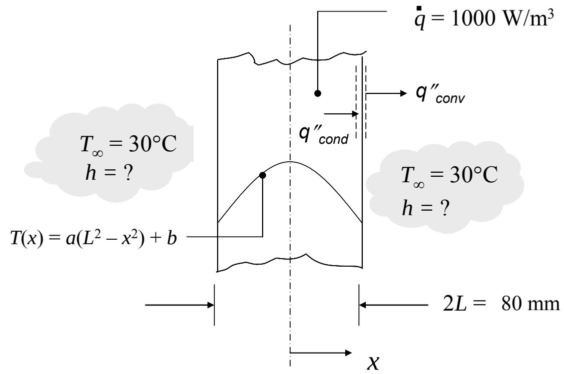
PROBLEM 2.10

KNOWN: Wall thickness. Thermal energy generation rate. Temperature distribution. Ambient fluid temperature.

FIND: Thermal conductivity. Convection heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Under the specified conditions, the heat equation, Equation 2.21, reduces to

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

With the given temperature distribution, $d^2T/dx^2 = -2a$. Therefore, solving for k gives

$$k = \frac{\dot{q}}{2a} = \frac{1000\ \text{W/m}^3}{2 \times 15°\text{C/m}^2} = 33.3\ \text{W/m} \cdot \text{K} \qquad <$$

The convection heat transfer coefficient can be found by applying the boundary condition at $x = L$ (or at $x = -L$),

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left[T(L) - T_\infty\right]$$

Therefore

$$h = \frac{-k\left.\frac{dT}{dx}\right|_{x=L}}{\left[T(L) - T_\infty\right]} = \frac{2kaL}{b - T_\infty} = \frac{2 \times 33.3\ \text{W/m} \cdot \text{K} \times 15°\text{C/m}^2 \times 0.04\ \text{m}}{40°\text{C} - 30°\text{C}} = 4\ \text{W/m}^2 \cdot \text{K} \qquad <$$

COMMENTS: (1) In Chapter 3, you will learn how to determine the temperature distribution. (2) The heat transfer coefficient could also have been found from an energy balance on the wall. With $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = 0$, we find $-2hA[T(L) - T_\infty] + 2\dot{q}LA = 0$. This yields the same result for h.