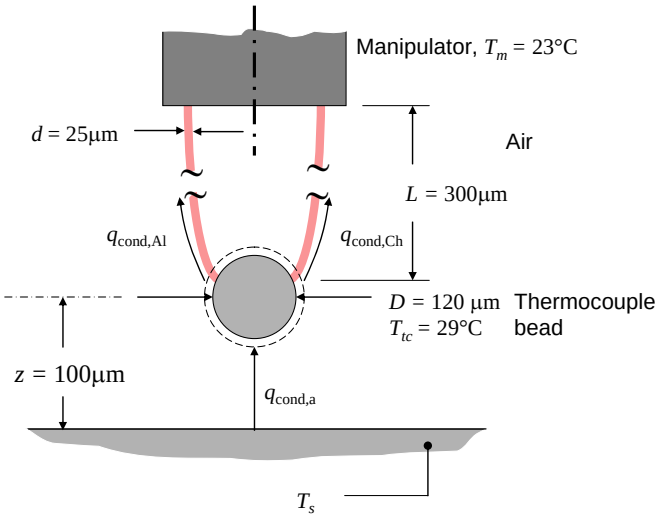
PROBLEM 4.25

KNOWN: Dimensions and temperature of thermocouple bead and wires. Manipulator temperature, distance between bead and surface.

FIND: Surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation and convection, (3) Isothermal thermocouple bead, (4) Air behaves as a semi-infinite medium, (5) Steady state conditions.

PROPERTIES: Table A.4, Air (310 K): k_a = 0.027 W/m·K.

ANALYSIS: An energy balance on the thermocouple bead yields

$$q_{\text{cond,air}} = q_{\text{cond,Al}} + q_{\text{cond,Ch}} \tag{1}$$

where the conduction heat transfer rates through the alumel and chromel wires are denoted as $q_{\text{cond,Al}}$ and $q_{\text{cond.Ch}}$, respectively. Conduction from the surface through the air to the thermocouple bead, $q_{\text{cond,air}}$, may be determined by use of the shape factor $S = (2\pi D)/(1 - D/4z)$of Case 1 of Table 4.1. Therefore, Equation (1) may be written as

$$Sk_a\left(T_s - T_{tc}\right) = k_{\text{Al}}\frac{\pi d^2}{4L}\left(T_{tc} - T_m\right) + k_{\text{Ch}}\frac{\pi d^2}{4L}\left(T_{tc} - T_m\right) \tag{2}$$

which may be rearranged to yield

$$T_s = \frac{1 - D/4z}{2Dk_a}\cdot\frac{d^2}{4L}\left[k_{\text{Al}}\left(T_{tc} - T_m\right) + k_{\text{Ch}}\left(T_{tc} - T_m\right)\right] + T_{tc}$$

$$= \frac{1 - 120/\left(4\times100\right)}{2\times120\times10^{-6}\,\text{m}\times0.027\ \text{W/m}\cdot\text{K}}\cdot\frac{\left(25\times10^{-6}\ \text{m}\right)^2}{4\times300\times10^{-6}\ \text{m}}\left[29\ \text{W/m}\cdot\text{K}\left(29-23\right)°\text{C} + 19\text{W/m}\cdot\text{K}\left(29-23\right)°\text{C}\right] + 29°\text{C}$$

$$= 45.2°\text{C} \quad <$$

Continued…

COMMENTS: The required surface temperature to induce the specified thermocouple temperature and its dependence on the separation distance, z, is shown below. As expected, the required surface temperature becomes greater as the separation distance increases.