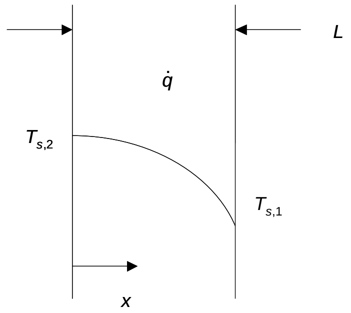
PROBLEM 3.88

KNOWN: One dimensional plane wall with uniform thermal energy generation and cold surface temperature $T_{s,1}$.

FIND: (a) Expression for the heat flux to the cold wall and hot surface temperature, (b) Comparison of the heat flux of part (a) with that associated with a plane wall with no energy generation and wall temperatures of $T_{s,1}$ and $T_{s,2}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer, (4) Uniform volumetric generation.

ANALYSIS: From Eq. (3.46) and the discussion beneath Eq. (3.49) the temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_{s,1} \tag{1}$$

(a) Equation (1) may be used to find an expression for the temperature gradient

$$\frac{\mathrm{d}T(x)}{\mathrm{d}x} = \frac{\dot{q}L^2}{2k}\left(-\frac{2x}{L^2}\right) = -\frac{\dot{q}x}{k}$$

Therefore, the heat flux at the cold ($x = L$) surface is

$$q''(x = L) = -k\left.\frac{dT}{dx}\right|_{x=L} = \dot{q}L \qquad <$$

The temperature of the hot surface may be found from Eq. (1) and is

$$T_{s,2} = T(x = 0) = \frac{\dot{q}L^2}{2k} + T_{s,1} \qquad <$$

(b) For the plane wall without energy generation, and with surface temperatures $T_{s,1}$ and $T_{s,2}$,

$$q'' = k\frac{\left(T_{s,2} - T_{s,1}\right)}{L} = k\left(\frac{\frac{\dot{q}L^2}{2k} + T_{s,1} - T_{s,1}}{L}\right) = \frac{\dot{q}L}{2} = \frac{1}{2}q''(x = L) \qquad <$$

Hence, the heat flux with uniform thermal energy generation is twice that which would be calculated for a plane wall without energy generation based upon the difference between the actual hot and cold surface temperatures.