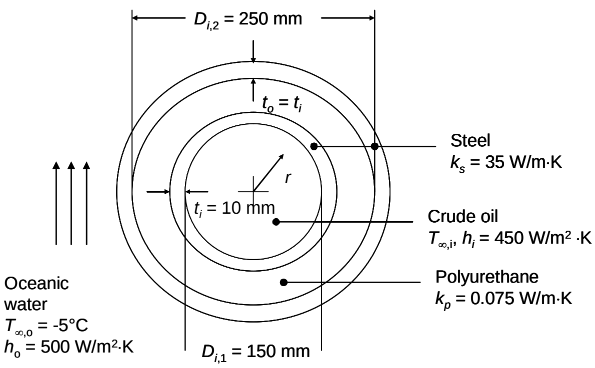
PROBLEM 3.47

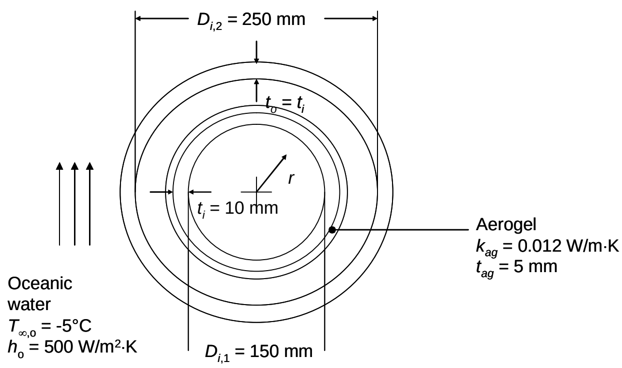
KNOWN: Dimensions of components of a pipe-in-pipe device. Thermal conductivity of materials, inner and outer heat transfer coefficients, outer fluid temperature.

FIND: (a) Maximum crude oil temperature to not exceed allowable service temperature of polyurethane. (b) Maximum crude oil temperature to not exceed allowable service temperature of polyurethane after insertion of aerogel layer.

SCHEMATIC:



Without Aerogel

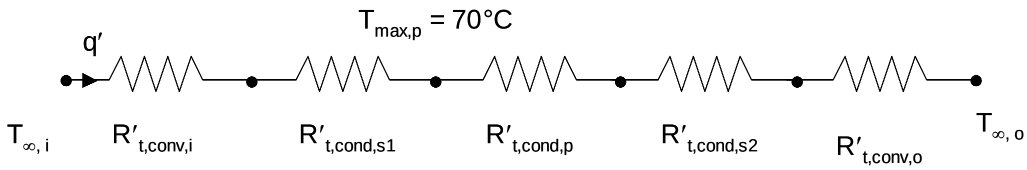


With Aerogel

ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible contact resistances, (3) Constant properties.

PROPERTIES: *Given*, Steel: k = 35 W/m·K; polyurethane: k = 0.075 W/m·K; aerogel: k = 0.012 W/m·K.

ANALYSIS: (a) The thermal resistance network for the case without the aerogel is shown below. The maximum polyurethane temperature occus at its inner surface.



Equating the heat rate per unit length of tubing from the crude oil to the inner surface of the polyurethane with the heat rate from the inner surface of the polyurethane to the oceanic waters yields

$$q' = \frac{T_{\infty,i} - T_{\max,p}}{R'_{t,\text{conv},i} + R'_{t,\text{cond},s1}} = \frac{T_{\max,p} - T_{\infty,o}}{R'_{t,\text{cond},p} + R'_{t,\text{cond},s2} + R'_{t,\text{conv},o}}$$

which may be rearranged to give

$$T_{\infty,i} = \frac{(T_{\max,p} - T_{\infty,o})(R'_{t,\text{conv},i} + R'_{t,\text{cond},s1})}{(R'_{t,\text{cond},p} + R'_{t,\text{cond},s2} + R'_{t,\text{conv},o})} + T_{\max,p} \qquad (1)$$

Continued…

PROBLEM 3.47 (Cont.)

The various thermal resistances are evaluated as follows.

$$R'_{t,\text{conv},i} = \frac{1}{450\text{W}/\text{m}^2 \cdot \text{K} \times \pi \times 0.150\text{m}} = 4.716 \times 10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} \;\; ; \;\; R'_{t,\text{cond},s1} = \frac{\ln\left[(150+20)/150\right]}{2\times\pi\times 35\text{W/m}\cdot\text{K}} = 569.2\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}}$$

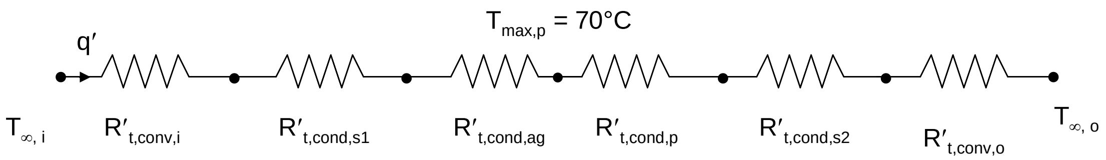
$$R'_{t,\text{cond},p} = \frac{\ln\left[250/(150+20)\right]}{2\times\pi\times 0.075\text{W/m}\cdot\text{K}} = 818.4\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} \;\; ; \;\; R'_{t,\text{cond},s2} = \frac{\ln\left[(250+20)/250\right]}{2\times\pi\times 35\text{W/m}\cdot\text{K}} = 350.0\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}}$$

$$R'_{t,\text{conv},o} = \frac{1}{500\text{W/m}^2 \cdot \text{K} \times \pi \times 0.270\text{m}} = 2.358\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}}$$

Substituting into Equation (1) yields

$$T_{\infty,i} = \frac{\left[70°\text{C} - (-5°\text{C})\right](4.716\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 569.2\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}})}{(818.4\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 350\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 2.358\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}})} + 70°\text{C} = 70.5°\text{C} \qquad <$$

(b) The thermal resistance network for the case with the aerogel is shown below.



The thermal resistance values are as before, except the conduction resistance per unit length in the polyurethane is decreased, since its thickness is reduced relative to part (a). In addition, the conduction resistance for the aerogel must be evaluated. These two resistances are:

$$R'_{t,\text{conv},ag} = \frac{\ln\left[(150+20+10)/(150+20)\right]}{2\times\pi\times 0.012\text{W/m}\cdot\text{K}} = 758\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} \;\; ; \;\; R'_{t,\text{cond},p} = \frac{\ln\left[250/180\right]}{2\times\pi\times 0.075\text{W/m}\cdot\text{K}} = 697\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}}$$

Incorporating the aerogel resistance, Equation (1) becomes

$$T_{\infty,i} = \frac{(T_{\max,p} - T_{\infty,o})(R'_{t,\text{conv},i} + R'_{t,\text{cond},s1} + R'_{t,\text{cond},ag})}{(R'_{t,\text{cond},p} + R'_{t,\text{cond},s2} + R'_{t,\text{conv},o})} + T_{\max,p}$$

Substituting values yields

Continued…

PROBLEM 3.47 (Cont.)

$$T_{\infty,i} = \frac{\left[70°\text{C} - (-5°\text{C})\right](4.716\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 569.2\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 758\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}})}{(697\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 350\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 2.358\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}})} + 70°\text{C}$$

$= 151.8°\text{C}$ <

COMMENTS: Assuming the dynamic viscosity of crude oil is similar to that of engine oil, we may evaluate the viscosity of the oil at the two maximum operating temperatures. From Table A-5 at T = 70.5°C = 343 K, μ = 0.046 N·s/m^2. At T = 151.8°C = 425 K, μ = 0.517 N·s/m^2. The viscosity of the oil with the aerogel insulation is 0.046/0.517 = 0.09, or only 9% of the viscosity of the oil without the aerogel. Savings in pumping costs and/or increases in oil production rates could be realized with use of the aerogel pipe-in-pipe concept.