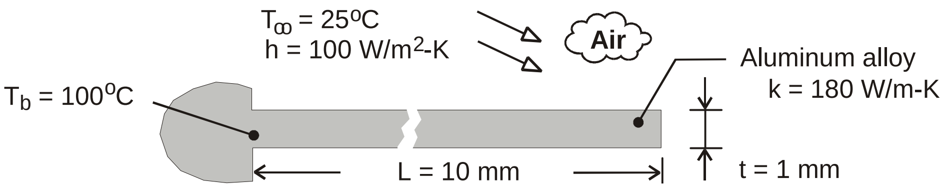
PROBLEM 3.133

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of fin length and thermal conductivity on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.77), (3.81) and (3.85), where $M \approx (2\,hw^2tk)^{1/2}\,(T_b - T_\infty) = (2 \times 100\ \text{W/m}^2\cdot\text{K} \times 0.001\text{m} \times 180\ \text{W/m}\cdot\text{K})^{1/2}\,(75°\text{C})\ w = 450\ w\ \text{W}$, $m \approx (2h/kt)^{1/2} = (200\ \text{W/m}^2\cdot\text{K}/180\ \text{W/m}\cdot\text{K} \times 0.001\text{m})^{1/2} = 33.3\text{m}^{-1}$, $mL \approx 33.3\text{m}^{-1} \times 0.010\text{m} = 0.333$, and $(h/mk) \approx (100\ \text{W/m}^2\cdot\text{K}/33.3\text{m}^{-1} \times 180\ \text{W/m}\cdot\text{K}) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.91), (3.86) and (3.88) yield

$$\eta_f \approx \frac{q_f'}{h(2L+t)\theta_b}, \quad \varepsilon_f \approx \frac{q_f'}{ht\,\theta_b}, \quad R_{t,f}' = \frac{\theta_b}{q_f'}$$

Case A: From Eq. (3.77), (3.91), (3.86), (3.88) and (3.75),

$$q_f' = \frac{M}{w}\,\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ \text{W/m}\,\frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\ \text{W/m} \qquad <$$

$$\eta_f = \frac{151\ \text{W/m}}{100\ \text{W/m}^2\cdot\text{K}\,(0.021\text{m})\,75°\text{C}} = 0.96 \qquad <$$

$$\varepsilon_f = \frac{151\ \text{W/m}}{100\ \text{W/m}^2\cdot\text{K}\,(0.001\text{m})\,75°\text{C}} = 20.1,\quad R_{t,f}' = \frac{75°\text{C}}{151\ \text{W/m}} = 0.50\ \text{m}\cdot\text{K/W} \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°\text{C} + \frac{75°\text{C}}{1.057 + (0.0167)\,0.340} = 95.6°\text{C} \qquad <$$

Case B: From Eqs. (3.81), (3.91), (3.86), (3.88) and (3.80)

$$q_f' = \frac{M}{w}\tanh mL = 450\ \text{W/m}\,(0.321) = 144\ \text{W/m} \qquad <$$

$$\eta_f = 0.92,\ \varepsilon_f = 19.2,\ R_{t,f}' = 0.52\ \text{m}\cdot\text{K/W} \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°\text{C} + \frac{75°\text{C}}{1.057} = 96.0°\text{C} \qquad <$$

Continued …..

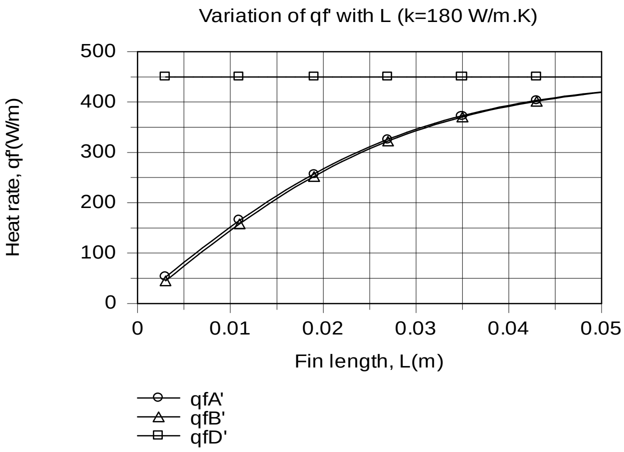
PROBLEM 3.133 (Cont.)

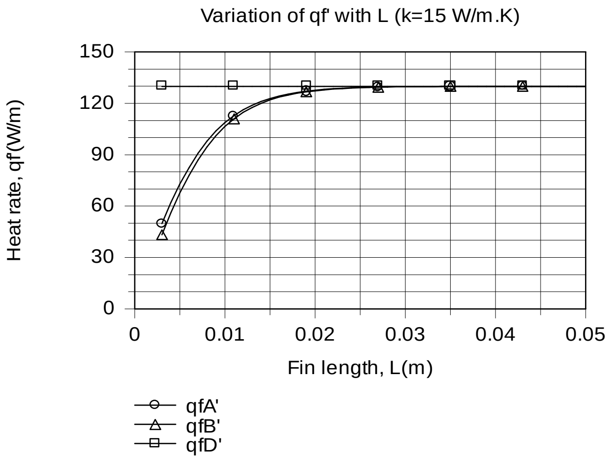
Case D (L $\rightarrow \infty$): From Eqs. (3.85), (3.91), (3.86), (3.88) and (3.84)

$$q'_f = \frac{M}{w} = 450\ \text{W/m} \quad <$$

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R'_{t,f} = 0.167\ \text{m}\cdot\text{K/W},\ T(L) = T_\infty = 25°\text{C} \quad <$$

(b) The effect of L on the heat rate is shown below for the aluminum and stainless steel fins.





For both materials, differences between the Case A and B results diminish with increasing L and are within 1% of each other at L ≈ 27 mm and L ≈ 13 mm for the aluminum and steel, respectively. At L = 3 mm, results differ by 14% and 13% for the aluminum and steel, respectively. The Case A and B results approach those of the infinite fin approximation more quickly for stainless steel due to the larger temperature gradients, |dT/dx|, for the smaller value of k.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q'_f, η_f and ε_f, as well as a larger value of $R'_{t,f}$) associated with insulating the tip.

Although $\eta_f = 0$ for the infinite fin, q'_f and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.