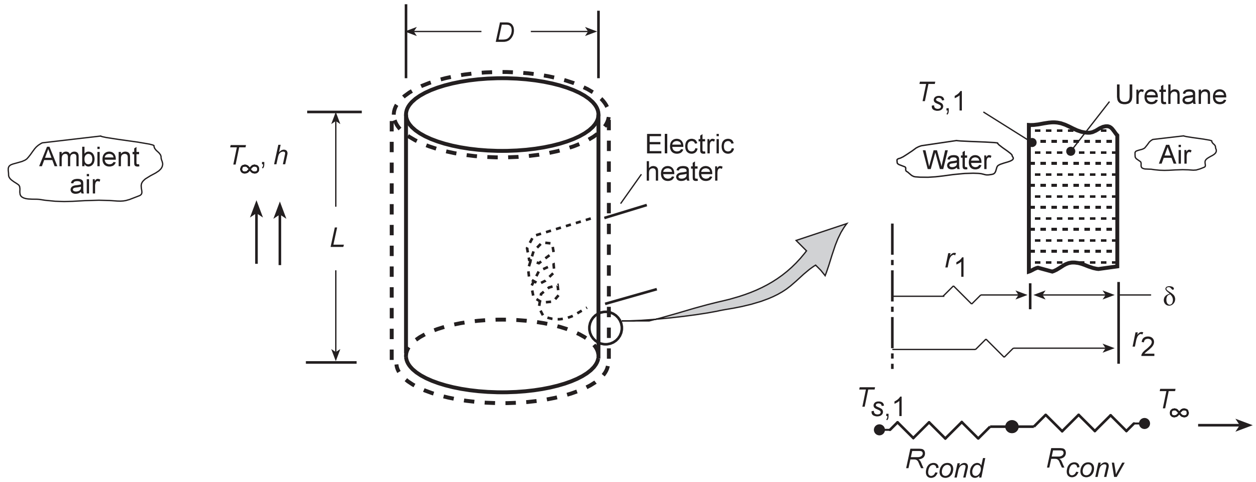
PROBLEM 3.46

KNOWN: Temperature and volume of hot water heater. Nature of heater insulating material. Ambient air temperature and convection coefficient. Unit cost of electric power.

FIND: Heater dimensions and insulation thickness for which annual cost of heat loss is less than \$50.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through side and end walls, (2) Conduction resistance dominated by insulation, (3) Inner surface temperature is approximately that of the water ($T_{s,1}$ = 55°C), (4) Constant properties, (5) Negligible radiation due to low emissivity foil covering on insulation.

PROPERTIES: *Table A.3*, Urethane Foam (T = 300 K): k = 0.026 W/m·K.

ANALYSIS: To minimize heat loss, tank dimensions which minimize the total surface area, $A_{s,t}$, should be selected. With $L = 4\forall/\pi D^2$, $A_{s,t} = \pi DL + 2\left(\pi D^2/4\right) = 4\forall/D + \pi D^2/2$, and the tank diameter for which $A_{s,t}$ is an extremum is determined from the requirement

$$dA_{s,t}/dD = -4\forall/D^2 + \pi D = 0$$

It follows that

$$D = \left(4\forall/\pi\right)^{1/3} \qquad \text{and} \qquad L = \left(4\forall/\pi\right)^{1/3}$$

With $d^2A_{s,t}/dD^2 = 8\forall/D^3 + \pi > 0$, the foregoing conditions yield the desired minimum in $A_{s,t}$. Hence, for $\forall$ = 100 gal × 0.00379 m³/gal = 0.379 m³,

$$D_{op} = L_{op} = 0.784\,\text{m}$$ $<$

For an annual cost of heat loss of \$50 and a unit electric power cost of \$0.18/kWh

$$Q_{annual} = \$50.00/\$0.18/\text{kWh} = 278\ \text{kWh}$$

The energy loss rate is therefore

$$q = Q_{annual}/(\text{hours per year}) = 278\times10^3\,\text{W}\cdot\text{h}/[(365\,\text{days})(24\ \text{h/day})] = 31.7\,\text{W}$$

Continued...

PROBLEM 3.46 (Cont.)

The total heat loss through the side and end walls is

$$q = \frac{T_{s,1} - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi k L_{op}} + \dfrac{1}{h 2\pi r_2 L_{op}}} + \frac{2\left(T_{s,1} - T_\infty\right)}{\dfrac{\delta}{k\left(\pi D_{op}^2/4\right)} + \dfrac{1}{h\left(\pi D_{op}^2/4\right)}}$$

With $r_1 = D_{op}/2 = 0.392$ m and $r_2 = r_1 + \delta$, everything is known except for the insulation thickness, δ.

$$q = 31.7\ \text{W} = \frac{(55-20)^\circ \text{C}}{\dfrac{\ln\left((0.392+\delta)/0.392\right)}{2\pi\left(0.026\ \text{W/m}\cdot\text{K}\right)0.784\ \text{m}} + \dfrac{1}{\left(2\ \text{W/m}^2\cdot\text{K}\right)2\pi\left(0.392\ \text{m}+\delta\right)0.784\ \text{m}}}$$

$$+ \frac{2(55-20)^\circ \text{C}}{\dfrac{\delta}{\left(0.026\ \text{W/m}\cdot\text{K}\right)\pi\left(0.784\ \text{m}\right)^2/4} + \dfrac{1}{\left(2\ \text{W/m}^2\cdot\text{K}\right)\pi\left(0.784\ \text{m}\right)^2/4}}$$

Solving by trial and error yields an insulation thickness of

$\delta = 68$ mm $<$

COMMENTS: Cylindrical containers of aspect ratio L/D = 1 are seldom used because of floor space constraints. Choosing L/D = 2, $\forall = \pi D^3/2$ and $D = (2\forall/\pi)^{1/3} = 0.623$ m. Hence, L = 1.245 m, r_1 = 0.312m and r_2 = 0.337 m. It follows that q = 34 W and C = \$53.62. The 7% increase in the annual cost of the heat loss is small, providing little justification for using the optimal heater dimensions.