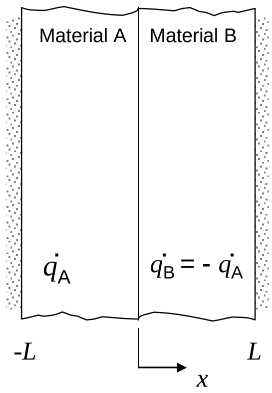
PROBLEM 2.67

KNOWN: Thickness of composite plane wall consisting of material A in left half and material B in right half. Exothermic reaction in material A and endothermic reaction in material B, with equal and opposite heat generation rates. External surfaces are insulated.

FIND: Sketch temperature and heat flux distributions for three thermal conductivity ratios, k_A/k_B.

SCHEMATIC:

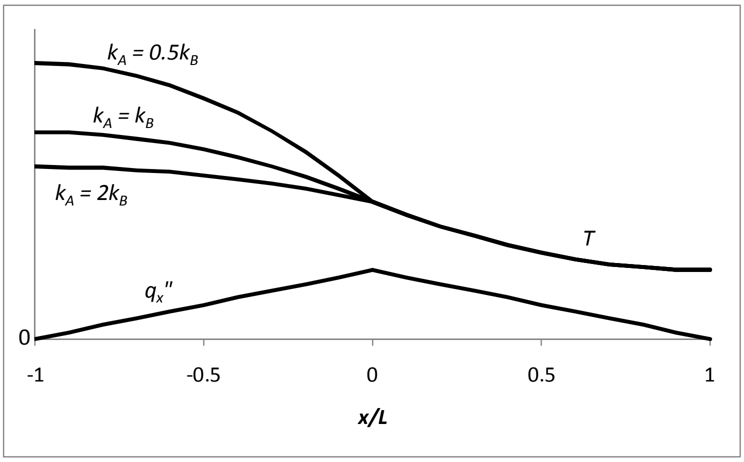


ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: From Equation 2.19 for steady-state, one-dimensional conduction, we find

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = -\dot{q} \quad \text{or} \quad \frac{\partial q_x''}{\partial x} = \dot{q}$$

From the second equation, with uniform heat generation rate, we see that q_x'' varies linearly with x, and its slope is $+\dot{q}_A$ in material A and $-\dot{q}_A$ in material B. Furthermore, since the wall is insulated on both exterior surfaces, the heat flux must be zero at $x = \pm L$. Thus, the heat flux is as shown in the graph below and does not depend on the thermal conductivities. The heat generated in the left half is conducting to the right and accumulating as it goes. Once it reaches the centerline, it begins to be consumed by the exothermic reaction and drops to zero at $x = L$.



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Since $q''_x = -k\dfrac{\partial T}{\partial x}$, the temperature gradient is negative everywhere, and its magnitude is greatest where the heat flux is greatest. Thus the slope of the temperature distribution is zero at $x = -L$, it becomes more negative as it reaches the center, and then becomes flatter again until it reaches a slope of zero at $x = L$. When $k_A = k_B$, the temperature distribution has equal and opposite slopes on either side of the centerline. If k_B is held fixed and k_A is varied, the results are as shown in the plot above. Since the temperature gradient is inversely proportional to the thermal conductivity, it is steeper in the region that has the smaller thermal conductivity. Physically, when thermal conductivity is larger, heat conducts more readily and causes the temperature to become more uniform.

If $\dot{q}_B = -2\dot{q}_A$, an energy balance on the wall gives:

$$\frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$$\frac{dE_{st}}{dt} = \dot{E}_g = (\dot{q}_A + \dot{q}_B)V = -\dot{q}_A V$$

where V is the volume. Since dE_{st}/dt is non-zero, the wall cannot be at steady-state. With the exothermic reaction greater than the endothermic reaction, the wall will continuously decrease in temperature. <

COMMENTS: (1) Given the information in the problem statement, it is not possible to calculate actual temperatures. There are an infinite number of correct solutions regarding temperature *values*, but only one correct solution regarding the *shape* of the temperature distribution. (2) Chemical reactions would cease if the temperature became too small. It would not be possible to continually cool the wall for the case when, initially, $\dot{q}_B = -2\dot{q}_A$.