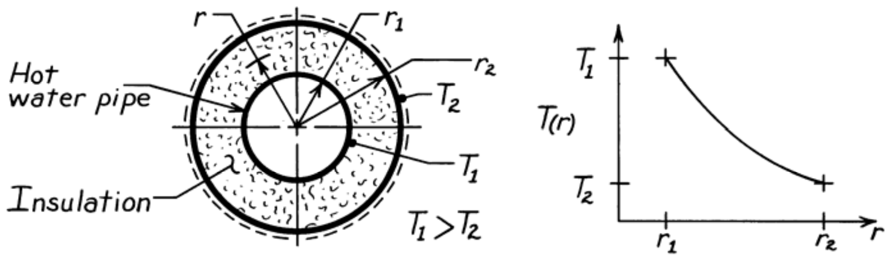
PROBLEM 2.3

KNOWN: Hot water pipe covered with thick layer of insulation.

FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k\left(2\pi r\ell\right)\frac{dT}{dr}$$

where $A_r = 2\pi r\ell$ and ℓ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $\dot{E}_{in} = \dot{E}_{out}$ since $\dot{E}_g = \dot{E}_{st} = 0.$ Hence

$$q_r = \text{Constant.}$$

That is, q_r is independent of radius (r). Since the thermal conductivity is also constant, it follows that

$$r\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius, r, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch.

COMMENTS: (1) Note that, while q_r is a constant and independent of r, q''_r is not a constant. How does $q''_r(r)$ vary with r? (2) Recognize that the radial temperature gradient, dT/dr, decreases with increasing radius.