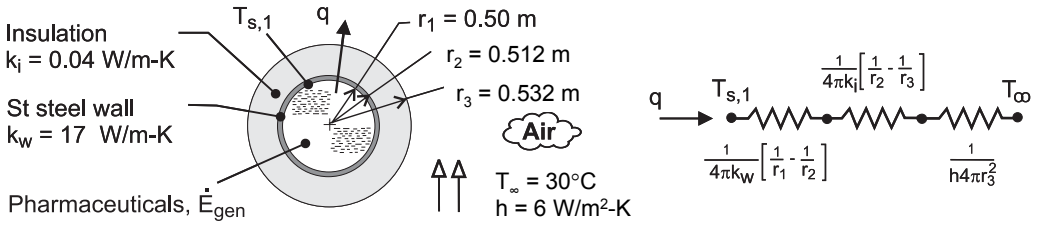
PROBLEM 3.70

KNOWN: Inner diameter, wall thickness and thermal conductivity of spherical vessel containing heat generating medium. Inner surface temperature without insulation. Thickness and thermal conductivity of insulation. Ambient air temperature and convection coefficient.

FIND: (a) Thermal energy generated within vessel, (b) Inner surface temperature of vessel with insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Neglect radiation due to relatively low emissivity of stainless steel in part (a). In part (b), insulation resistance dominates.

ANALYSIS: (a) From an energy balance performed at an instant for a control surface about the pharmaceuticals, $\dot{E}_g = q$, in which case, without the insulation

$$\dot{E}_g = q = \frac{T_{s,1} - T_\infty}{\frac{1}{4\pi k_w}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\frac{1}{4\pi r_2^2 h}} = \frac{(50-30)°C}{\frac{1}{4\pi(17\,W/m\cdot K)}\left(\frac{1}{0.50m}-\frac{1}{0.512m}\right)+\frac{1}{4\pi(0.512m)^2\,6\,W/m^2\cdot K}}$$

$$\dot{E}_g = q = \frac{20°C}{\left(2.19\times10^{-4}+5.06\times10^{-2}\right)K/W} = 394\,W \qquad <$$

(b) With the insulation,

$$T_{s,1} = T_\infty + q\left[\frac{1}{4\pi k_w}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\frac{1}{4\pi k_i}\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\frac{1}{4\pi r_3^2 h}\right]$$

$$T_{s,1} = 30°C + 394\,W\left[2.19\times10^{-4}+\frac{1}{4\pi(0.04)}\left(\frac{1}{0.512}-\frac{1}{0.532}\right)+\frac{1}{4\pi(0.532)^2\,6}\right]\frac{K}{W}$$

$$T_{s,1} = 30°C + 394\,W\left[2.19\times10^{-4}+0.146+0.047\right]\frac{K}{W} = 106°C \qquad <$$

COMMENTS: The thermal resistance associated with the vessel wall is negligible, and without the insulation the dominant resistance is due to convection. The thermal resistance of the insulation is approximately three times that due to convection. Radiation may not be negligible, and would have the effect of increasing the heat loss rate (for fixed inner surface temperature) or decreasing the inner surface temperature (for fixed heat loss rate).