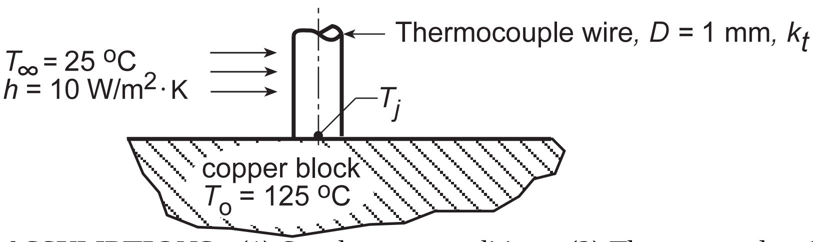
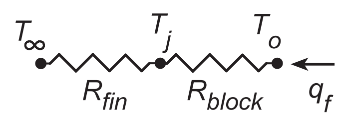
PROBLEM 4.30

KNOWN: Long constantan wire butt-welded to a large copper block forming a thermocouple junction on the surface of the block.

FIND: (a) The measurement error (T_j - T_o) for the thermocouple for prescribed conditions, and (b) Compute and plot (T_j - T_o) for h = 5, 10 and 25 W/m^2·K for block thermal conductivity $15 \le k \le 400$ W/m·K. When is it advantageous to use smaller diameter wire?

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) Thermocouple wire behaves as a fin with constant heat transfer coefficient, (3) Copper block has uniform temperature, except in the vicinity of the junction.

PROPERTIES: *Table A-1*, Copper (pure, 400 K), k_b = 393 W/m·K; Constantan (350 K), $k_t \approx 25$ W/m·K.

ANALYSIS: The thermocouple wire behaves as a long fin permitting heat to flow from the surface thereby depressing the sensing junction temperature below that of the block T_o. In the block, heat flows into the circular region of the wire-block interface; the thermal resistance to heat flow within the block is approximated as a disk of diameter D on a semi-infinite medium (k_b, T_o). The thermocouple-block combination can be represented by a thermal circuit as shown above. The thermal resistance of the fin follows from the heat rate expression for an infinite fin, $R_{fin} = (hPk_tA_c)^{-1/2}$.

From Table 4.1, the shape factor for the disk-on-a-semi-infinite medium is given as S = 2D and hence $R_{block} = 1/k_bS = 1/2k_bD$. From the thermal circuit,

$$T_o - T_j = \frac{R_{block}}{R_{fin} + R_{block}}(T_o - T_\infty) = \frac{1.27}{1273 + 1.27}(125 - 25)^\circ C \approx 0.001(125 - 25)^\circ C = 0.1^\circ C . <$$

with $P = \pi D$ and $A_c = \pi D^2/4$ and the thermal resistances as

$$R_{fin} = \left(10\, W/m^2 \cdot K\,(\pi/4)\, 25\, W/m \cdot K \times \left(1 \times 10^{-3}\, m\right)^3\right)^{-1/2} = 1273\, K/W$$

$$R_{block} = (1/2) \times 393\, W/m \cdot K \times 10^{-3}\, m = 1.27\, K/W .$$

(b) We keyed the above equations into the IHT workspace, performed a sweep on k_b for selected values of h and created the plot shown. When the block thermal conductivity is low, the error (T_o - T_j) is larger, increasing with increasing convection coefficient. A smaller diameter wire will be advantageous for low values of k_b and higher values of h.