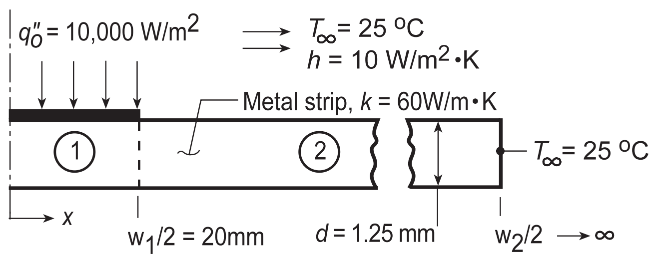
PROBLEM 3.115

KNOWN: Thin plastic film being bonded to a metal strip by laser heating method; strip dimensions and thermophysical properties are prescribed as are laser heating flux and convection conditions.

FIND: (a) Expression for temperature distribution for the region with the plastic strip, $-w_1/2 \le x \le w_1/2$, (b) Temperature at the center (x = 0) and the edge of the plastic strip ($x = \pm w_1/2$) when the laser flux is 10,000 W/m^2; (c) Plot the temperature distribution for the strip and point out special features.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction only, (3) Plastic film has negligible thermal resistance, (4) Upper and lower surfaces have uniform convection coefficients, (5) Edges of metal strip are at air temperature (T_∞), that is, strip behaves as infinite fin so that $w_2 \to \infty$, (6) All the incident laser heating flux q''_o is absorbed by the film, (7) Negligible radiation heat transfer.

PROPERTIES: Metal strip (given): ρ = 7850 kg/m^3, c_p = 435 J/kg·m^3, k = 60 W/m·K.

ANALYSIS: (a) The strip-plastic film arrangement can be modeled as an infinite fin of uniform cross section a portion of which is exposed to the laser heat flux on the upper surface. The general solutions for the two regions of the strip, in terms of $\theta \equiv T(x) - T_\infty$, are

$$0 \le x \le w_1/2 \qquad \theta_1(x) = C_1 e^{+mx} + C_2 e^{-mx} + M/m^2 \qquad (1)$$

$$M = q''_o P/2kA_c = q''_o/kd \qquad m = (2h/kd)^{1/2} \qquad (2,3)$$

$$w_1/2 \le x \le \infty \qquad \theta_2(x) = C_3 e^{+mx} + C_4 e^{-mx}. \qquad (4)$$

Four boundary conditions can be identified to evaluate the constants:

At x = 0: $$\frac{d\theta_1}{dx}(0) = 0 = C_1 m e^0 - C_2 m e^{-0} + 0 \quad \to \quad C_1 = C_2 \qquad (5)$$

At x = w_1/2: $$\theta(w_1/2) = \theta_2(w_1/2)$$

$$C_1 e^{+mw_1/2} + C_2 e^{-mw_1/2} + M/m^2 = C_3 e^{+mw_1/2} + C_4 e^{-mw_1/2} \qquad (6)$$

At x = w_1/2: $$d\theta_1(w_1/2)/dx = d\theta_2(w_1/2)/dx$$

$$mC_1 e^{+mw_1/2} - mC_2 e^{-mw_1/2} + 0 = mC_3 e^{+mw_1/2} - mC_4 e^{-mw_1/2} \qquad (7)$$

At x $\to \infty$: $$\theta_2(\infty) = 0 = C_3 e^{\infty} + C_4 e^{-\infty} \quad \to \quad C_3 = 0 \qquad (8)$$

With $C_3 = 0$ and $C_1 = C_2$, combine Eqs. (6 and 7) to eliminate C_4 to find

$$C_1 = C_2 = -\frac{M/m^2}{2e^{mw_1/2}}. \qquad (9)$$

and using Eq. (6) with Eq. (9) find

$$C_4 = M/m^2 \sinh(mw_1/2) e^{-mx_1/2} \qquad (10)$$

Continued...

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Hence, the temperature distribution in the region (1) under the plastic film, $0 \le x \le w_1/2$, is

$$\theta_1(x) = -\frac{M/m^2}{2e^{mw_1/2}}\left(e^{+mx} + e^{-mx}\right) + \frac{M}{m^2} = \frac{M}{m^2}\left(1 - e^{-mw_1/2}\cosh mx\right) \qquad (11) \quad <$$

and for the region (2), $x \ge w_1/2$,

$$\theta_2(x) = \frac{M}{m^2}\sinh\left(mw_1/2\right)e^{-mx} \qquad (12)$$

(b) Substituting numerical values into the temperature distribution expression above, $\theta_1(0)$ and $\theta_1(w_1/2)$ can be determined. First evaluate the following parameters:

$$M = 10,000\ \mathrm{W/m^2} \Big/ 60\ \mathrm{W/m\cdot K} \times 0.00125\ \mathrm{m} = 133,333\ \mathrm{K/m^2}$$

$$m = \left(2 \times 10\ \mathrm{W/m^2\cdot K} \Big/ 60\ \mathrm{W/m\cdot K} \times 0.00125\ \mathrm{m}\right)^{1/2} = 16.33\ \mathrm{m^{-1}}$$

Hence, for the midpoint $x = 0$,

$$\theta_1(0) = \frac{133,333\ \mathrm{K/m^2}}{\left(16.33\ \mathrm{m^{-1}}\right)^2}\left[1 - \exp\left(-16.33\ \mathrm{m^{-1}} \times 0.020\ \mathrm{m}\right) \times \cosh(0)\right] = 139.3\ \mathrm{K}$$

$$T_1(0) = \theta_1(0) + T_\infty = 139.3\ \mathrm{K} + 25^\circ\mathrm{C} = 164.3^\circ\mathrm{C}. \quad <$$

For the position $x = w_1/2 = 0.020$ m,

$$\theta_1(w_1/2) = 500.0\left[1 - 0.721\cosh\left(16.33\ \mathrm{m^{-1}} \times 0.020\ \mathrm{m}\right)\right] = 120.1\ \mathrm{K}$$

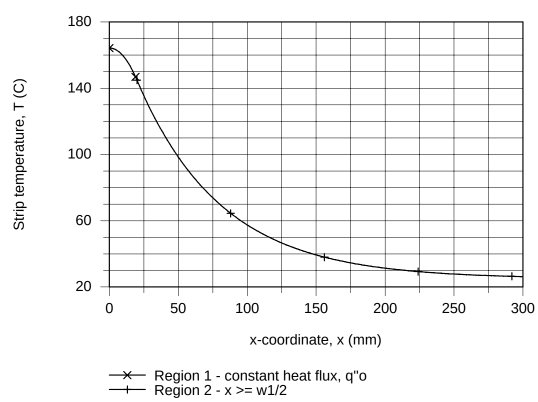
$$T_1(w_1/2) = 120.1\ \mathrm{K} + 25^\circ\mathrm{C} = 145.1^\circ\mathrm{C}. \quad <$$

(c) The temperature distributions, $\theta_1(x)$ and $\theta_2(x)$, are shown in the plot below. Using IHT, Eqs. (11) and (12) were entered into the workspace and a graph created. The special features are noted:

(1) No gradient at midpoint, $x = 0$; symmetrical distribution.

(2) No discontinuity of gradient at $w_1/2$ (20 mm).

(3) Temperature excess and gradient approach zero with increasing value of x.



COMMENTS: How wide must the strip be in order to satisfy the infinite fin approximation such that $\theta_2(x \to \infty) = 0$? For x = 200 mm, find $\theta_2(200\ \mathrm{mm}) = 6.3^\circ$C; this would be a poor approximation. When x = 300 mm, $\theta_2(300\ \mathrm{mm}) = 1.2^\circ$C; hence when $w_2/2 = 300$ mm, the strip is a reasonable approximation to an infinite fin.