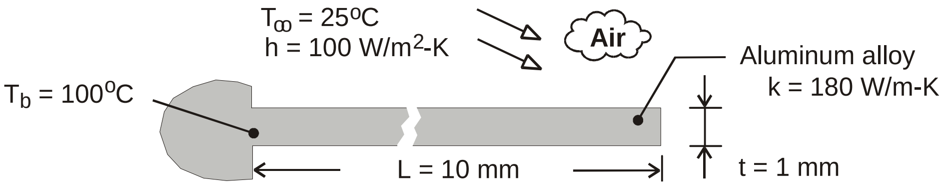
PROBLEM 3.131

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of convection coefficient and thermal conductivity on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.77), (3.81) and (3.85), where $M \approx (2\ hw^2tk)^{1/2}\ (T_b - T_\infty) = (2 \times 100\ W/m^2 \cdot K \times 0.001m \times 180\ W/m \cdot K)^{1/2}\ (75°C)\ w$ = 450 w W, $m \approx (2h/kt)^{1/2} = (200\ W/m^2 \cdot K/180\ W/m \cdot K \times 0.001m)^{1/2} = 33.3m^{-1}$, $mL \approx 33.3m^{-1} \times 0.010m = 0.333$, and $(h/mk) \approx (100\ W/m^2 \cdot K/33.3m^{-1} \times 180\ W/m \cdot K) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.91), (3.86) and (3.88) yield

$$\eta_f \approx \frac{q_f'}{h(2L+t)\theta_b}, \quad \varepsilon_f \approx \frac{q_f'}{ht\theta_b}, \quad R_{t,f}' = \frac{\theta_b}{q_f'}$$

Case A: From Eq. (3.77), (3.91), (3.86), (3.88) and (3.75),

$$q_f' = \frac{M}{w}\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\,W/m\,\frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\,W/m \qquad <$$

$$\eta_f = \frac{151\,W/m}{100\,W/m^2 \cdot K(0.021m)75°C} = 0.96 \qquad <$$

$$\varepsilon_f = \frac{151\,W/m}{100\,W/m^2 \cdot K(0.001m)75°C} = 20.2,\ R_{t,f}' = \frac{75°C}{151\,W/m} = 0.50\,m \cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°C + \frac{75°C}{1.057 + (0.0167)0.340} = 95.6°C \qquad <$$

Case B: From Eqs. (3.81), (3.91), (3.86), (3.88) and (3.80)

$$q_f' = \frac{M}{w}\tanh mL = 450\,W/m(0.321) = 144\,W/m \qquad <$$

$$\eta_f = 0.92,\ \varepsilon_f = 19.3,\ R_{t,f}' = 0.52\ m \cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°C + \frac{75°C}{1.057} = 96.0°C \qquad <$$

Continued …

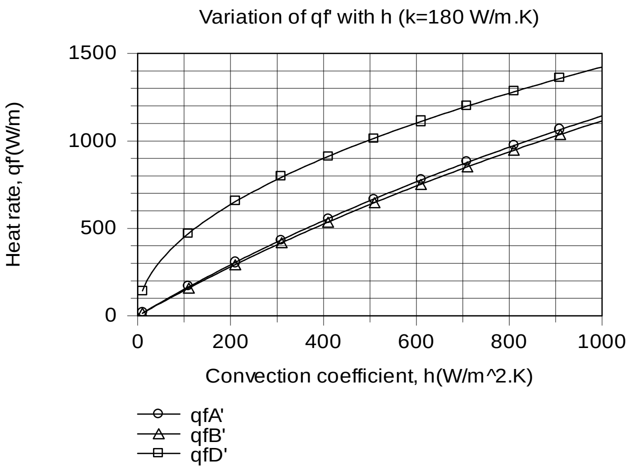
PROBLEM 3.131 (Cont.)

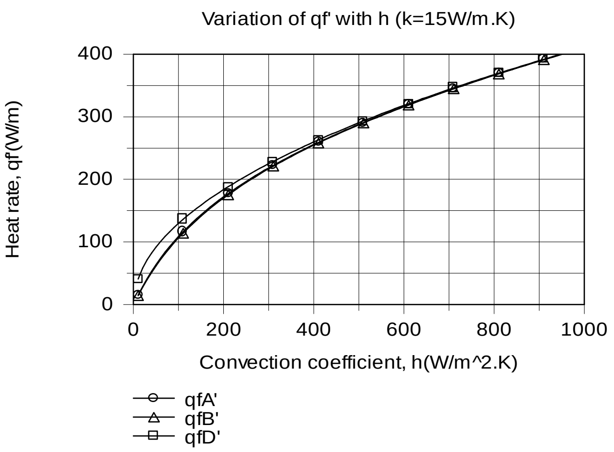
Case D ($L \rightarrow \infty$): From Eqs. (3.85), (3.91), (3.86), (3.88) and (3.84)

$$q_f' = \frac{M}{w} = 450\ \mathrm{W/m} \qquad <$$

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R_{t,f}' = 0.167\ \mathrm{m \cdot K/W},\ T(L) = T_\infty = 25°\mathrm{C} \qquad <$$

(b) The effect of h on the heat rate is shown below for the aluminum and stainless steel fins.





For both materials, there is little difference between the Case A and B results over the entire range of h. The difference (percentage) increases with decreasing h and increasing k, but even for the worst case condition ($h = 10\ \mathrm{W/m^2 \cdot K}$, $k = 180\ \mathrm{W/m \cdot K}$), the heat rate for Case A (15.7 W/m) is only slightly larger than that for Case B (14.9 W/m). For aluminum, the heat rate is significantly over-predicted by the infinite fin approximation over the entire range of h. For stainless steel, it is over-predicted for small values of h, but results for all three cases are within 1% for $h > 500\ \mathrm{W/m^2 \cdot K}$.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q_f', η_f and ε_f, as well as a larger value of $R_{t,f}'$) associated with insulating the tip.

Although $\eta_f = 0$ for the infinite fin, q_f' and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.