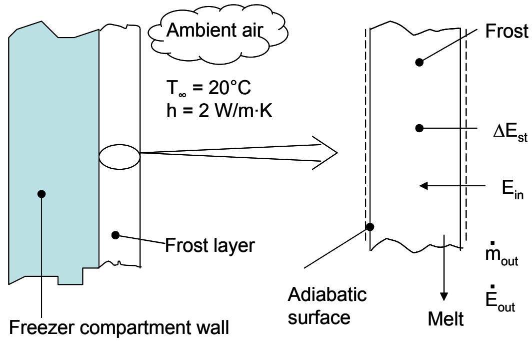
PROBLEM 1.65

KNOWN: Frost formation of 3-mm thickness on a freezer compartment. Surface exposed to convection process with ambient air.

FIND: Time required for the frost to melt, t_m.

SCHEMATIC:



ASSUMPTIONS: (1) Frost is isothermal at the fusion temperature, T_f, (2) The water melt falls away from the exposed surface, (3) Frost exchanges radiation with surrounding frost, so net radiation exchange is negligible, and (4) Backside surface of frost formation is adiabatic.

PROPERTIES: Frost, $\rho_f = 770\,\text{kg/m}^3$, $h_{sf} = 334\,\text{kJ/kg}$.

ANALYSIS: The time t_m required to melt a 2-mm thick frost layer may be determined by applying a mass balance and an energy balance (Eq 1.12b) over the differential time interval dt to a control volume around the frost layer.

$$dm_{st} = -\dot{m}_{out}dt \qquad dE_{st} = \left(\dot{E}_{in} - \dot{E}_{out}\right)dt \qquad (1a,b)$$

With h_f as the enthalpy of the melt and h_s as the enthalpy of frost, we have

$$dE_{st} = dm_{st}h_s \qquad \dot{E}_{out}dt = \dot{m}_{out}h_f\,dt \qquad (2a,b)$$

Combining Eqs. (1a) and (2a,b), Eq. (1b) becomes (with $h_{sf} = h_f - h_S$)

$$\dot{m}_{out}h_{sf}\,dt = \dot{E}_{in}dt = q''_{conv}A_s dt$$

Integrating both sides of the equation with respect to time, find

$$\rho_f A_s h_{sf} x_o = h A_s \left(T_\infty - T_f\right) t_m$$

$$t_m = \frac{\rho_f h_{sf} x_o}{h\left(T_\infty - T_f\right)}$$

$$t_m = \frac{700\,\text{kg/m}^3 \times 334\times 10^3\,\text{J/kg}\times 0.003\text{m}}{2\,\text{W/m}^2\cdot\text{K}\left(20-0\right)\text{K}} = 17{,}540\text{ s} = 4.9\text{ h}$$ <

COMMENTS: (1) The energy balance could be formulated intuitively by recognizing that the total heat *in* by convection during the time interval $t_m\left(q''_{conv}\cdot t_m\right)$ must be equal to the total latent energy for melting the frost layer $\left(\rho\, x_o h_{sf}\right)$. This equality is directly comparable to the derived expression above for t_m.