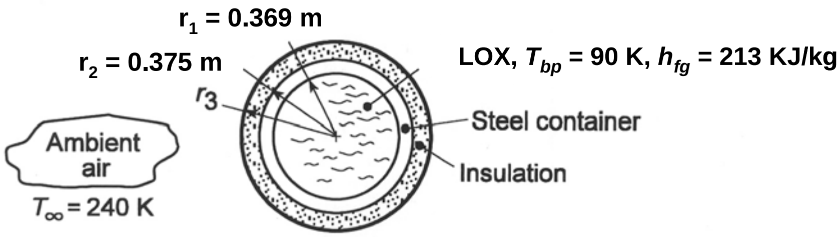
PROBLEM 3.68

KNOWN: Dimensions of spherical, stainless steel liquid oxygen (LOX) storage container. Boiling point and latent heat of fusion of LOX. Environmental temperature.

FIND: Thermal isolation system which maintains boil-off below 1 kg/day.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible thermal resistances associated with internal and external convection, conduction in the container wall, and contact between wall and insulation, (3) Negligible radiation at exterior surface (due to low emissivity insulation selected), (4) Constant insulation thermal conductivity.

PROPERTIES: *Table A.1*, 304 Stainless steel (T = 100 K): k_s = 9.2 W/m·K; *Table A.3*, Reflective, aluminum foil-glass paper insulation (T = 150 K): k_i = 0.000017 W/m·K (see choice of insulation below).

ANALYSIS: The heat gain associated with a loss of 1 kg/day is

$$q = \dot{m}h_{fg} = \frac{1\,\text{kg/day}}{86,400\,\text{s/day}}\left(2.13\times10^{5}\ \text{J/kg}\right) = 2.47\,\text{W}$$

With an overall temperature difference of $\left(T_\infty - T_{bp}\right)$ = 150 K, the corresponding total thermal resistance is

$$R_{tot} = \frac{\Delta T}{q} = \frac{150\,\text{K}}{2.47\,\text{W}} = 60.7\,\text{K/W}$$

The conduction resistance of the steel wall is

$$R_{t,cond,s} = \frac{1}{4\pi k_s}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi\left(9.2\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.369\,\text{m}} - \frac{1}{0.375\,\text{m}}\right) = 3.75\times10^{-4}\ \text{K/W}$$

With a typical combined radiation and convection heat transfer coefficient of h = 10 W/m^2·K, the resistance between the surface and the environment can be estimated as

$$R_{conv,rad} = \frac{1}{hA_s} = \frac{1}{10\ \text{W/m}^2\cdot\text{K}\times4\pi\left(0.375\,\text{m}\right)^2} = 0.0566\,\text{K/W}$$

It is clear that these resistances are insufficient, and reliance must be placed on the insulation. A special insulation of very low thermal conductivity should be selected. The best choice is a highly reflective foil/glass matted insulation which was developed for cryogenic applications. It follows that

$$R_{t,cond,i} = 60.7\,\text{K/W} = \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) = \frac{1}{4\pi\left(0.000017\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.375\,\text{m}} - \frac{1}{r_3}\right)$$

which yields r_3 = 0.37681 m. The minimum insulation thickness is therefore δ = (r_3 - r_2) = 1.8 mm.

COMMENTS: The heat loss could be reduced well below the maximum allowable by adding more insulation. Also, in view of weight restrictions associated with launching space vehicles, consideration should be given to fabricating the LOX container from a lighter material.