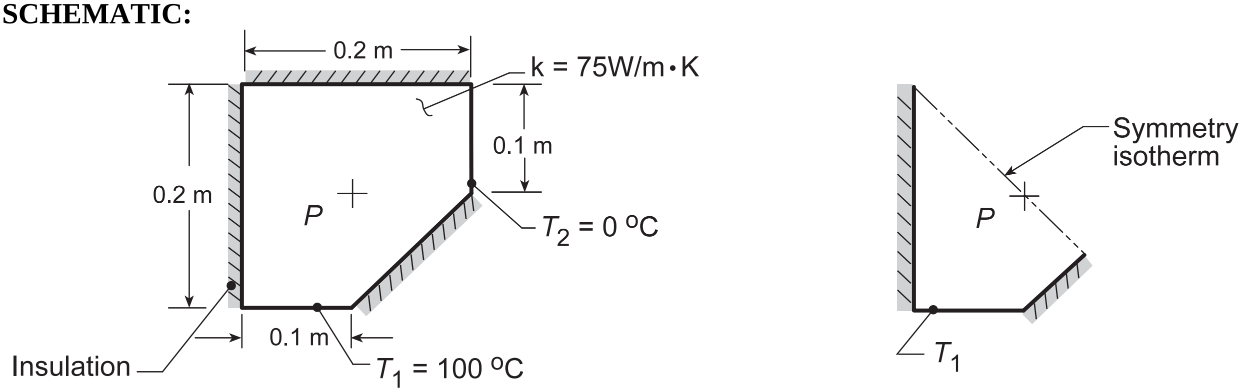
PROBLEM 4S.3

KNOWN: Structural member with known thermal conductivity subjected to a temperature difference.

FIND: (a) Temperature at a prescribed point P, (b) Heat transfer per unit length of the strut, (c) Sketch the 25, 50 and 75°C isotherms, and (d) Same analysis on the shape but with adiabatic-isothermal boundary conditions reversed.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: (a) When constructing the flux plot, note that the line of symmetry which passes through the point P is an isotherm as shown above. It follows that

$$T(P) = (T_1 + T_2)/2 = (100 + 0)^\circ C/2 = 50^\circ C.$$ <

(b) The flux plot on the symmetrical section is now constructed to obtain the shape factor from which the heat rate is determined. That is, from Equation 4S.6 and 4S.7,

$$q = kS(T_1 - T_2) \quad \text{and} \quad S = M\ell/N. \qquad (1,2)$$

From the plot of the symmetrical section,

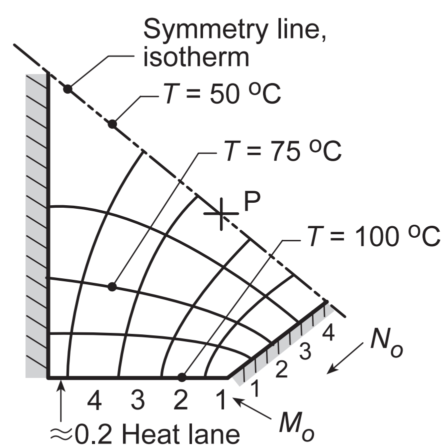
$$S_o = 4.2\ell/4 = 1.05\ell.$$

For the full section of the strut,

$$M = M_o = 4.2$$

but N = 2N_o = 8. Hence,

$$S = S_o/2 = 0.53\ell$$



and with $q' = q/\ell$, giving

$$q'/\ell = 75\,W/m\cdot K \times 0.53(100 - 0)^\circ C = 3975\,W/m.$$ <

(c) The isotherms for T = 50, 75 and 100°C are shown on the flux plot. The T = 25°C isotherm is symmetric with the T = 75°C isotherm.

(d) By reversing the adiabatic and isothermal boundary conditions, the two-dimensional shape appears as shown in the sketch below. The symmetrical element to be flux plotted is the same as for the strut, except the symmetry line is now an adiabat.

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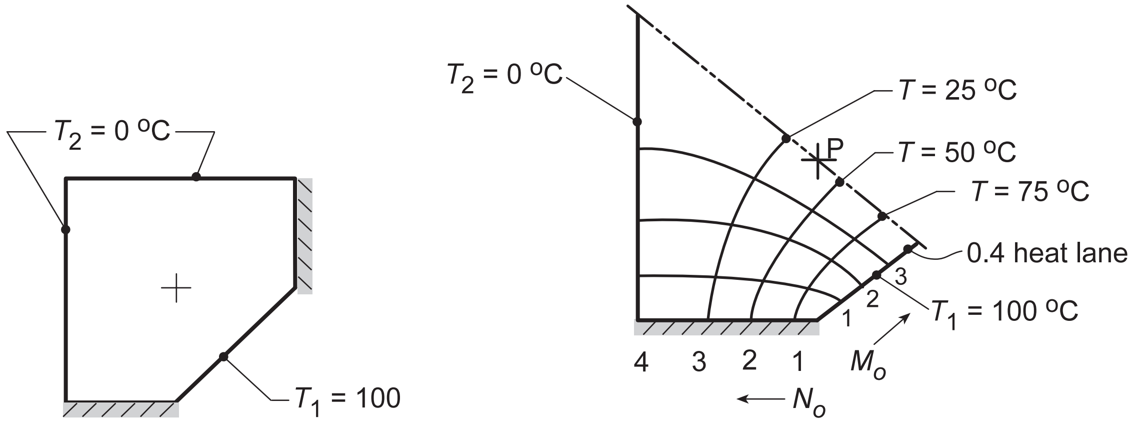
PROBLEM 4S.3 (Cont.)



From the flux plot, find M_o = 3.4 and N_o = 4, and from Equation (2)

$$S_o = M_o\ell/N_o = 3.4\ell/4 = 0.85\ell \qquad S = 2S_o = 1.70\ell$$

and the heat rate per unit length from Equation (1) is

$$q' = 75\,\text{W/m}\cdot\text{K} \times 1.70(100-0)^\circ\text{C} = 12,750\,\text{W/m}$$ <

From the flux plot, estimate that

T(P) ≈ 40°C. <

COMMENTS: (1) By inspection of the shapes for parts (a) and (b), it is obvious that the heat rate for the latter will be greater. The calculations show the heat rate is greater by more than a factor of three.

(2) By comparing the flux plots for the two configurations, and corresponding roles of the adiabats and isotherms, would you expect the shape factor for parts (a) to be the reciprocal of part (b)?