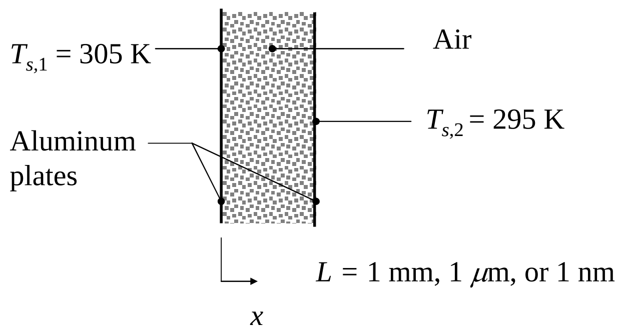
PROBLEM 3.173

KNOWN: Size and temperatures of parallel aluminum plates. Spacing between the plates. Air between the plates.

FIND: The conduction heat transfer through the air.

SCHEMATIC:



ASSUMPTIONS: (1) Ideal gas behavior.

PROPERTIES: Table A.4 (T = 300 K): Air; c_p = 1007 J/kg·K, k = 0.0263 W/m·K. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol, $d = 0.372 \times 10^{-9}$ m.

ANALYSIS: For air, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315 \text{ kJ/kmol}\cdot\text{K}}{28.97 \text{ kg/kmol}} = 0.287 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.007\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.720\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad \gamma = \frac{c_p}{c_v} = \frac{1.007}{0.720} = 1.399$$

From Equation 2.11 the mean free path of air is

$$\lambda_{\text{mfp}} = \frac{k_B T}{\sqrt{2}\pi d^2 p} = \frac{1.381\times10^{-23} \text{ J/K}\times 300 \text{ K}}{\sqrt{2}\pi\left(0.372 \times 10^{-9} \text{ m}\right)^2\left(1.0133\times10^5 \text{ N/m}^2\right)} = 66.5\times10^{-9} \text{ m} = 66.5 \text{ nm}$$

For L = 1 mm,

$$R_{t,m-m} = \frac{L}{kA} = \frac{1\times10^{-3}\text{m}}{0.0263 \text{ W/m}\cdot\text{K}\times10\times10^{-3}\text{m}\times10\times10^{-3}\text{m}} = 380.2 \text{ K/W}$$

$$R_{t,m-s} = \frac{\lambda_{\text{mfp}}}{kA}\left[\frac{2-\alpha_t}{\alpha_t}\right]\left[\frac{9\gamma-5}{\gamma+1}\right] = \frac{66.5\times10^{-9} \text{ m}}{0.0263 \text{ W/m}\cdot\text{K}\times100\times10^{-6} \text{ m}^2}\left[\frac{2-0.92}{0.92}\right]\left[\frac{9\times1.399-5}{1.399+1}\right]$$
$$= 0.09392 \text{ K/W}$$

Hence, the conduction heat rate is

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$$q = \frac{T_{s,1} - T_{s,2}}{\left(R_{t,m-m} + R_{t,m-s}\right)} = \frac{305\text{K} - 295\text{K}}{\left(380.2\ \text{K/W} + 0.09392\ \text{K/W}\right)} = 0.0263\ \text{W} \qquad <$$

For $L = 1\ \mu$m,

$$R_{t,m-m} = \frac{L}{kA} = \frac{1 \times 10^{-6}\,\text{m}}{0.0263\ \text{W/m}\cdot\text{K} \times 10 \times 10^{-3}\,\text{m} \times 10 \times 10^{-3}\,\text{m}} = 0.3802\ \text{K/W}$$

$$R_{t,m-s} = 0.09392\ \text{K/W}$$

Hence, the conduction heat rate is

$$q = \frac{T_{s,1} - T_{s,2}}{\left(R_{t,m-m} + R_{t,m-s}\right)} = \frac{305\text{K} - 295\text{K}}{\left(0.3802\ \text{K/W} + 0.09392\ \text{K/W}\right)} = 21.09\ \text{W} \qquad <$$

For $L = 10$ nm,

$$R_{t,m-m} = \frac{L}{kA} = \frac{10 \times 10^{-9}\,\text{m}}{0.0263\ \text{W/m}\cdot\text{K} \times 10 \times 10^{-3}\,\text{m} \times 10 \times 10^{-3}\,\text{m}} = 0.0038\ \text{K/W}$$

$$R_{t,m-s} = 0.09392\ \text{K/W}$$

Hence, the conduction heat rate is

$$q = \frac{T_{s,1} - T_{s,2}}{\left(R_{t,m-m} + R_{t,m-s}\right)} = \frac{305\text{K} - 295\text{K}}{\left(0.0038\ \text{K/W} + 0.09392\ \text{K/W}\right)} = 102.3\ \text{W} \qquad <$$

COMMENT: If the molecule-surface collision resistance were to be neglected, the heat rates would be q = 0.0263 W, 26.3 W, and 2632 W for the L = 1 mm, 1 μm and 10 nm plate spacings, respectively. Hence, molecule-surface collisions are negligible for large plate spacings, and dominant at small plate spacings.