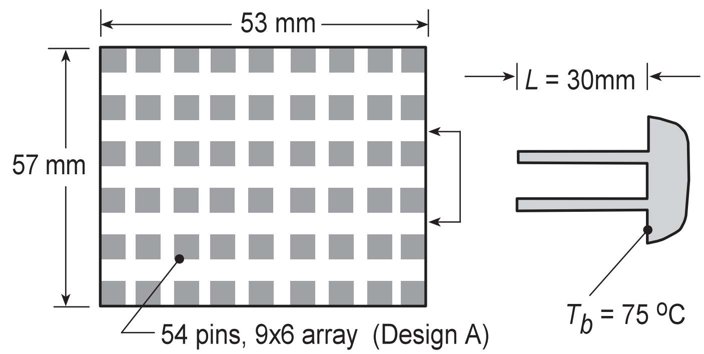
Problem 3.151

KNOWN: Two finned heat sinks, Designs A and B, prescribed by the number of fins in the array, N, fin dimensions of square cross-section, w, and length, L, with different convection coefficients, h.

FIND: Determine which fin arrangement is superior. Calculate the heat rate, q_f, efficiency, η_f, and effectiveness, ε_f, of a single fin, as well as, the total heat rate, q_t, and overall efficiency, η_o, of the array. Also, compare the total heat rates per unit volume.

SCHEMATIC:



Design	Fin dimensions Cross section w x w (mm)	Length L (mm)	Number of fins	Convection coefficient ($W/m^2 \cdot K$)
A	3 x 3	30	6 x 9	125
B	1 x 1	7	14 x 17	375

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Convection coefficient is uniform over fin and prime surfaces, (4) Fin tips experience convection, and (5) Constant properties.

ANALYSIS: Following the treatment of Section 3.6.5, the overall efficiency of the array, Eq. (3.103), is

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b} \tag{1}$$

where A_t is the total surface area, the sum of the exposed portion of the base (prime area) plus the fin surfaces, Eq. 3.104,

$$A_t = N \cdot A_f + A_b \tag{2}$$

where the surface area of a single fin and the prime area are

$$A_f = 4(L \times w) + w^2 \tag{3}$$

$$A_b = b1 \times b2 - N \cdot A_c \tag{4}$$

Combining Eqs. (1) and (2), the total heat rate for the array is

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \tag{5}$$

where η_f is the efficiency of a single fin. From Table 3.4, Case A, for the tip condition with convection, the single fin efficiency based upon Eq. 3.91,

$$\eta_f = \frac{q_f}{hA_f\theta_b} \tag{6}$$

Continued...

where

$$q_f = M\frac{\sinh(mL)+(h/mk)\cosh(mL)}{\cosh(mL)+(h/mk)\sinh(mL)} \tag{7}$$

$$M=(hPkA_c)^{1/2}\theta_b \qquad m=(hP/kA_c)^{1/2} \qquad P=4w \qquad A_c=w^2 \qquad (8,9,10)$$

The single fin effectiveness, from Eq. 3.86,

$$\varepsilon_f = \frac{q_f}{hA_c\theta_b} \tag{11}$$

Additionally, we want to compare the performance of the designs with respect to the array volume,

$$q_f''' = q_t/\forall = q_t/(b1 \cdot b2 \cdot L) \tag{12}$$

The above analysis was organized for easy treatment with equation-solving software. Solving Eqs. (1) through (11) simultaneously with appropriate numerical values, the results are tabulated below.

Design	q_t (W)	q_f (W)	η_o	η_f	ε_f	q_f''' (W/m^3)
A	113	1.80	0.804	0.779	31.9	1.25×10^6
B	165	0.475	0.909	0.873	25.3	7.81×10^6

COMMENTS: (1) Both designs have good efficiencies and effectiveness. Clearly, Design B is superior because the heat rate is nearly 50% larger than Design A for the same board footprint. Further, the space requirement for Design B is four times less ($\forall = 2.12\times10^{-5}$ vs. 9.06×10^{-5} m^3) and the heat rate per unit volume is 6 times greater.

(2) Design A features 54 fins compared to 238 fins for Design B. Also very significant to the performance comparison is the magnitude of the convection coefficient which is 3 times larger for Design B. Estimating convection coefficients for fin arrays (and tube banks) is discussed in Chapter 7.6. Of concern is how the upstream fins alter the flow past the downstream fins and whether the convection coefficient is uniform over the array.

(3) The *IHT Extended Surfaces Model*, for a *Rectangular Pin Fin Array* could have been used to solve this problem.