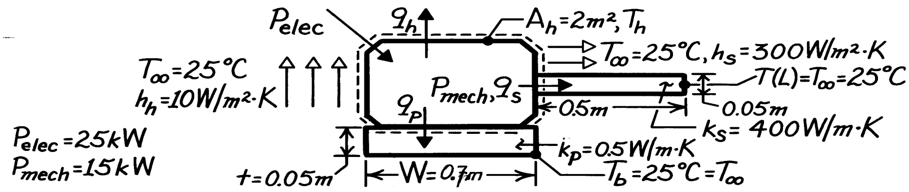
PROBLEM 3.117

KNOWN: Electric power input and mechanical power output of a motor. Dimensions of housing, mounting pad and connecting shaft needed for heat transfer calculations. Temperature of ambient air, tip of shaft, and base of pad.

FIND: Housing temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pad and shaft, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Conservation of energy yields

$$P_{elec} - P_{mech} - q_h - q_p - q_s = 0$$

$$q_h = h_h A_h (T_h - T_\infty), \qquad q_p = k_p W^2 \frac{(T_h - T_\infty)}{t}, \qquad q_s = M \frac{\cosh mL - \theta_L/\theta_b}{\sinh mL}$$

$$\theta_L = 0, \qquad mL = \left(4h_s L^2 / k_s D\right)^{1/2}, \qquad M = \left(\frac{\pi^2}{4} D^3 h_s k_s\right)^{1/2} (T_h - T_\infty).$$

Hence

$$q_s = \frac{\left(\left[\pi^2/4\right] D^3 h_s k_s\right)^{1/2} (T_h - T_\infty)}{\tanh\left(4h_s L^2/k_s D\right)^{1/2}}$$

Substituting, and solving for (T_h - T_∞),

$$T_h - T_\infty = \frac{P_{elec} - P_{mech}}{h_h A_h + k_p W^2/t + \left(\left(\pi^2/4\right) D^3 h_s k_s\right)^{1/2} / \tanh\left(4h_s L^2/k_s D\right)^{1/2}}$$

$$\left(\left(\pi^2/4\right) D^3 h_s k_s\right)^{1/2} = 6.08 \text{ W/K}, \qquad \left(4h_s L^2/k_s D\right)^{1/2} = 3.87, \qquad \tanh mL = 0.999$$

$$T_h - T_\infty = \frac{(25-15)\times 10^3 \text{ W}}{\left[10\times 2 + 0.5(0.7)^2/0.05 + 6.08/0.999\right]\text{W/K}} = \frac{10^4 \text{ W}}{(20+4.90+6.15)\text{W/K}}$$

$$T_h - T_\infty = 322.1\text{K} \qquad T_h = 347.1^\circ\text{C}$$ <

COMMENTS: (1) T_h is large enough to provide significant heat loss by radiation from the housing. Assuming an emissivity of 0.8 and surroundings at 25°C, $q_{rad} = \varepsilon A_h \left(T_h^4 - T_{sur}^4\right)$ = 4347 W, which compares with $q_{conv} = hA_h (T_h - T_\infty) = 5390$ W. Radiation has the effect of decreasing T_h. (2) The infinite fin approximation, q_s = M, is excellent.