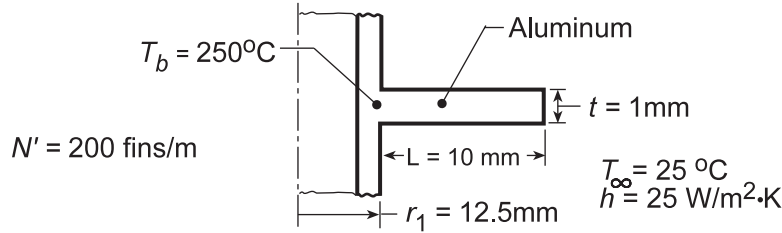


PROBLEM 3.158

KNOWN: Dimensions and base temperature of an annular, aluminum fin of rectangular profile. Ambient air conditions.

FIND: (a) Fin heat loss, (b) Heat loss per unit length of tube with 200 fins spaced at 5 mm increments.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and contact resistance, (5) Uniform convection coefficient.

PROPERTIES: Table A-1, Aluminum, pure ($T \approx 400$ K): $k = 240$ W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.20 are

$$r_{2c} = r_2 + t/2 = (12.5 \text{ mm} + 10 \text{ mm}) + 0.5 \text{ mm} = 23 \text{ mm} = 0.023 \text{ m}$$

$$r_{2c}/r_1 = 1.84 \quad L_c = L + t/2 = 10.5 \text{ mm} = 0.0105 \text{ m}$$

$$A_p = L_c t = 0.0105 \text{ m} \times 0.001 \text{ m} = 1.05 \times 10^{-5} \text{ m}^2$$

$$L_c^{3/2} (h/kA_p)^{1/2} = (0.0105 \text{ m})^{3/2} \left(\frac{25 \text{ W/m}^2 \cdot \text{K}}{240 \text{ W/m} \cdot \text{K} \times 1.05 \times 10^{-5} \text{ m}^2} \right)^{1/2} = 0.15.$$

Hence, the fin effectiveness is $\eta_f \approx 0.97$, and from Eq. 3.91 and Fig. 3.6, the fin heat rate is

$$q_f = \eta_f q_{\max} = \eta_f h A_{f(\text{ann})} \theta_b = 2\pi \eta_f h (r_{2c}^2 - r_1^2) \theta_b$$

$$q_f = 2\pi \times 0.97 \times 25 \text{ W/m}^2 \cdot \text{K} \times \left[(0.023 \text{ m})^2 - (0.0125 \text{ m})^2 \right] 225^\circ \text{C} = 12.8 \text{ W}. \quad <$$

(b) Recognizing that there are $N = 200$ fins per meter length of the tube, the total heat rate considering contributions due to the fin and base (unfinned) surfaces is

$$q' = N' q_f + h(1 - N't) 2\pi r_1 \theta_b$$

$$q' = 200 \text{ m}^{-1} \times 12.8 \text{ W} + 25 \text{ W/m}^2 \cdot \text{K} (1 - 200 \text{ m}^{-1} \times 0.001 \text{ m}) \times 2\pi \times (0.0125 \text{ m}) 225^\circ \text{C}$$

$$q' = (2560 \text{ W} + 353 \text{ W})/\text{m} = 2.91 \text{ kW/m}. \quad <$$

COMMENTS: Note that, while covering only 20% of the tube surface area, the tubes account for more than 85% of the total heat dissipation.