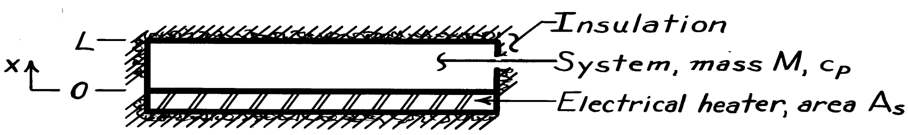
PROBLEM 2.54

KNOWN: One-dimensional system, initially at a uniform temperature T_i, is suddenly exposed to a uniform heat flux at one boundary, while the other boundary is insulated.

FIND: (a) Proper form of heat equation and boundary and initial conditions, (b) Temperature distributions for following conditions: initial condition ($t \le 0$), and several times after heater is energized; will a steady-state condition be reached; (c) Heat flux at x = 0, L/2, L as a function of time; (d) Expression for uniform temperature, T_f, reached after heater has been switched off following an elapsed time, t_e, with the heater on.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal heat generation, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation follows from Eq. 2.21. Also, the appropriate boundary and initial conditions are:

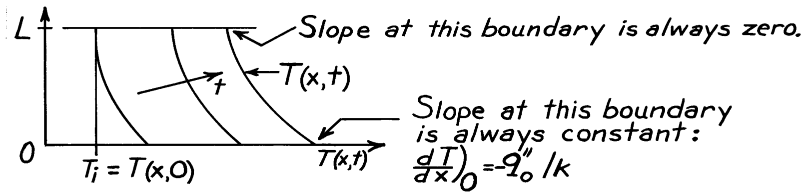
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Initial condition: $T(x,0) = T_i$ Uniform temperature

Boundary conditions: $x = 0$ $\quad q''_o = -k\partial T/\partial x)_0$

$x = L$ $\quad \partial T/\partial x)_L = 0$

(b) The temperature distributions are as follows:



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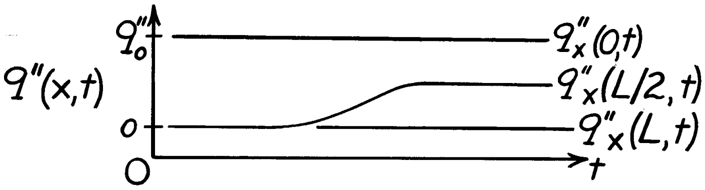
No steady-state condition will be reached since $\dot{E}_{in} = \dot{E}_{st}$ and $\dot{E}_{in}$ is constant.

(c)The heat flux as a function of time for positions x = 0, L/2 and L is as follows:



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(d) If the heater is energized until $t = t_e$ and then switched off, the system will eventually reach a uniform temperature, T_f. Perform an energy balance on the system, Eq. 1.12b, for an interval of time $\Delta t = t_e$,

$$E_{in} = E_{st} \qquad E_{in} = Q_{in} = \int_0^{t_e} q''_o A_s dt = q''_o A_s t_e \qquad E_{st} = Mc\left(T_f - T_i\right)$$

It follows that $\quad q''_o A_s t_e = Mc\left(T_f - T_i\right) \quad$ or $\quad T_f = T_i + \dfrac{q''_o A_s t_e}{Mc}.$ <