^{-1} = \left(50\,W/m^2 \cdot K \times \pi \times 0.375\,m \times 4\,m\right)^{-1} = 0.004244\,K/W$$

using Case 6 of Table 4.1,

$$R_{cond(2D)} = \left(Sk\right)^{-1} = \left(17.176\,m \times 27\,W/m \cdot K\right)^{-1} = 0.00216\,K/W$$

$$R_{conv,2} = \left(h_2 \times 4wL\right)^{-1} = \left(4\,W/m^2 \cdot K \times 6\,m \times 4\,m\right)^{-1} = 0.0104\,K/W$$

Hence,

$$q = \frac{(300 - 25)^\circ C}{0.0168\,K/W} = 16.4\,kW \quad <$$

$$T_1 = T_{\infty,1} - qR_{conv,1} = 300^\circ C - 69.5^\circ C = 230.5^\circ C \quad <$$

$$T_2 = T_{\infty,2} + qR_{conv,2} = 25^\circ C + 170.6^\circ C = 195.6^\circ C \quad <$$

COMMENTS: The largest resistance is associated with convection at the outer surface, and the conduction resistance is much smaller than both convection resistances. Hence, $(T_2 - T_{\infty,2}) > (T_{\infty,1} - T_1) >> (T_1 - T_2)$.