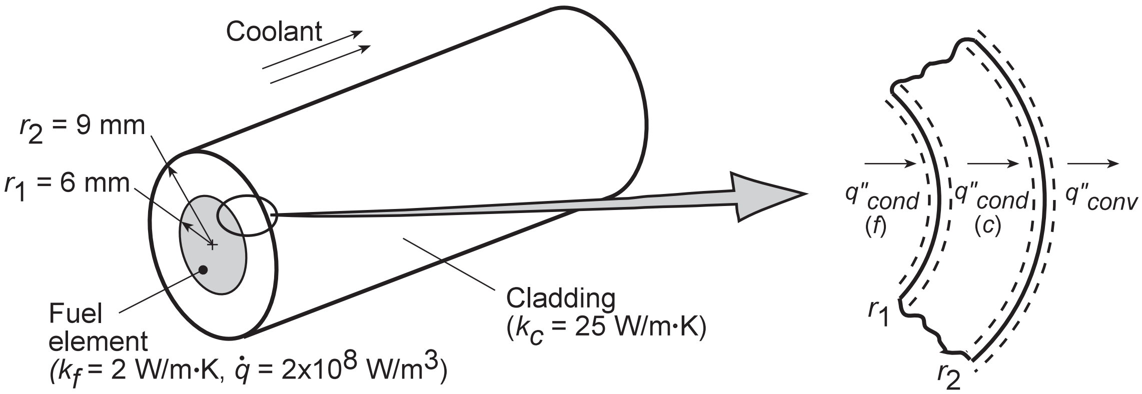
PROBLEM 3.98

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding, (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.54 and 3.28, the heat equations for the fuel (f) and cladding (c) are

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_f}{dr}\right)=-\frac{\dot{q}}{k_f} \qquad \left(0 \le r \le r_1\right) \qquad\qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0 \qquad \left(r_1 \le r \le r_2\right)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr}=-\frac{\dot{q}r}{2k_f}+\frac{C_1}{k_f r} \qquad T_f=-\frac{\dot{q}r^2}{4k_f}+\frac{C_1}{k_f}\ln r+C_2 \tag{1,2}$$

$$\frac{dT_c}{dr}=\frac{C_3}{k_c r} \qquad T_c=\frac{C_3}{k_c}\ln r+C_4 \tag{3,4}$$

The corresponding boundary conditions are:

$$dT_f/dr\big)_{r=0}=0 \qquad\qquad T_f\left(r_1\right)=T_c\left(r_1\right) \tag{5,6}$$

$$-k_f\left.\frac{dT_f}{dr}\right)_{r=r_1}=-k_c\left.\frac{dT_c}{dr}\right)_{r=r_1} \qquad -k_c\left.\frac{dT_c}{dr}\right)_{r=r_2}=h\left[T_c\left(r_2\right)-T_\infty\right] \tag{7,8}$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f=-\frac{\dot{q}r^2}{4k_f}+C_2 \tag{9}$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f}+C_2=\frac{C_3\ln r_1}{k_c}+C_4 \tag{10}$$

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Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \tag{11}$$

Finally, from Eq. (8), $-\dfrac{C_3}{r_2} = h\left[\dfrac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \tag{12}$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \tag{13}$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \tag{14}$$ $<$

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty\,. \tag{15}$$ $<$

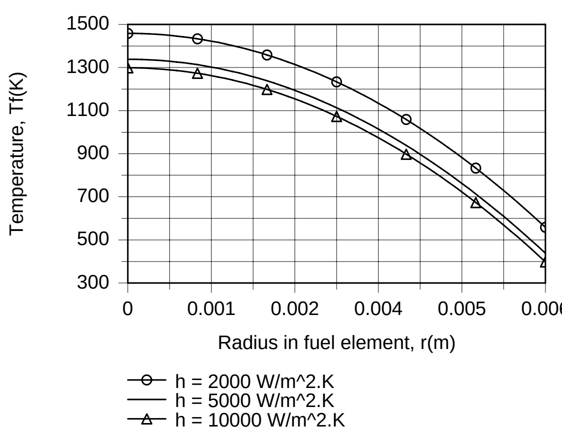
(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for h = 2000 W/m^2·K is

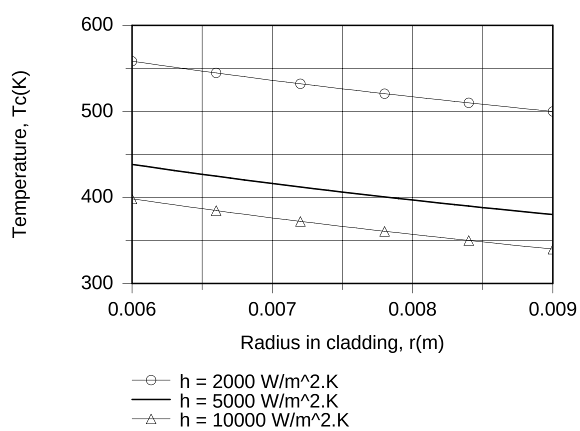
$$T_f(0) = \frac{2\times10^8\ \mathrm{W/m^3}\times(0.006\,\mathrm{m})^2}{4\times 2\ \mathrm{W/m\cdot K}} + \frac{2\times10^8\ \mathrm{W/m^3}\times(0.006\,\mathrm{m})^2}{2\times 25\ \mathrm{W/m\cdot K}}\ln\frac{0.009\,\mathrm{m}}{0.006\,\mathrm{m}}$$

$$+\frac{2\times10^8\ \mathrm{W/m^3}\,(0.006\,\mathrm{m})^2}{2\times(0.009\,\mathrm{m})\,2000\ \mathrm{W/m^2\cdot K}} + 300\,\mathrm{K}$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\,\mathrm{K} = 1458\,\mathrm{K}\,.$$ $<$

(c) Temperature distributions for the prescribed values of h are as follows:





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PROBLEM 3.98 (Cont.)

Clearly, the ability to control the maximum fuel temperature by increasing h is limited, and even for $h \rightarrow \infty$, $T_f(0)$ exceeds 1000 K. The overall temperature drop, $T_f(0) - T_\infty$, is influenced principally by the low thermal conductivity of the fuel material.

COMMENTS: For the prescribed conditions, Eq. (14) yields, $T_f(0) - T_f(r_1) = \dot{q}r_1^2/4k_f = (2\times10^8$ $W/m^3)(0.006\ m)^3/8\ W/m\cdot K = 900$ K, in which case, with no cladding and $h \rightarrow \infty$, $T_f(0) = 1200$ K. To reduce $T_f(0)$ below 1000 K for the prescribed material, it is necessary to reduce $\dot{q}$.