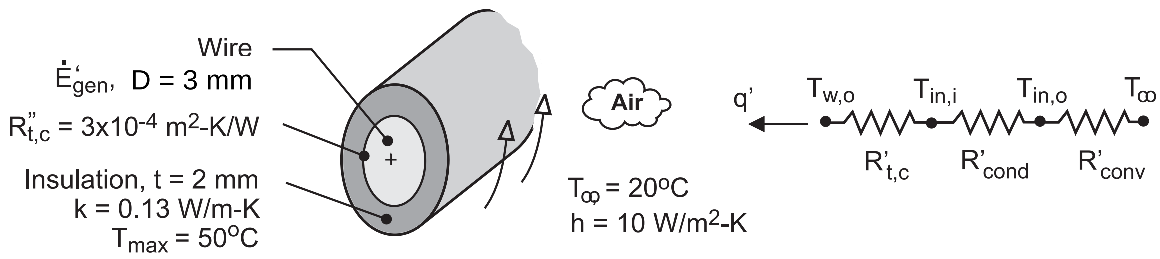
PROBLEM 3.54

KNOWN: Diameter of electrical wire. Thickness and thermal conductivity of rubberized sheath. Contact resistance between sheath and wire. Convection coefficient and ambient air temperature. Maximum allowable sheath temperature.

FIND: Maximum allowable power dissipation per unit length of wire. Critical radius of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible radiation exchange with surroundings.

ANALYSIS: The maximum insulation temperature corresponds to its inner surface and is independent of the contact resistance. From the thermal circuit, we may write

$$\dot{E}'_g = q' = \frac{T_{in,i} - T_\infty}{R'_{cond} + R'_{conv}} = \frac{T_{in,i} - T_\infty}{\left[\ell n\left(r_{in,o}/r_{in,i}\right)/2\pi k\right] + \left(1/2\pi r_{in,o} h\right)}$$

where $r_{in,i} = D/2 = 0.0015\text{m}$, $r_{in,o} = r_{in,i} + t = 0.0035\text{m}$, and $T_{in,i} = T_{max} = 50°\text{C}$ yields the maximum allowable power dissipation. Hence,

$$\dot{E}'_{g,max} = \frac{\left(50-20\right)°\text{C}}{\dfrac{\ell n\, 2.333}{2\pi \times 0.13\,\text{W/m}\cdot\text{K}} + \dfrac{1}{2\pi\left(0.0035\text{m}\right)10\,\text{W/m}^2\cdot\text{K}}} = \frac{30°\text{C}}{\left(1..37 + 4.54\right)\text{m}\cdot\text{K/W}} = 5.37\,\text{W/m} \qquad <$$

The critical insulation radius is also unaffected by the contact resistance and is given by

$$r_{cr} = \frac{k}{h} = \frac{0.13\,\text{W/m}\cdot\text{K}}{10\,\text{W/m}^2\cdot\text{K}} = 0.013\text{m} = 13\,\text{mm} \qquad <$$

Hence, $r_{in,o} < r_{cr}$ and $E'_{g,max}$ could be increased by increasing $r_{in,o}$ up to a value of 13 mm (t = 12 mm).

COMMENTS: The contact resistance affects the temperature of the wire, and for $q' = \dot{E}'_{g,max} = 5.37\,\text{W/m}$, the outer surface temperature of the wire is $T_{w,o} = T_{in,i} + q' R'_{t,c} = 50°\text{C} + \left(5.37\,\text{W/m}\right)\left(3\times10^{-4}\text{m}^2\cdot\text{K/W}\right)/\pi\left(0.003\text{m}\right) = 50.2°\text{C}$. Hence, the temperature change across the contact resistance is negligible.