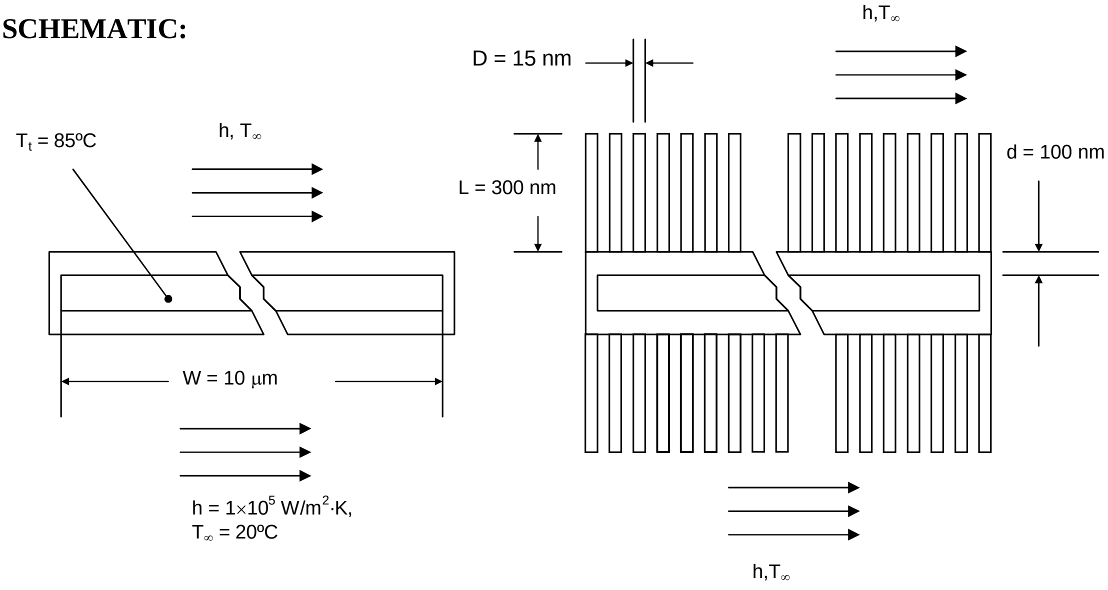
# PROBLEM 3.145

**KNOWN:** Dimensions of electronics package and finned nano-heat sink. Temperature and heat transfer coefficient of coolant.

**FIND:** Maximum heat rate to maintain temperature below 85°C for finned and un-finned packages.

**SCHEMATIC:**

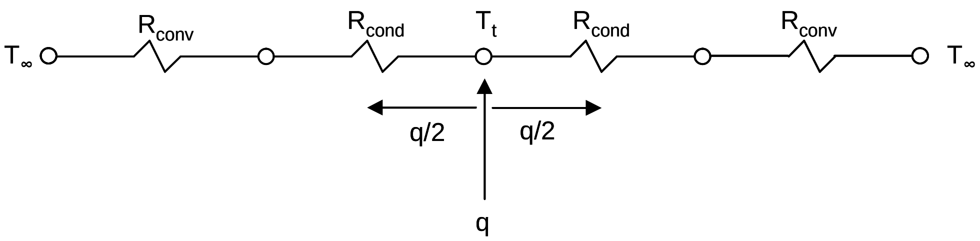



Unfinned Nano-finned

**ASSUMPTIONS:** (1) Steady-state, (2) Negligible temperature variation across fin thickness, (3) Constant properties, (4) Uniform heat transfer coefficient, (5) Negligible contact resistance, (6) Negligible heat loss from edges of package.

**PROPERTIES:** Table A.2, Silicon carbide (T ≈ 300 K): k = 490 W/m·K.

**ANALYSIS:** (a) The thermal circuit for the un-finned package is




where $$R_{cond} = \frac{d}{kA} = \frac{100 \times 10^{-9}\ \text{m}}{490\ \text{W/m} \cdot \text{K} \times (10 \times 10^{-6}\ \text{m})^2} = 2.04\ \text{K/W}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{10^5\ \text{W/m}^2 \cdot \text{K} \times (10 \times 10^{-6}\ \text{m})^2} = 1 \times 10^5\ \text{K/W}$$

Thus $$q = 2\frac{(T_t - T_\infty)}{R_{cond} + R_{conv}} = 2\frac{(85°\text{C} - 20°\text{C})}{(2.04 + 10^5)\ \text{K/W}} = 1.30 \times 10^{-3}\ \text{W}$$ <

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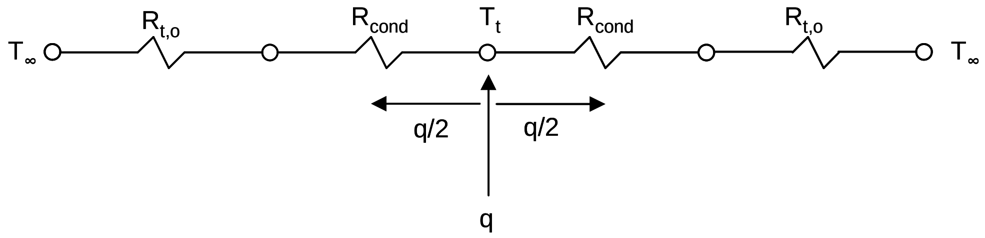
## PROBLEM 3.145 (Cont.)

For the finned nano-heat sink, the convection resistance is replaced by a fin array thermal resistance:




From Equations 3.108, 3.107, and 3.104

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \quad \eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f), \quad A_t = NA_f + A_b$$

where $A_f = \pi D L_c = \pi D(L + D/4) = \pi \times 15 \times 10^{-9}\ \text{m} \times (300 + 15/4) \times 10^{-9}\ \text{m} = 1.43 \times 10^{-14}\ \text{m}^2$,
$A_b = W^2 - N\pi D^2/4 = (10 \times 10^{-6}\ \text{m})^2 - 40{,}000 \times \pi \times (15 \times 10^{-9}\ \text{m})^2/4 = 9.29 \times 10^{-11}\ \text{m}^2$, and
$A_t = 40{,}000 \times 1.43 \times 10^{-14}\ \text{m}^2 + 9.29 \times 10^{-11}\ \text{m}^2 = 6.65 \times 10^{-10}\ \text{m}^2$. Then with
$mL_c = (4h/kD)^{1/2}L_c = (4 \times 10^5\ \text{W/m}^2\cdot\text{K} / 490\ \text{W/m}\cdot\text{K} \times 15 \times 10^{-9}\ \text{m})^{1/2} \times 304 \times 10^{-9}\ \text{m} = 7.09 \times 10^{-2}$,

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = \frac{\tanh(7.09 \times 10^{-2})}{7.09 \times 10^{-2}} = 0.998$$

It follows that

$$\eta_o = 1 - \frac{40{,}000 \times 1.43 \times 10^{-14}\ \text{m}^2}{6.65 \times 10^{-10}\ \text{m}^2}(1 - 0.998) = 0.999$$

and

$$R_{t,o} = \frac{1}{0.999 \times 10^5\ \text{W/m}^2 \cdot \text{K} \times 6.65 \times 10^{-10}\ \text{m}^2} = 1.50 \times 10^4\ \text{K/W}$$

Therefore

$$q = 2\frac{(T_t - T_\infty)}{R_{cond} + R_{t,o}} = 2\frac{(85°\text{C} - 20°\text{C})}{2.04\ \text{K/W} + 1.50 \times 10^4\ \text{K/W}} = 8.64 \times 10^{-3}\ \text{W} \qquad <$$

**COMMENTS:** (1) The conduction resistance of the silicon carbide sheets is negligible. (2) The fins increase the allowable heat rate significantly. (3) We have neglected the contact resistance between the electronics and the silicon carbide sheets. If it dominates, the fins will not be effective in increasing the allowable heat rate. Little is known about contact resistance at the nanoscale.