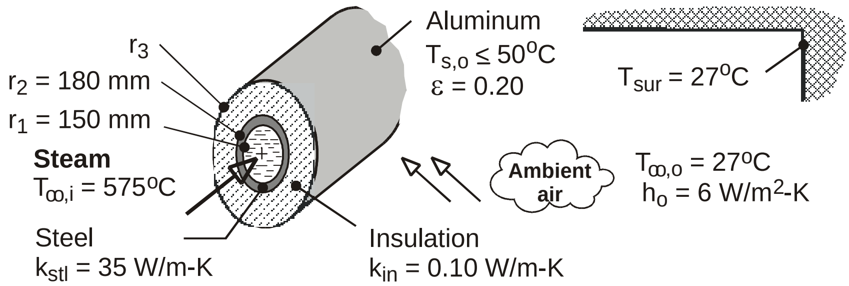
PROBLEM 3.51

KNOWN: Diameter, wall thickness and thermal conductivity of steel tubes. Temperature of steam flowing through the tubes. Thermal conductivity of insulation and emissivity of aluminum sheath. Temperature of ambient air and surroundings. Convection coefficient at outer surface and maximum allowable surface temperature.

FIND: (a) Minimum required insulation thickness ($r_3 - r_2$) and corresponding heat loss per unit length, (b) Effect of insulation thickness on outer surface temperature and heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction, (3) Negligible contact resistances at the material interfaces, (4) Negligible steam side convection resistance ($T_{\infty,i} = T_{s,i}$), (5) Negligible conduction resistance for aluminum sheath, (6) Constant properties, (7) Large surroundings.

ANALYSIS: (a) To determine the insulation thickness, an energy balance must be performed at the outer surface, where $q' = q'_{conv,o} + q'_{rad}$. With $q'_{conv,o} = 2\pi r_3 h_o \left(T_{s,o} - T_{\infty,o}\right)$, $q'_{rad} = 2\pi r_3 \ \varepsilon\sigma \left(T_{s,o}^4 - T_{sur}^4\right)$, $q' = \left(T_{s,i} - T_{s,o}\right) / \left(R'_{cond,st} + R'_{cond,ins}\right)$, $R'_{cond,st} = \ell n\left(r_2 / r_1\right) / 2\pi k_{st}$, and $R'_{cond,ins} = \ell n\left(r_3 / r_2\right) / 2\pi k_{ins}$, it follows that

$$\frac{2\pi\left(T_{s,i} - T_{s,o}\right)}{\dfrac{\ell n\left(r_2 / r_1\right)}{k_{st}} + \dfrac{\ell n\left(r_3 / r_2\right)}{k_{ins}}} = 2\pi r_3 \left[h_o\left(T_{s,o} - T_{\infty,o}\right) + \varepsilon\sigma\left(T_{s,o}^4 - T_{sur}^4\right)\right]$$

$$\frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\ \text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(r_3/0.18\right)}{0.10\ \text{W/m}\cdot\text{K}}} = 2\pi r_3\left[6\ \text{W/m}^2\cdot\text{K}\left(323 - 300\right)\text{K} + 0.20 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(323^4 - 300^4\right)\text{K}^4\right]$$

A trial-and-error solution yields r_3 = 0.394 m = 394 mm, in which case the insulation thickness is

$$t_{ins} = r_3 - r_2 = 214\,\text{mm} \qquad <$$

The heat rate is then

$$q' = \frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\,\text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(0.394/0.18\right)}{0.10\,\text{W/m}\cdot\text{K}}} = 420\,\text{W/m} \qquad <$$

(b) The effects of r_3 on $T_{s,o}$ and q' have been computed and are shown below.

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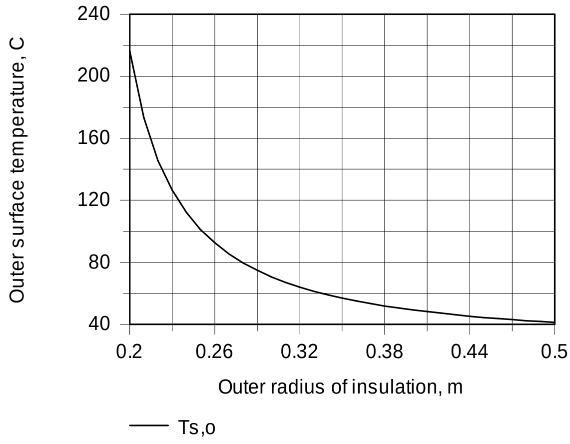
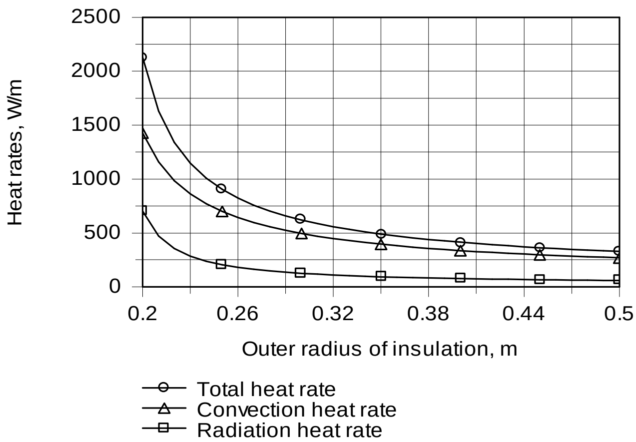
PROBLEM 3.51 (Cont.)





Beyond $r_3 \approx 0.40$ m, there are rapidly diminishing benefits associated with increasing the insulation thickness.

COMMENTS: Note that the thermal resistance of the insulation is much larger than that for the tube wall. For the conditions of Part (a), the radiation coefficient is h_r = 1.37 W/m, and the heat loss by radiation is less than 25% of that due to natural convection $\left(q'_{rad} = 78\,\mathrm{W/m},\ q'_{conv,o} = 342\ \mathrm{W/m}\right)$.