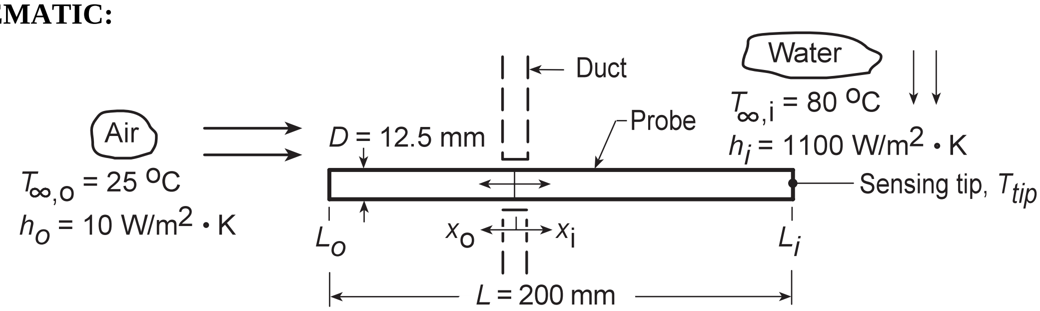
PROBLEM 3.121

KNOWN: Temperature sensing probe of thermal conductivity k, length L and diameter D is mounted on a duct wall; portion of probe L_i is exposed to water stream at $T_{\infty,i}$ while other end is exposed to ambient air at $T_{\infty,o}$; convection coefficients h_i and h_o are prescribed.

FIND: (a) Expression for the measurement error, $\Delta T_{err} = T_{tip} - T_{\infty,i}$, (b) For prescribed $T_{\infty,i}$ and $T_{\infty,o}$, calculate ΔT_{err} for immersion to total length ratios of 0.225, 0.425, and 0.625, (c) Compute and plot the effects of probe thermal conductivity and water velocity (h_i) on ΔT_{err}.

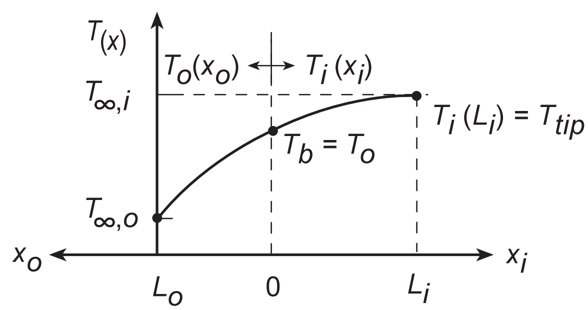
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in probe, (3) Probe is thermally isolated from the duct, (4) Convection coefficients are uniform over their respective regions.

PROPERTIES: Probe material (given): k = 177 W/m·K.

ANALYSIS: (a) To derive an expression for $\Delta T_{err} = T_{tip} - T_{\infty,i}$, we need to determine the temperature distribution in the immersed length of the probe $T_i(x)$. Consider the probe to consist of two regions: $0 \le x_i \le L_i$, the immersed portion, and $0 \le x_o \le (L - L_i)$, the ambient-air portion where the origin corresponds to the location of the duct wall. Use the results for the temperature distribution and fin heat rate of Case A, Table 3.4:



Temperature distribution in region i:

$$\frac{\theta_i}{\theta_{b,i}} = \frac{T_i(x_i) - T_{\infty,i}}{T_o - T_{\infty,i}} = \frac{\cosh\left(m_i\left(L_i - x_i\right)\right) + \left(h_i/m_i k\right)\sinh\left(L_i - x_i\right)}{\cosh\left(m_i L_i\right) + \left(h_i/m_i k\right)\sinh\left(m_i L_i\right)} \quad (1)$$

and the tip temperature, $T_{tip} = T_i(L_i)$ at $x_i = L_i$, is

$$\frac{T_{tip} - T_{\infty,i}}{T_o - T_{\infty,i}} = A = \frac{\cosh(0) + \left(h_i/m_i k\right)\sinh(0)}{\cosh\left(m_i L_i\right) + \left(h_i/m_i k\right)\sinh\left(m_i L_i\right)} \quad (2)$$

and hence

$$\Delta T_{err} = T_{tip} - T_{\infty,i} = A\left(T_o - T_{\infty,i}\right) \quad (3)$$ <

where T_o is the temperature at $x_i = x_o = 0$ which at present is unknown, but can be found by setting the fin heat rates equal, that is,

$$q_{f,o} = -q_{f,i} \quad (4)$$

Continued...

PROBLEM 3.121 (Cont.)

$$\left(h_o P k A_c\right)^{1/2} \theta_{b,o} \cdot B = -\left(h_i P k A_c\right)^{1/2} \theta_{b,i} \cdot C$$

Solving for T_o, find

$$\frac{\theta_{b,o}}{\theta_{b,i}} = \frac{T_o - T_{\infty,o}}{T_o - T_{\infty,i}} = -\frac{\left(h_i P k A_c\right)^{1/2} C}{\left(h_o P k A_c\right)^{1/2} B} = -\left(\frac{h_i}{h_o}\right)^{1/2} \frac{C}{B}$$

$$T_o = \left[T_{\infty,o} + \left(\frac{h_i}{h_o}\right)^{1/2} \frac{C}{B} T_{\infty,i}\right] \Bigg/ \left[1 + \left(\frac{h_i}{h_o}\right)^{1/2} \frac{C}{B}\right] \qquad (5)$$

where the constants B and C are,

$$B = \frac{\sinh\left(m_o L_o\right) + \left(h_o / m_o k\right)\cosh\left(m_o L_o\right)}{\cosh\left(m_o L_o\right) + \left(h_o / m_o k\right)\sinh\left(m_o L_o\right)} \qquad (6)$$

$$C = \frac{\sinh\left(m_i L_i\right) + \left(h_i / m_i k\right)\cosh\left(m_i L_i\right)}{\cosh\left(m_i L_i\right) + \left(h_i / m_i k\right)\sinh\left(m_i L_i\right)} \qquad (7)$$

(b) To calculate the immersion error for prescribed immersion lengths, L_i/L = 0.225, 0.425 and 0.625, we use Eq. (3) as well as Eqs. (2, 6, 7 and 5) for A, B, C, and T_o, respectively. Results of these calculations are summarized below.

L_i/L	L_o (mm)	L_i (mm)	A	B	C	T_o (°C)	ΔT_{err} (°C)	
0.225	155	45	0.2328	0.5865	0.9731	76.7	-0.76	<
0.425	115	85	0.0396	0.4639	0.992	77.5	-0.10	<
0.625	75	125	0.0067	0.3205	0.9999	78.2	-0.01	<

(c) The probe behaves as a fin having ends exposed to the cool ambient air and the hot ambient water whose temperature is to be measured. As shown above, the probe is more accurate when more of its length is exposed to the water. If the thermal conductivity is *decreased*, heat transfer along the probe length is likewise decreased, the tip temperature will be closer to the water temperature. If the velocity of the water *decreases*, the convection coefficient will decrease, and the difference between the tip and water temperatures will increase.