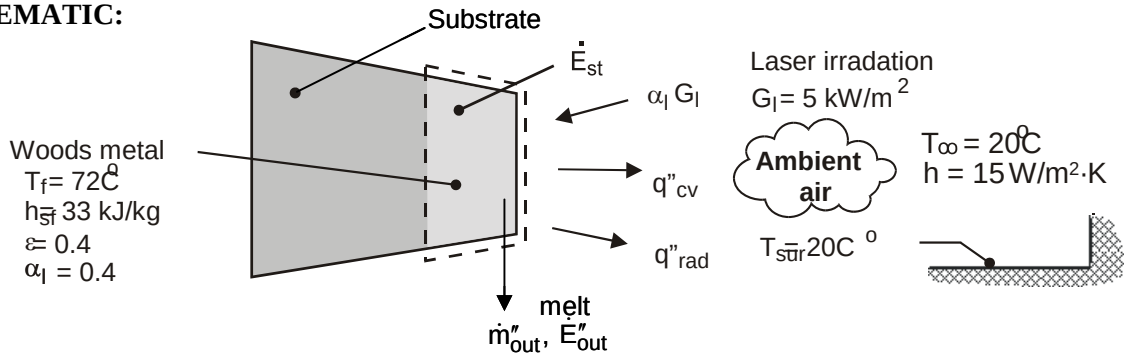
PROBLEM 1.66

KNOWN: Vertical slab of Woods metal initially at its fusion temperature, T_f, joined to a substrate. Exposed surface is irradiated with laser source, $G_\ell\,(W/m^2)$.

FIND: Instantaneous rate of melting per unit area, $\dot{m}''_m$ (kg/s·m^2), and the material removed in a period of 2 s, (a) Neglecting heat transfer from the irradiated surface by convection and radiation exchange, and (b) Allowing for convection and radiation exchange.

SCHEMATIC:



ASSUMPTIONS: (1) Woods metal slab is isothermal at the fusion temperature, T_f, and (2) The melt runs off the irradiated surface.

ANALYSIS: (a) The instantaneous rate of melting per unit area may be determined by applying a mass balance and an energy balance (Equation 1.12c) on the metal slab at an instant of time neglecting convection and radiation exchange from the irradiated surface.

$$\dot{m}''_{st} = \dot{m}''_{in} - \dot{m}''_{out} \qquad \dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \tag{1a,b}$$

With h_f as the enthalpy of the melt and h_s as the enthalpy of the solid, we have

$$\dot{E}''_{st} = \dot{m}''_{st} h_s \qquad \dot{E}''_{out} = \dot{m}''_{out} h_f \tag{2a,b}$$

Combining Equations (1a) and (2a,b), Equation (1b) becomes (with $h_{sf} = h_f - h_s$)

$$\dot{m}''_{out} h_{sf} = \dot{E}''_{in} = \alpha_l G_l$$

Thus the rate of melting is

$$\dot{m}''_{out} = \alpha_l G_l / h_{sf} = 0.4 \times 5000\,W/m^2 \,/ 33{,}000\,J/kg = 60.6 \times 10^{-3}\,kg/s \times m^2 \quad <$$

The material removed in a 2s period per unit area is

$$M''_{2s} = \dot{m}''_{out} \times \Delta t = 121\,g/m^2 \quad <$$

(b) The energy balance considering convection and radiation exchange with the surroundings yields

$$\dot{m}''_{out} h_{sf} = \alpha_l G_l - q''_{cv} - q''_{rad}$$

$$q''_{cv} = h\left(T_f - T_\infty\right) = 15\,W/m^2 \cdot K\left(72 - 20\right)K = 780\ W/m^2$$

$$q''_{rad} = \varepsilon\sigma\left(T_f^4 - T_\infty^4\right) = 0.4 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K\left(\left[72 + 273\right]^4 - \left[20 + 273\right]^4\right)K^4 = 154\,W/m^2$$

$$\dot{m}''_{out} = 32.3 \times 10^{-3}\,kg/s \cdot m^2 \qquad M_{2s} = 64\,g/m^2 \quad <$$

COMMENTS: (1) The effects of heat transfer by convection and radiation reduce the estimate for the material removal rate by a factor of two. The heat transfer by convection is nearly 5 times larger than by radiation exchange.

Continued…

PROBLEM 1.66 (Cont.)

(2) Suppose the work piece were horizontal, rather than vertical, and the melt puddled on the surface rather than ran off. How would this affect the analysis?

(3) Lasers are common heating sources for metals processing, including the present application of melting (heat transfer with phase change), as well as for heating work pieces during milling and turning (laser-assisted machining).