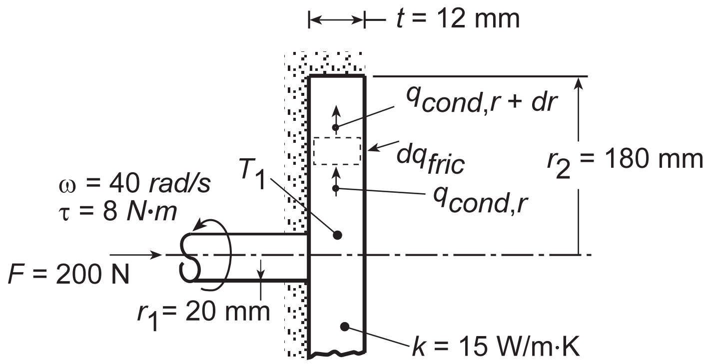
PROBLEM 3.127

KNOWN: Dimensions of disc/shaft assembly. Applied angular velocity, force, and torque. Thermal conductivity and inner temperature of disc.

FIND: (a) Expression for the friction coefficient μ, (b) Radial temperature distribution in disc, (c) Value of μ for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant k, (4) Uniform disc contact pressure p, (5) All frictional heat dissipation is transferred to shaft from base of disc.

ANALYSIS: (a) The normal force acting on a differential ring extending from r to r+dr on the contact surface of the disc may be expressed as $dF_n = p2\pi r dr$. Hence, the tangential force is $dF_t = \mu p 2\pi r dr$, in which case the torque may be expressed as

$$d\tau = 2\pi\mu p r^2 dr$$

For the entire disc, it follows that

$$\tau = 2\pi\mu p\int_o^{r_2} r^2 dr = \frac{2\pi}{3}\mu p r_2^3$$

where $p = F/\pi r_2^2$. Hence,

$$\mu = \frac{3}{2}\frac{\tau}{F r_2}$$ <

(b) Performing an energy balance on a differential control volume in the disc, it follows that

$$q_{cond,r} + dq_{fric} - q_{cond,r+dr} = 0$$

With $dq_{fric} = \omega d\tau = 2\mu F\omega\left(r^2/r_2^2\right)dr$, $q_{cond,r+dr} = q_{cond,r} + \left(dq_{cond,r}/dr\right)dr$, and $q_{cond,r} = -k\left(2\pi r t\right)dT/dr$, it follows that

$$2\mu F\omega\left(r^2/r_2^2\right)dr + 2\pi k t\frac{d\left(r dT/dr\right)}{dr}dr = 0$$

or

$$\frac{d\left(r dT/dr\right)}{dr} = -\frac{\mu F\omega}{\pi k t r_2^2}r^2$$

Integrating twice,

Continued...

$$\frac{dT}{dr} = -\frac{\mu F\omega}{3\pi k t r_2^2} r^2 + \frac{C_1}{r}$$

$$T = -\frac{\mu F\omega}{9\pi k t r_2^2} r^3 + C_1 \ell n r + C_2$$

Since the disc is well insulated at $r = r_2$, $dT/dr|_{r_2} = 0$ and

$$C_1 = \frac{\mu F\omega r_2}{3\pi k t}$$

With $T(r_1) = T_1$, it also follows that

$$C_2 = T_1 + \frac{\mu F\omega}{9\pi k t r_2^2} r_1^3 - C_1 \ell n r_1$$

Hence,

$$T(r) = T_1 - \frac{\mu F\omega}{9\pi k t r_2^2}\left(r^3 - r_1^3\right) + \frac{\mu F\omega r_2}{3\pi k t} \ell n \frac{r}{r_1} \qquad <$$

(c) For the prescribed conditions,

$$\mu = \frac{3}{2} \frac{8 N \cdot m}{200 N (0.18 m)} = 0.333 \qquad <$$

Since the maximum temperature occurs at $r = r_2$,

$$T_{max} = T(r_2) = T_1 - \frac{\mu F\omega r_2}{9\pi k t}\left[1 - \left(\frac{r_1}{r_2}\right)^3\right] + \frac{\mu F\omega r_2}{3\pi k t} \ell n \left(\frac{r_2}{r_1}\right)$$

With $\left(\mu F\omega r_2 / 3\pi k t\right) = \left(0.333 \times 200 N \times 40 rad/s \times 0.18 m / 3\pi \times 15 W/m \cdot K \times 0.012 m\right) = 282.7^\circ C$,

$$T_{max} = 80^\circ C - \frac{282.7^\circ C}{3}\left[1 - \left(\frac{0.02}{0.18}\right)^3\right] + 282.7^\circ C \ell n \left(\frac{0.18}{0.02}\right)$$

$$T_{max} = 80^\circ C - 94.1^\circ C + 621.1^\circ C = 607^\circ C \qquad <$$

COMMENTS: The maximum temperature is excessive, and the disks should be actively cooled (by convection) at their outer surfaces.