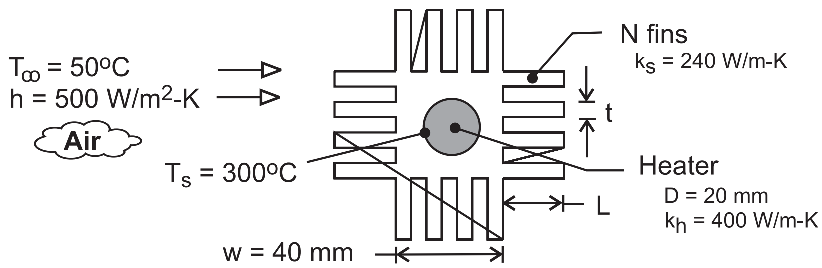
PROBLEM 4.36

KNOWN: Dimensions and thermal conductivities of a heater and a finned sleeve. Convection conditions on the sleeve surface.

FIND: (a) Heat rate per unit length, (b) Generation rate and centerline temperature of heater, (c) Effect of fin parameters on heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Constant properties, (3) Negligible contact resistance between heater and sleeve, (4) Uniform convection coefficient at outer surfaces of sleeve, (5) Uniform heat generation, (6) Negligible radiation.

ANALYSIS: (a) From the thermal circuit, the desired heat rate is

$$q' = \frac{T_s - T_\infty}{R'_{cond(2D)} + R'_{t,o}} = \frac{T_s - T_\infty}{R'_{tot}}$$

The two-dimensional conduction resistance, may be estimated from Eq. (4.21) and Case 6 of Table 4.2

$$R'_{cond(2D)} = \frac{1}{S' k_s} = \frac{\ln(1.08w/D)}{2\pi k_s} = \frac{\ln(2.16)}{2\pi(240\,W/m\cdot K)} = 5.11\times10^{-4}\,m\cdot K/W$$

The thermal resistance of the fin array is given by Equation (3.103), where η_o and A_t are given by Equations. (3.107) and (3.104) and η_f is given by Equation (3.94). With $L_c = L + t/2 = 0.022$ m, m = $(2h/k_s t)^{1/2} = 32.3\ m^{-1}$ and $mL_c = 0.710$,

$$\eta_f = \frac{\tanh\ mL_c}{mL_c} = \frac{0.61}{0.71} = 0.86$$

$$A'_t = NA'_f + A'_b = N(2L + t) + (4w - Nt) = 0.704m + 0.096m = 0.800m$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}(1 - \eta_f) = 1 - \frac{0.704m}{0.800m}(0.14) = 0.88$$

$$R'_{t,o} = (\eta_o h A'_t)^{-1} = \left(0.88 \times 500\,W/m^2\cdot K \times 0.80m\right)^{-1} = 2.84\times10^{-3} m\cdot K/W$$

$$q' = \frac{(300-50)°C}{\left(5.11\times10^{-4} + 2.84\times10^{-3}\right) m\cdot K/W} = 74,600\,W/m$$ <

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(b) Equation (3.60) may be used to determine $\dot{q}$, if h is replaced by an overall coefficient based on the surface area of the heater. From Equation (3.37),

$$U_s A_s' = U_s \pi D = \left(R_{tot}'\right)^{-1} = \left(3.35\times10^{-3}\,m\cdot K/W\right)^{-1} = 298\,W/m\cdot K$$

$$U_s = 298\,W/m\cdot K/\left(\pi\times0.02m\right) = 4750\,W/m^2\cdot K$$

$$\dot{q} = 4\,U_s\left(T_s - T_\infty\right)/D = 4\left(4750\,W/m^2\cdot K\right)\left(250°C\right)/0.02m = 2.38\times10^8\,W/m^3 \quad <$$

From Equation (3.58) the centerline temperature is

$$T(0) = \frac{\dot{q}\left(D/2\right)^2}{4k_h} + T_s = \frac{2.38\times10^8\,W/m^3\left(0.01m\right)^2}{4\left(400\,W/m\cdot K\right)} + 300°C = 315°C \quad <$$

(c) Subject to the prescribed constraints, the following results have been obtained for parameter variations corresponding to 16 ≤ N ≤ 40, 2 ≤ t ≤ 8 mm and 20 ≤ L ≤ 40 mm.

N	t(mm)	L(mm)	η_f	$q'(W/m)$
16	4	20	0.86	74,400
16	8	20	0.91	77,000
28	4	20	0.86	107,900
32	3	20	0.83	115,200
40	2	20	0.78	127,800
40	2	40	0.51	151,300

Clearly there is little benefit to simply increasing t, since there is no change in A_t' and only a marginal increase in η_f. However, due to an attendant increase in A_t', there is significant benefit to increasing N for fixed t (no change in η_f) and additional benefit in concurrently increasing N while decreasing t. In this case the effect of increasing A_t' exceeds that of decreasing η_f. The same is true for increasing L, although there is an upper limit at which diminishing returns would be reached. The upper limit to L could also be influenced by manufacturing constraints.

COMMENTS: Without the sleeve, the heat rate would be $q' = \pi Dh\left(T_s - T_\infty\right) = 7850\,W/m$, which is well below that achieved by using the increased surface area afforded by the sleeve.