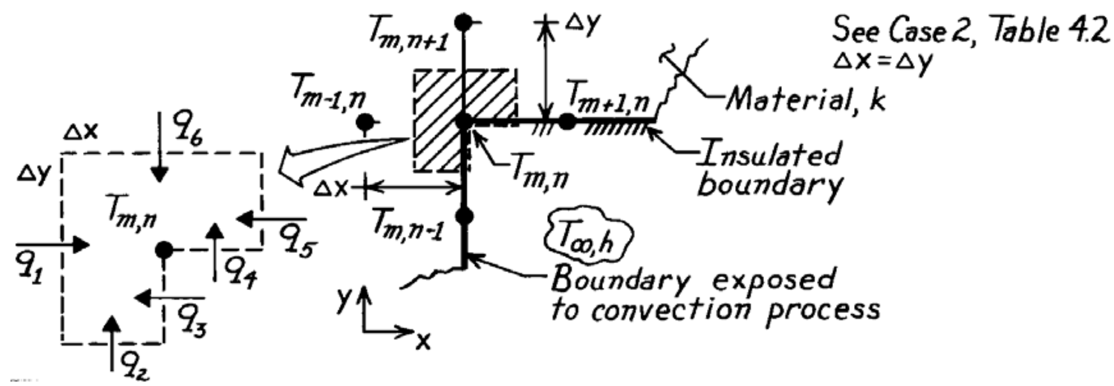
PROBLEM 4.38

KNOWN: Internal corner of a two-dimensional system with prescribed convection boundary conditions.

FIND: Finite-difference equations for these situations: (a) Horizontal boundary is perfectly insulated and vertical boundary is subjected to a convection process (T_∞,h), (b) Both boundaries are perfectly insulated; compare result with Eq. 4.41.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) No internal generation.

ANALYSIS: Consider the nodal network shown above and also as Case 2, Table 4.2. Having defined the control volume – the shaded area of unit thickness normal to the page – next identify the heat transfer processes. Finally, perform an energy balance wherein the processes are expressed using appropriate rate equations.

(a) With the horizontal boundary insulated and the vertical boundary subjected to a convection process, the energy balance results in the following finite-difference equation:

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 0$$

$$k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + h\left[\frac{\Delta y}{2}\cdot 1\right]\left(T_\infty - T_{m,n}\right)$$

$$+0 + k\left[\frac{\Delta y}{2}\cdot 1\right]\frac{T_{m+1,n} - T_{m,n}}{\Delta x} + k(\Delta x \cdot 1)\frac{T_{m,n+1} - T_{m,n}}{\Delta y} = 0.$$

Letting $\Delta x = \Delta y$, and regrouping, find

$$2\left(T_{m-1,n} + T_{m,n+1}\right) + \left(T_{m+1,n} + T_{m,n-1}\right) + \frac{h\Delta x}{k}T_\infty - \left[6 + \frac{h\Delta x}{k}\right]T_{m,n} = 0. \qquad <$$

(b) With both boundaries insulated, the energy balance would have $q_3 = q_4 = 0$. The same result would be obtained by letting h = 0 in the previous result. Hence,

$$2\left(T_{m-1,n} + T_{m,n+1}\right) + \left(T_{m+1,n} + T_{m,n-1}\right) - 6\ T_{m,n} = 0. \qquad <$$

Note that this expression compares exactly with Equation 4.41 when h = 0, which corresponds to insulated boundaries.