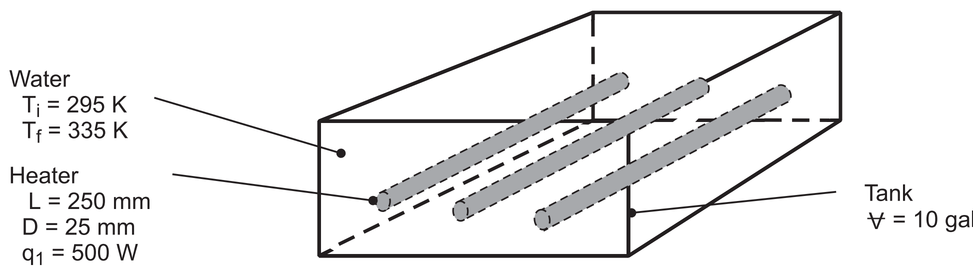
PROBLEM 1.51

KNOWN: Initial temperature of water and tank volume. Power dissipation, emissivity, length and diameter of submerged heaters. Expressions for convection coefficient associated with natural convection in water and air.

FIND: (a) Time to raise temperature of water to prescribed value, (b) Heater temperature shortly after activation and at conclusion of process, (c) Heater temperature if activated in air.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss from tank to surroundings, (2) Water is *well-mixed* (at a uniform, but time varying temperature) during heating, (3) Negligible changes in thermal energy storage for heaters, (4) Constant properties, (5) Surroundings afforded by tank wall are large relative to heaters.

ANALYSIS: (a) Application of conservation of energy to a closed system (the water) at an instant, Equation (1.12c), with

$$\dot{E}_{st} = dU_t/dt, \quad \dot{E}_{in} = 3q_1, \quad \dot{E}_{out} = 0, \text{ and } \dot{E}_g = 0,$$

$$\text{yields } \frac{dU_t}{dt} = 3q_1 \text{ and } \rho\forall c\frac{dT}{dt} = 3q_1$$

Hence, $\int_0^t dt = \left(\rho\forall c/3q_1\right)\int_{T_i}^{T_f} dT$

$$t = \frac{990 \text{ kg/m}^3 \times 10\text{gal}\left(3.79\times10^{-3}\text{m}^3/\text{gal}\right)4180\text{J/kg}\cdot\text{K}}{3\times500 \text{ W}}\left(335-295\right)\text{K} = 4180 \text{ s}$$ <

(b) From Equation (1.3a), the heat rate by convection from each heater is

$$q_1 = Aq_1'' = Ah\left(T_s - T\right) = \left(\pi DL\right)370\left(T_s - T\right)^{4/3}$$

Hence, $T_s = T + \left(\dfrac{q_1}{370\pi DL}\right)^{3/4} = T + \left(\dfrac{500 \text{ W}}{370 \text{ W/m}^2\cdot\text{K}^{4/3}\times\pi\times0.025 \text{ m}\times0.250 \text{ m}}\right)^{3/4} = \left(T+24\right)\text{K}$

With water temperatures of $T_i \approx 295$ K and $T_f = 335$ K shortly after the start of heating and at the end of heating, respectively, $T_{s,i} = 319$ K and $T_{s,f} = 359$ K <

(c) From Equation (1.10), the heat rate in air is

$$q_1 = \pi DL\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

Substituting the prescribed values of q_1, D, L, $T_\infty = T_{sur}$ and ε, an iterative solution yields

$$T_s = 830 \text{ K}$$ <

COMMENTS: In part (c) it is presumed that the heater can be operated at $T_s = 830$ K without experiencing burnout. The much larger value of T_s for air is due to the smaller convection coefficient. However, with q_{conv} and q_{rad} equal to 59 W and 441 W, respectively, a significant portion of the heat dissipation is effected by radiation.