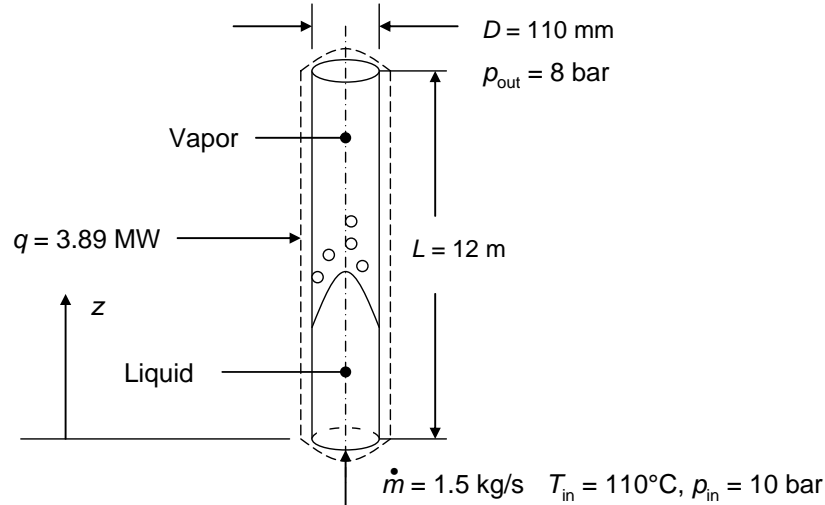# PROBLEM 1.37

KNOWN: Flow of water in a vertical tube. Tube dimensions. Mass flow rate. Inlet pressure and temperature. Heat rate. Outlet pressure.

FIND: (a) Outlet temperature, (b) change in combined thermal and flow work, (c) change in mechanical energy, and (d) change in total energy of the water from the inlet to the outlet of the tube.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible change in mechanical energy. (3) Uniform velocity distributions at the tube inlet and outlet.

PROPERTIES: Table A.6 water ($T = 110°C$): $\rho = 950$ kg/m^3, ($T = (179.9°C + 110 °C)/2 = 145°C$): $c_p = 4300$ J/kg·K, $\rho = 919$ kg/m^3. Other properties are taken from Moran, M.J. and Shapiro, H.N., *Fundamentals of Engineering Thermodynamics*, 6th Edition, John Wiley & Sons, Hoboken, 2008 including ($p_{sat} = 10$ bar): $T_{sat} = 179.9°C$, $i_f = 762.81$ kJ/kg; ($p = 8$ bar, $i = 3056$ kJ/kg): $T = 300°C$, $\upsilon = 0.3335$ m^3/kg.

ANALYSIS: (a) The steady-flow energy equation, in the absence of work (other than flow work), is

$$\dot{m}\left(u + pv + \tfrac{1}{2}V^2 + gz\right)_{in} - \dot{m}\left(u + pv + \tfrac{1}{2}V^2 + gz\right)_{out} + q = 0$$
$$\dot{m}\left(i + \tfrac{1}{2}V^2 + gz\right)_{in} - \dot{m}\left(i + \tfrac{1}{2}V^2 + gz\right)_{out} + q = 0 \tag{1}$$

Neglecting the change in mechanical energy yields

$$\dot{m}(i_{in} - i_{out}) + q = 0$$

The inlet enthalpy is

$$i_{in} = i_{f,in} + c_p(T_{in} - T_{sat}) = 762.81 \text{ kJ/kg} + 4.3 \text{ kJ/kg} \cdot \text{K} \times (110 - 179.9)°C = 462.2 \text{ kJ/kg}$$

Thus the outlet enthalpy is

$$i_{out} = i_{in} + q/\dot{m} = 462.2 \text{ kJ/kg} + 3890 \text{ kW} / 1.5 \text{ kg/s} = 3056 \text{ kJ/kg}$$

and the outlet temperature can be found from thermodynamic tables at $p = 8$ bars, $i = 3056$ kJ/kg, for which

$$T_{\text{out}} = 300°C$$

(b) The change in the combined thermal and flow work energy from inlet to outlet:

$$E_{i,\text{out}} - E_{i,\text{in}} = \dot{m}(i_{\text{out}} - i_{\text{in}}) = q = 3.89 \text{ MW} \qquad <$$

(c) The change in mechanical energy can now be calculated. First, the outlet specific volume can be found from thermodynamic tables at $T_{\text{out}} = 300°C$, $p_{\text{out}} = 8$ bars, $\upsilon = 0.3335$ m^3/kg. Next, the conservation of mass principle yields

$$V_{\text{in}} = \frac{4\dot{m}}{\rho\pi D^2} = \frac{4 \times 1.5 \text{ kg/s}}{950 \text{ kg/m}^3 \times \pi \times (0.110 \text{ m})^2} = 0.166 \text{ m/s} \; ; \; V_{\text{out}} = \frac{\upsilon 4\dot{m}}{\pi D^2} = \frac{0.3335 \text{ m}^3/\text{kg} \times 4 \times 1.5 \text{ kg/s}}{\pi \times (0.110 \text{ m})^2} = 52.6 \text{ m/s}$$

The change in mechanical energy from inlet to outlet is:

$$E_{m,\text{out}} - E_{m,\text{in}} = \dot{m}(\tfrac{1}{2}V^2 + gz)_{\text{out}} - \dot{m}(\tfrac{1}{2}V^2 + gz)_{\text{in}}$$

$$= 1.5 \text{ kg/s} \times \left(\tfrac{1}{2}\left[(52.6 \text{ m/s})^2 - (0.166 \text{ m/s})^2\right] + 9.8 \text{ m/s}^2 \times 12 \text{ m}\right) = 2.25 \text{ kW} \qquad <$$

(d) The change in the total energy is the summation of the thermal, flow work, and mechanical energy change or

$$E_{\text{in}} - E_{\text{out}} = 3.89 \text{ MW} + 2.25 \text{ kW} = 3.89 \text{ MW} \qquad <$$

COMMENTS: (1) The change in mechanical energy, consisting of kinetic and potential energy components, is negligible compared to the change in thermal and flow work energy. (2) The average heat flux at the tube surface is $q'' = q/(\pi DL) = 3.89$ MW $/(\pi \times 0.110 \text{ m} \times 12 \text{ m}) = 0.94$ MW/m^2, which is very large. (3) The change in the velocity of the water is inversely proportional to the change in the density. As such, the outlet velocity is very large, and large pressure drops will occur in the vapor region of the tube relative to the liquid region of the tube.