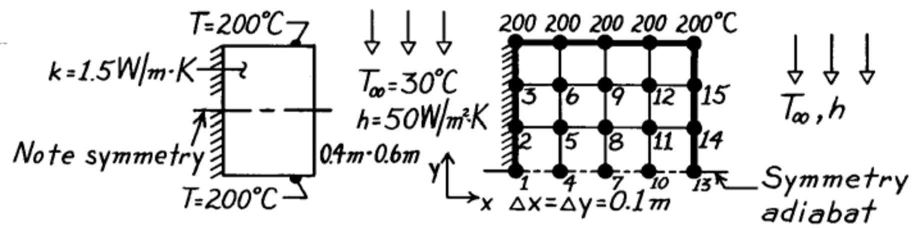
PROBLEM 4.73

KNOWN: Bar of rectangular cross-section subjected to prescribed boundary conditions.

FIND: Using a numerical technique with a grid spacing of 0.1m, determine the temperature distribution and the heat transfer rate from the bar to the fluid.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: The nodal network has $\Delta x = \Delta y = 0.1$m. Note the adiabat corresponding to system symmetry. The finite-difference equations for each node can be written using either Eq. 4.29, for interior nodes, or Eq. 4.42, for a plane surface with convection. In the case of adiabatic surfaces, Eq. 4.42 is used with h = 0. Note that

$$\frac{h\Delta x}{k} = \frac{50\,\text{W/m}^2\cdot\text{K}\times 0.1\text{m}}{1.5\ \text{W/m}\cdot\text{K}} = 3.333.$$

Node	*Finite-Difference Equations*
1	$-4T_1 + 2T_2 + 2T_4 = 0$
2	$-4T_2 + T_1 + T_3 + 2T_5 = 0$
3	$-4T_3 + 200 + 2T_6 + T_2 = 0$
4	$-4T_4 + T_1 + 2T_5 + T_7 = 0$
5	$-4T_5 + T_2 + T_6 + T_8 + T_4 = 0$
6	$-4T_6 + T_5 + T_3 + 200 + T_9 = 0$
7	$-4T_7 + T_4 + 2T_8 + T_{10} = 0$
8	$-4T_8 + T_7 + T_5 + T_9 + T_{11} = 0$
9	$-4T_9 + T_8 + T_6 + 200 + T_{12} = 0$
10	$-4T_{10} + T_7 + 2T_{11} + T_{13} = 0$
11	$-4T_{11} + T_{10} + T_8 + T_{12} + T_{14} = 0$
12	$-4T_{12} + T_{11} + T_9 + 200 + T_{15} = 0$
13	$2T_{10} + T_{14} + 6.666\times 30 - 10.666\ T_{13} = 0$
14	$2T_{11} + T_{13} + T_{15} + 6.666\times 30 - 2(3.333+2)T_{14} = 0$
15	$2T_{12} + T_{14} + 200 + 6.666\times 30 - 2(3.333+2)\ T_{15} = 0$

Using the matrix inversion method, Section 4.5.2, the above equations can be written in the form [A] [T] = [C] where [A] and [C] are shown on the next page. Using a stock matrix inversion routine, the temperatures [T] are determined.

Continued …

PROBLEM 4.73 (Cont.)

$$
[A]=\begin{bmatrix}
-4 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & -4 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & -4 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -4 & 1 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -10.66 & 2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & -10.66 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & -10.66
\end{bmatrix}
$$

$$
[C]=\begin{bmatrix} 0\\ 0\\ -200\\ 0\\ 0\\ -200\\ 0\\ 0\\ -200\\ 0\\ 0\\ -200\\ -200\\ -200\\ -400 \end{bmatrix}
\qquad
[T]=\begin{bmatrix} T_1\\ T_2\\ T_3\\ T_4\\ T_5\\ T_6\\ T_7\\ T_8\\ T_9\\ T_{10}\\ T_{11}\\ T_{12}\\ T_{13}\\ T_{14}\\ T_{15} \end{bmatrix}
=\begin{bmatrix} 153.9\\ 159.7\\ 176.4\\ 148.0\\ 154.4\\ 172.9\\ 129.4\\ 137.0\\ 160.7\\ 95.6\\ 103.5\\ 132.8\\ 45.8\\ 48.7\\ 67.0 \end{bmatrix}\left({}^\circ C\right)
$$

Considering symmetry, the heat transfer rate to the fluid is twice the convection rate from the surfaces of the control volumes exposed to the fluid. Using Newton's law of cooling, considering a unit thickness of the bar, find

$$q_{conv} = 2\left[h\cdot\frac{\Delta y}{2}\cdot\left(T_{13}-T_\infty\right)+h\cdot\Delta y\cdot\left(T_{14}-T_\infty\right)+h\cdot\Delta y\left(T_{15}-T_\infty\right)+h\cdot\frac{\Delta y}{2}\left(200-T_\infty\right)\right]$$

$$q_{conv} = 2h\cdot\Delta y\left[\frac{1}{2}\left(T_{13}-T_\infty\right)+\left(T_{14}-T_\infty\right)+\left(T_{15}-T_\infty\right)+\frac{1}{2}\left(200-T_\infty\right)\right]$$

$$q_{conv} = 2\times 50\frac{\text{W}}{\text{m}^2\cdot\text{K}}\times 0.1\text{m}\left[\frac{1}{2}\left(45.8-30\right)+\left(48.7-30\right)+\left(67.0-30\right)+\frac{1}{2}\left(200-30\right)\right]$$

$$q_{conv} = 1487 \text{ W/m.}$$ <