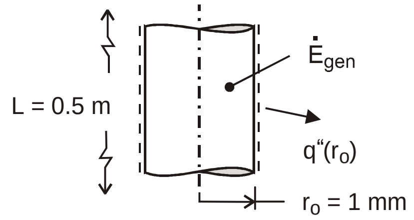
PROBLEM 2.68

KNOWN: Radius and length of coiled wire in hair dryer. Electric power dissipation in the wire, and temperature and convection coefficient associated with air flow over the wire.

FIND: (a) Form of heat equation and conditions governing transient, thermal behavior of wire during start-up, (b) Volumetric rate of thermal energy generation in the wire, (c) Sketch of temperature distribution at selected times during start-up, (d) Variation with time of heat flux at r = 0 and r = r_o.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties, (3) Uniform volumetric heating, (4) Negligible radiation from surface of wire.

ANALYSIS: (a) The general form of the heat equation for cylindrical coordinates is given by Eq. 2.26. For one-dimensional, radial conduction and constant properties, the equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{q}}{k}=\frac{\rho c_p}{k}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

The initial condition is $T(r,0)=T_i$ <

The boundary conditions are: $\partial T/\partial r|_{r=0}=0$ <

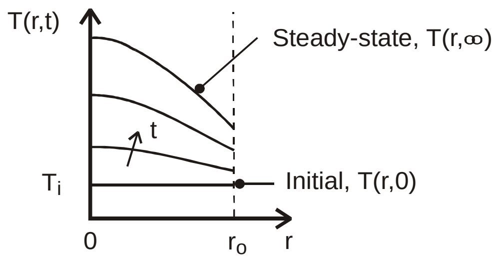
$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}=h\left[T(r_o,t)-T_\infty\right]$$ <

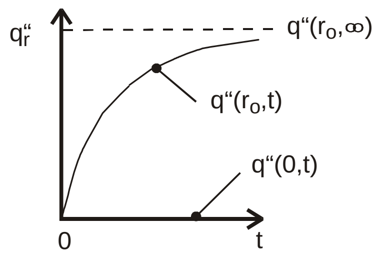
(b) The volumetric rate of thermal energy generation is

$$\dot{q}=\frac{\dot{E}_g}{\forall}=\frac{P_{elec}}{\pi r_o^2 L}=\frac{500\ \text{W}}{\pi(0.001\text{m})^2(0.5\text{m})}=3.18\times10^8\ \text{W/m}^3$$ <

Under steady-state conditions, all of the thermal energy generated within the wire is transferred to the air by convection. Performing an energy balance for a control surface about the wire, $-\dot{E}_{out}+\dot{E}_g=0$, it follows that $-2\pi r_o L\ q''(r_o, t\to\infty)+P_{elec}=0$. Hence,

$$q''(r_o, t\to\infty)=\frac{P_{elec}}{2\pi r_o L}=\frac{500\ \text{W}}{2\pi(0.001\text{m})0.5\text{m}}=1.59\times10^5\ \text{W/m}^2$$ <





COMMENTS: The symmetry condition at r = 0 imposes the requirement that $\partial T/\partial r|_{r=0}=0$, and hence $q''(0,t)=0$ throughout the process. The temperature at r_o, and hence the convection heat flux, increases steadily during the start-up, and since conduction to the surface must be balanced by convection from the surface at all times, $|\partial T/\partial r|_{r=r_o}$ also increases during the start-up.