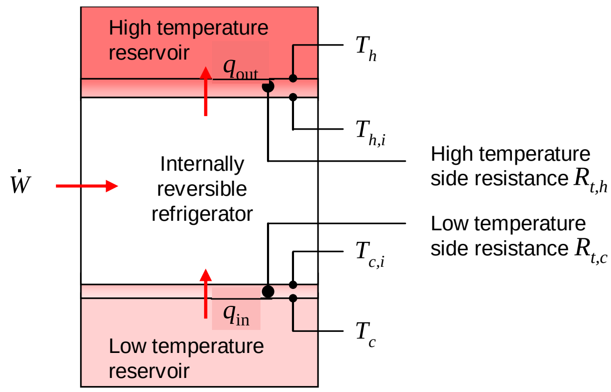
# PROBLEM 1.38

**KNOWN:** Hot and cold reservoir temperatures of an internally reversible refrigerator. Thermal resistances between refrigerator and hot and cold reservoirs.

**FIND:** Expressions for modified Coefficient of Performance and power input of refrigerator.

**SCHEMATIC:**




**ASSUMPTIONS**: (1) Refrigerator is internally reversible, (2) Steady-state operation.

**ANALYSIS:** Heat is transferred from the low temperature reservoir (the refrigerated space) at $T_c$ to the refrigerator unit, through the resistance $R_{t,c}$, with $T_c > T_{c,i}$. Heat is rejected from the refrigerator unit to the higher temperature reservoir (the surroundings), through the resistance $R_{t,h}$, with $T_{h,i} > T_h$. The heat input and output rates can be expressed in a manner analogous to Equations 1.18a and 1.18b.

$$q_{\text{in}} = (T_c - T_{c,i}) / R_{t,c} \tag{1}$$

$$q_{\text{out}} = (T_{h,i} - T_h) / R_{t,h} \tag{2}$$

Equations (1) and (2) can be solved for the internal temperatures, to yield

$$T_{h,i} = T_h + q_{\text{out}} R_{t,h} = T_h + q_{\text{in}} R_{t,h} \left( \frac{1 + \text{COP}_m}{\text{COP}_m} \right) \tag{3}$$

$$T_{c,i} = T_c - q_{\text{in}} R_{t,c} \tag{4}$$

In Equation (3), $q_{\text{out}}$ has been expressed as

$$q_{\text{out}} = q_{\text{in}} \left( \frac{1 + \text{COP}_m}{\text{COP}_m} \right) \tag{5}$$

using the definition of $\text{COP}_m$ given in the problem statement. The modified Coefficient of Performance can then be expressed as

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**PROBLEM 1.38 (Cont.)**

$$\mathrm{COP}_m = \frac{T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{T_c - q_{\text{in}} R_{t,c}}{T_h + q_{\text{in}} R_{t,h}\left(\dfrac{1+\mathrm{COP}_m}{\mathrm{COP}_m}\right) - T_c + q_{\text{in}} R_{t,c}}$$

Manipulating this expression,

$$\left(T_h - T_c + q_{\text{in}} R_{t,c}\right)\mathrm{COP}_m + q_{\text{in}} R_{t,h}(1+\mathrm{COP}_m) = T_c - q_{\text{in}} R_{t,c}$$

Solving for $\mathrm{COP}_m$ results in

$$\mathrm{COP}_m = \frac{T_c - q_{\text{in}} R_{\text{tot}}}{T_h - T_c + q_{\text{in}} R_{\text{tot}}}$$ <

From the definition of $\mathrm{COP}_m$, the power input can be determined:

$$\dot{W} = \frac{q_{\text{in}}}{\mathrm{COP}_m} = q_{\text{in}} \frac{T_h - T_c + q_{\text{in}} R_{\text{tot}}}{T_c - q_{\text{in}} R_{\text{tot}}}$$ <

**COMMENTS:** As $q_{\text{in}}$ or $R_{\text{tot}}$ goes to zero, the Coefficient of Performance approaches the maximum Carnot value, $\mathrm{COP}_m = \mathrm{COP}_C = T_c /(T_h - T_c)$.