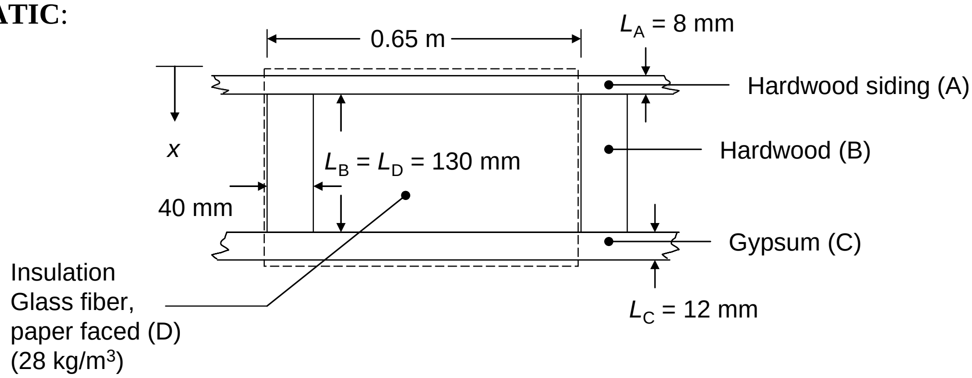
PROBLEM 3.16

KNOWN: Dimensions and materials associated with a composite wall (2.5 m × 6.5 m, 10 studs each 2.5 m high).

FIND: Wall thermal resistance.

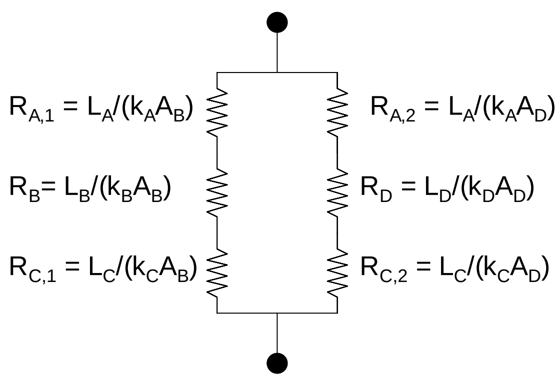
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Planes parallel to *x* are adiabatic, (3) Constant properties, (4) Negligible contact resistance.

PROPERTIES: Table A-3 ($T \approx 300$ K): Hardwood siding, k_A = 0.094 W/m·K; Hardwood, k_B = 0.16 W/m·K; Gypsum, k_C = 0.17 W/m·K; Insulation (glass fiber paper faced, 28 kg/m^3), k_D = 0.038 W/m·K.

ANALYSIS: Using the adiabatic surface assumption, the thermal circuit associated with a single unit (enclosed by dashed lines) of the wall is as shown to the right. The various resistances are



$$R_{A,1} = (L_A/k_A A_B) = \frac{0.008\ \text{m}}{0.094\ \text{W/m}\cdot\text{K}\ (0.04\ \text{m} \times 2.5\ \text{m})} = 0.8511\ \text{K/W}$$

$$R_B = (L_B/k_B A_B) = \frac{0.13\ \text{m}}{0.16\ \text{W/m}\cdot\text{K}\ (0.04\ \text{m} \times 2.5\ \text{m})} = 8.125\ \text{K/W}$$

$$R_{C,1} = (L_C/k_C A_B) = \frac{0.012\ \text{m}}{0.17\ \text{W/m}\cdot\text{K}\ (0.04\ \text{m} \times 2.5\ \text{m})} = 0.7059\ \text{K/W}$$

$$R_{A,2} = (L_A/k_A A_D) = \frac{0.008\ \text{m}}{0.094\ \text{W/m}\cdot\text{K}\ (0.61\ \text{m} \times 2.5\ \text{m})} = 0.0558\ \text{K/W}$$

$$R_D = (L_D/k_D A_D) = \frac{0.13\ \text{m}}{0.038\ \text{W/m}\cdot\text{K}\ (0.61\ \text{m} \times 2.5\ \text{m})} = 2.243\ \text{K/W}$$

$$R_{C,2} = (L_C/k_C A_D) = \frac{0.012\ \text{m}}{0.17\ \text{W/m}\cdot\text{K}\ (0.61\ \text{m} \times 2.5\ \text{m})} = 0.0463\ \text{K/W}$$

Continued…

The total unit resistance is

$$R_{tot,1} = \left(\frac{1}{R_{A,1}+R_B+R_{C,1}} + \frac{1}{R_{A,2}+R_D+R_{C,2}} \right)^{-1} = \left(\frac{1}{0.8511+8.125+0.7059} + \frac{1}{0.0558+2.243+0.0463} \right)^{-1}$$

$$= 1.888 \text{ K/W}$$

With 10 such units in parallel, the total wall resistance is $R_{tot} = (10 \times 1/R_{tot,1})^{-1} = 0.1888$ K/W. <

COMMENTS: (1) Contact resistance will increase the overall wall resistance relative to that calculated here. (2) The total wall resistance assuming isothermal surfaces normal to the *x* direction is 0.1854 K/W, which is within 2 % of the value found in this solution.