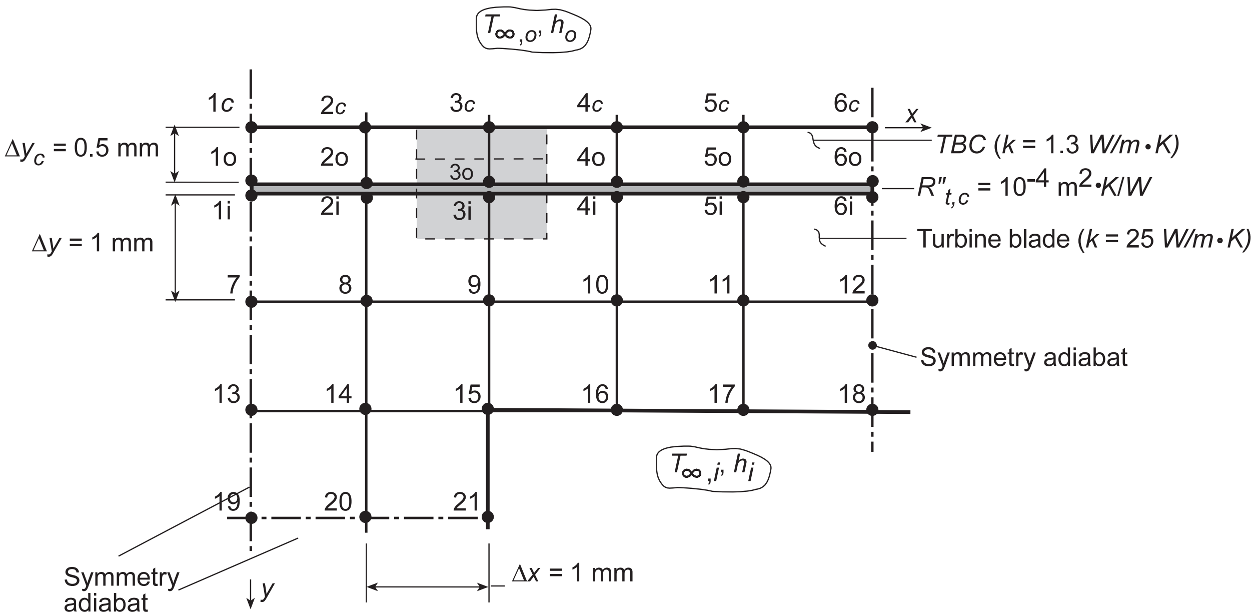
PROBLEM 4.68

KNOWN: Dimensions and operating conditions for a gas turbine blade with embedded channels.

FIND: Effect of applying a zirconia, thermal barrier coating.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: Preserving the nodal network of Example 4.3 and adding surface nodes for the TBC, finite-difference equations previously developed for nodes 7 through 21 are still appropriate, while new equations must be developed for nodes 1c-6c, 1o-6o, and 1i-6i. Considering node 3c as an example, an energy balance yields

$$h_o \Delta x\left(T_{\infty,o} - T_{3c}\right) + \frac{k_c\left(\Delta y_c/2\right)}{\Delta x}\left(T_{2c} - T_{3c}\right) + \frac{k_c\left(\Delta y_c/2\right)}{\Delta x}\left(T_{4c} - T_{3c}\right) + \frac{k_c \Delta x}{\Delta y_c}\left(T_{3o} - T_{3c}\right) = 0$$

or, with Δx = 1 mm and Δy_c = 0.5 mm,

$$0.25\left(T_{2c} + T_{4c}\right) + 2T_{3o} - \left(2.5 + \frac{h_o \Delta x}{k_c}\right)T_{3c} = -\frac{h_o \Delta x}{k_c}T_{\infty,o}$$

Similar expressions may be obtained for the other 5 nodal points on the outer surface of the TBC.

Applying an energy balance to node 3o at the inner surface of the TBC, we obtain

$$\frac{k_c \Delta x}{\Delta y_c}\left(T_{3c} - T_{3o}\right) + \frac{k_c\left(\Delta y_c/2\right)}{\Delta x}\left(T_{2o} - T_{3o}\right) + \frac{k_c\left(\Delta y_c/2\right)}{\Delta x}\left(T_{4o} - T_{3o}\right) + \frac{\Delta x}{R''_{t,c}}\left(T_{3i} - T_{3o}\right) = 0$$

or,

$$2T_{3c} + 0.25\left(T_{2o} + T_{4o}\right) + \frac{\Delta x}{k_c R''_{t,c}}T_{3i} - \left(2.5 + \frac{\Delta x}{k_c R''_{t,c}}\right)T_{3o} = 0$$

Similar expressions may be obtained for the remaining nodal points on the inner surface of the TBC (outer region of the contact resistance).

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Applying an energy balance to node 3i at the outer surface of the turbine blade, we obtain

$$\frac{\Delta x}{R''_{t,c}}\left(T_{3o}-T_{3i}\right)+\frac{k\left(\Delta y/2\right)}{\Delta x}\left(T_{2i}-T_{3i}\right)+\frac{k\left(\Delta y/2\right)}{\Delta x}\left(T_{4i}-T_{3i}\right)+\frac{k\Delta x}{\Delta y}\left(T_9-T_{3i}\right)=0$$

or,

$$\frac{\Delta x}{kR''_{t,c}}T_{3o}+0.5\left(T_{2,i}+T_{4,i}\right)+T_9-\left(2+\frac{\Delta x}{kR''_{t,c}}\right)T_{3i}=0$$

Similar expressions may be obtained for the remaining nodal points on the inner region of the contact resistance.

The 33 finite-difference equations were entered into the workspace of IHT from the keyboard, and for h_o = 1000 W/m^2·K, $T_{\infty,o}$ = 1700 K, h_i = 200 W/m^2·K and $T_{\infty,i}$ = 400 K, the following temperature field was obtained, where coordinate (x,y) locations are in mm and temperatures are in °C.

y\x	0	1	2	3	4	5
0	1536	1535	1534	1533	1533	1532
0.5	1473	1472	1471	1469	1468	1468
0.5	1456	1456	1454	1452	1451	1451
1.5	1450	1450	1447	1446	1444	1444
2.5	1446	1445	1441	1438	1437	1436
3.5	1445	1443	1438	0	0	0

Note the significant reduction in the turbine blade temperature, as, for example, from a surface temperature of T_1 = 1526 K without the TBC to T_{1i} = 1456 K with the coating. Hence, the coating is serving its intended purpose.

COMMENTS: (1) Significant additional benefits may still be realized by increasing h_i. (2) The foregoing solution may be used to determine the temperature field without the TBC by setting $k_c \rightarrow \infty$ and $R''_{t,c} \rightarrow 0$.