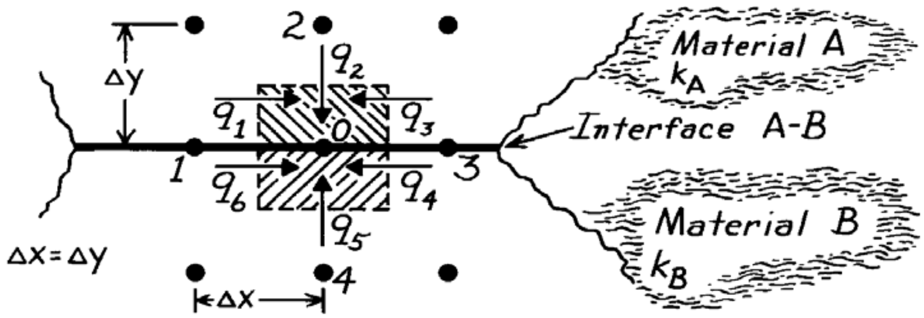
PROBLEM 4.47

KNOWN: Nodal point on boundary between two materials.

FIND: Finite-difference equation for steady-state conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) No internal heat generation, (5) Negligible thermal contact resistance at interface.

ANALYSIS: The control volume is defined about nodal point 0 as shown above. The conservation of energy requirement has the form

$$\sum_{i=1}^{6} q_i = q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 0$$

since all heat rates are shown as *into* the CV. Each heat rate can be written using Fourier's law,

$$k_A \cdot \frac{\Delta y}{2} \cdot \frac{T_1 - T_0}{\Delta x} + k_A \cdot \Delta x \cdot \frac{T_2 - T_0}{\Delta y} + k_A \cdot \frac{\Delta y}{2} \cdot \frac{T_3 - T_0}{\Delta x}$$
$$+ k_B \cdot \frac{\Delta y}{2} \cdot \frac{T_3 - T_0}{\Delta x} + k_B \cdot \Delta x \cdot \frac{T_4 - T_0}{\Delta y} + k_B \cdot \frac{\Delta y}{2} \cdot \frac{T_1 - T_0}{\Delta x} = 0.$$

Recognizing that $\Delta x = \Delta y$ and regrouping gives the relation,

$$-T_0 + \frac{1}{4} T_1 + \frac{k_A}{2(k_A + k_B)} T_2 + \frac{1}{4} T_3 + \frac{k_B}{2(k_A + k_B)} T_4 = 0. \qquad <$$

COMMENTS: Note that when $k_A = k_B$, the result agrees with Equation 4.29 which is appropriate for an interior node in a medium of fixed thermal conductivity.