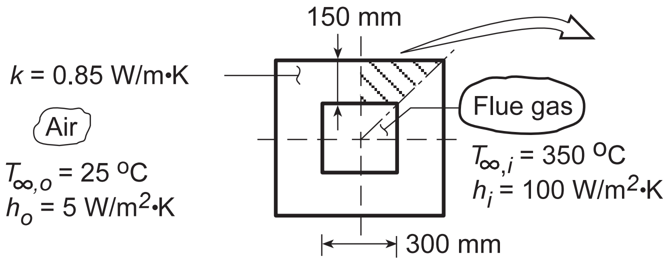
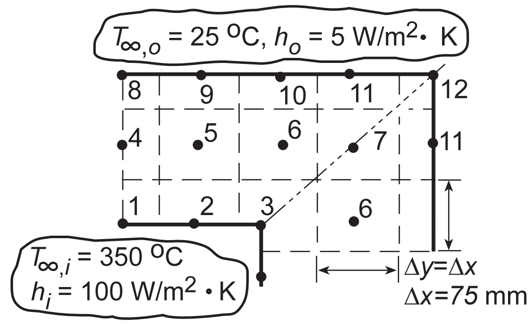
PROBLEM 4.92

KNOWN: Flue of square cross section with prescribed geometry, thermal conductivity and inner and outer surface convective conditions.

FIND: (a) Heat loss per unit length, q', by convection to the air, (b) Effect of grid spacing and convection coefficients on temperature field; show isotherms.

SCHEMATIC:





Schematic (a)

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Taking advantage of symmetry, the nodal network for a 75 mm grid spacing is shown in schematic (a). To obtain the heat rate, we need first to determine the temperatures T_i. Recognize that there are four types of nodes: interior (4-7), plane surface with convection (1, 2, 8-11), internal corner with convection (3), and external corner with convection (12). Using the appropriate relations from Table 4.2, the finite-difference equations are

Node		Equation
1	$\left(2T_4 + T_2 + T_2\right) + \frac{2h_i\Delta x}{k} T_{\infty,i} - 2\left(\frac{h_i\Delta x}{k} + 2\right) T_1 = 0$	4.42
2	$\left(2T_5 + T_3 + T_1\right) + \frac{2h_i\Delta x}{k} T_{\infty,i} - 2\left(\frac{h_i\Delta x}{k} + 2\right) T_2 = 0$	4.42
3	$2\left(T_6 + T_6\right) + \left(T_2 + T_2\right) + \frac{2h_i\Delta x}{k} T_{\infty,i} - 2\left(3 + \frac{h_i\Delta x}{k}\right) T_3 = 0$	4.41
4	$\left(T_8 + T_5 + T_1 + T_5\right) - 4T_4 = 0$	4.29
5	$\left(T_9 + T_6 + T_2 + T_4\right) - 4T_5 = 0$	4.29
6	$\left(T_{10} + T_7 + T_3 + T_5\right) - 4T_6 = 0$	4.29
7	$\left(T_{11} + T_{11} + T_6 + T_6\right) - 4T_7 = 0$	4.29
8	$\left(2T_4 + T_9 + T_9\right) + \frac{2h_o\Delta x}{k} T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right) T_8 = 0$	4.42
9	$\left(2T_5 + T_{10} + T_8\right) + \frac{2h_o\Delta x}{k} T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right) T_9 = 0$	4.42
10	$\left(2T_6 + T_{11} + T_9\right) + \frac{2h_o\Delta x}{k} T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right) T_{10} = 0$	4.42
11	$\left(2T_7 + T_{12} + T_{10}\right) + \frac{2h_o\Delta x}{k} T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right) T_{11} = 0$	4.42
12	$\left(T_{11} + T_{11}\right) + \frac{2h_o\Delta x}{k} T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 1\right) T_{12} = 0$	4.43

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PROBLEM 4.92 (Cont.)

Substituting $T_{\infty,o}$ = 350°C, h_o = 100 W/m^2·K, $T_{\infty,i}$ = 25°C, h_i = 5 W/m^2·K, Δx = 0.075 m, and k = 0.85 W/m·K and solving the preceding equations simultaneously using, for example, IHT, yields

T_1 = 340.4°C, T_2 = 339.5°C, T_3 = 329.1°C, T_4 = 256.5°C, T_5 = 251.4°C, T_6 = 231.5°C, T_7 = 182.3°C, T_8 = 182.6°C, T_9 = 178.3°C, T_{10} = 163.1°C, T_{11} = 133.1°C, T_{12} = 100.0°C. <

The heat loss to the outside air for the upper surface (Nodes 8 through 12) is of the form

$$q' = h_o \Delta x \left[\frac{1}{2}\left(T_8 - T_{\infty,o}\right) + \left(T_9 - T_{\infty,o}\right) + \left(T_{10} - T_{\infty,o}\right) + \left(T_{11} - T_{\infty,o}\right) + \frac{1}{2}\left(T_{12} - T_{\infty,o}\right) \right]$$

$$q' = 5\,\mathrm{W/m^2 \cdot K} \times 0.075\,\mathrm{m} \left[\frac{1}{2}(182.6 - 25) + (178.3 - 25) + (163.1 - 25) + (133.1 - 25) + \frac{1}{2}(100.0 - 25) \right] {}^\circ\mathrm{C} = 193.4\,\mathrm{W/m}$$

Hence, for the entire flue cross-section, considering symmetry,

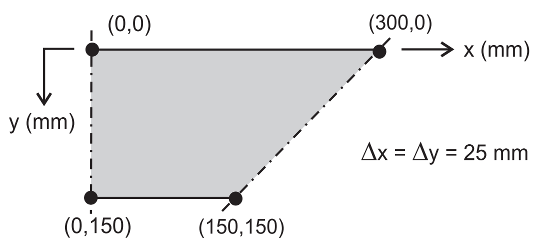
$$q'_{tot} = 8 \times q' = 8 \times 193.4\,\mathrm{W/m} = 1.55\,\mathrm{kW/m}$$ <

The convection heat rate at the inner surface is

$$q'_{tot} = 8 \times h_i \Delta x \left[\frac{1}{2}\left(T_{\infty,i} - T_1\right) + \left(T_{\infty,i} - T_2\right) + \frac{1}{2}\left(T_{\infty,i} - T_3\right) \right] = 1.55\,\mathrm{kW/m}$$

which is the same as the heat loss from the upper surface, as it must be. <

(b) Using the *Finite-Difference Equations* option from the *Tools* portion of the IHT menu, the following two-dimensional temperature field was computed for the grid shown in the schematic below, where x and y are in mm and the temperatures are in °C.



y\x	0	25	50	75	100	125	150	175	200	225	250	275	300
0	180.7	180.2	178.4	175.4	171.1	165.3	158.1	149.6	140.1	129.9	119.4	108.7	98.0
25	204.2	203.6	201.6	198.2	193.3	186.7	178.3	168.4	157.4	145.6	133.4	121.0	
50	228.9	228.3	226.2	222.6	217.2	209.7	200.1	188.4	175.4	161.6	147.5		
75	255.0	254.4	252.4	248.7	243.1	235.0	223.9	209.8	194.1	177.8			
100	282.4	281.8	280.1	276.9	271.6	263.3	250.5	232.8	213.5				
125	310.9	310.5	309.3	307.1	303.2	296.0	282.2	257.5					
150	340.0	340.0	339.6	339.1	337.9	335.3	324.7						

Agreement between the temperature fields for the (a) and (b) grids is good, with the largest differences occurring at the interior and exterior corners. Ten isotherms generated using *FEHT* are shown on the symmetric section below. Note how the heat flow is nearly normal to the flue wall around the mid-section. In the corner regions, the isotherms are curved and we'd expect that grid size might influence the accuracy of the results. Convection heat transfer to the inner surface is

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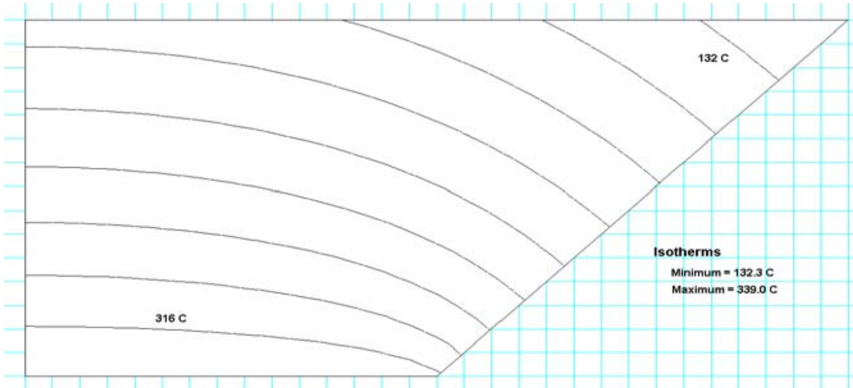
PROBLEM 4.92 (Cont.)



$$q' = 8h_i\Delta x\left[\left(T_{\infty,i} - T_1\right)/2 + \left(T_{\infty,i} - T_2\right) + \left(T_{\infty,i} - T_3\right) + \left(T_{\infty,i} - T_4\right)\right.$$
$$\left. + \left(T_{\infty,i} - T_5\right) + \left(T_{\infty,i} - T_6\right) + \left(T_{\infty,i} - T_7\right)/2\right] = 1.52\,\text{kW/m}$$

and the agreement with results of the coarse grid is excellent.

The heat rate increases with increasing h_i and h_o, while temperatures in the wall increase and decrease, respectively, with increasing h_i and h_o.

COMMENTS. (1) Gauss-Seidel iteration may be used to solve this system of equations. Following the procedures of Appendix D, the system of equations is rewritten in the proper form. Note that diagonal dominance is present; hence, no re-ordering is necessary.

$$T_1^k = 0.09239T_2^{k-1} + 0.09239T_4^{k-1} + 285.3$$
$$T_2^k = 0.04620T_1^k + 0.04620T_3^{k-1} + 0.09239T_5^{k-1} + 285.3$$
$$T_3^k = 0.08457T_2^k + 0.1692T_6^{k-1} + 261.2$$
$$T_4^k = 0.25T_1^k + 0.50T_5^{k-1} + 0.25T_8^{k-1}$$
$$T_5^k = 0.25T_2^k + 0.25T_4^k + 0.25T_6^{k-1} + 0.25T_9^{k-1}$$
$$T_6^k = 0.25T_3^k + 0.25T_5^k + 0.25T_7^{k-1} + 0.25T_9^{k-1}$$
$$T_7^k = 0.50T_6^k + 0.50T_{11}^{k-1}$$
$$T_8^k = 0.4096T_4^k + 0.4096T_9^{k-1} + 4.52$$
$$T_9^k = 0.4096T_5^k + 0.2048T_8^k + 0.2048T_{10}^{k-1} + 4.52$$
$$T_{10}^k = 0.4096T_6^k + 0.2048T_9^k + 0.2048T_{11}^{k-1} + 4.52$$
$$T_{11}^k = 0.4096T_7^k + 0.2048T_{10}^k + 0.2048T_{12}^{k-1} + 4.52$$
$$T_{12}^k = 0.6939T_{11}^k + 7.65$$

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PROBLEM 4.92 (Cont.)

The initial estimates (k = 0) are carefully chosen to minimize calculation labor; let $\varepsilon < 1.0$.

k	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}
0	340	330	315	250	225	205	195	160	150	140	125	110
1	338.9	336.3	324.3	237.2	232.1	225.4	175.2	163.1	161.7	155.6	130.7	98.3
2	338.3	337.4	328.0	241.4	241.5	226.6	178.6	169.6	170.0	158.9	130.4	98.1
3	338.8	338.4	328.2	247.7	245.7	230.6	180.5	175.6	173.7	161.2	131.6	98.9
4	339.4	338.8	328.9	251.6	248.7	232.9	182.3	178.7	176.0	162.9	132.8	99.8
5	339.8	339.2	329.3	254.0	250.5	234.5	183.7	180.6	177.5	164.1	133.8	100.5
6	340.1	339.4	329.7	255.4	251.7	235.7	184.7	181.8	178.5	164.7	134.5	101.0
7	340.3	339.5	329.9	256.4	252.5	236.4	185.5	182.7	179.1	165.6	135.1	101.4

The heat loss to the outside air for the upper surface (Nodes 8 through 12) is of the form

$$q' = h_o \Delta x\left[\frac{1}{2}\left(T_8 - T_{\infty,o}\right) + \left(T_9 - T_{\infty,o}\right) + \left(T_{10} - T_{\infty,o}\right) + \left(T_{11} - T_{\infty,o}\right) + \frac{1}{2}\left(T_{12} - T_{\infty,o}\right)\right]$$

$$q' = 5\,\mathrm{W/m^2\cdot K} \times 0.075\,\mathrm{m}\left[\frac{1}{2}(182.7-25) + (179.1-25) + (165.6-25) + (135.1-25) + \frac{1}{2}(101.4-25)\right]{}^\circ\mathrm{C} = 195\,\mathrm{W/m}$$

Hence, for the entire flue cross-section, considering symmetry,

$$q'_{tot} = 8 \times q' = 8 \times 195\,\mathrm{W/m} = 1.57\,\mathrm{kW/m} \qquad <$$

The convection heat rate at the inner surface is

$$q'_{tot} = 8 \times h_i \Delta x\left[\frac{1}{2}\left(T_{\infty,i} - T_1\right) + \left(T_{\infty,i} - T_2\right) + \frac{1}{2}\left(T_{\infty,i} - T_3\right)\right] = 8 \times 190.5\,\mathrm{W/m} = 1.52\,\mathrm{kW/m}$$

which is within 2.5% of the foregoing result. The convection heat rates would be identical when $\varepsilon = 0$.

(2) For this problem the Gauss-Seidel iteration method is cumbersome, time-consuming and inaccurate unless many more iterations are included. For relatively small systems of simultaneous equations such as in this problem, enhanced accuracy can usually be obtained with far less effort through use of a numerical solver such as available on many handheld calculators, IHT or some other commercial code.