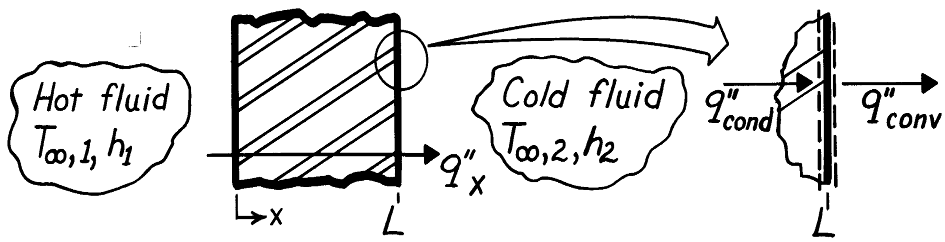
PROBLEM 3.1

KNOWN: One-dimensional, plane wall separating hot and cold fluids at $T_{\infty,1}$ and $T_{\infty,2}$, respectively.

FIND: Temperature distribution, T(x), and heat flux, q''_x, in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k and L.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation, (5) No generation.

ANALYSIS: For the foregoing conditions, the general solution to the heat diffusion equation is of the form, Equation 3.2,

$$T(x) = C_1 x + C_2. \tag{1}$$

The constants of integration, C_1 and C_2, are determined by using surface energy balance conditions at x = 0 and x = L, Equation 2.34, and as illustrated above,

$$-k\frac{dT}{dt}\bigg]_{x=0} = h_1\left[T_{\infty,1} - T(0)\right] \qquad -k\frac{dT}{dx}\bigg]_{x=L} = h_2\left[T(L) - T_{\infty,2}\right]. \tag{2,3}$$

For the boundary condition at x = 0, Equation (2), use Equation (1) to find

$$-k\left(C_1 + 0\right) = h_1\left[T_{\infty,1} - \left(C_1 \cdot 0 + C_2\right)\right] \tag{4}$$

and for the boundary condition at x = L to find

$$-k\left(C_1 + 0\right) = h_2\left[\left(C_1 L + C_2\right) - T_{\infty,2}\right]. \tag{5}$$

Multiply Eq. (4) by h_2 and Eq. (5) by h_1, and add the equations to obtain C_1. Then substitute C_1 into Eq. (4) to obtain C_2. The results are

$$C_1 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{k\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} \qquad C_2 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{h_1\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} + T_{\infty,1}$$

$$T(x) = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}\left[\frac{x}{k} + \frac{1}{h_1}\right] + T_{\infty,1}.$$ <

From Fourier's law, the heat flux is a constant and of the form

$$q''_x = -k\frac{dT}{dx} = -k\,C_1 = +\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}.$$ <