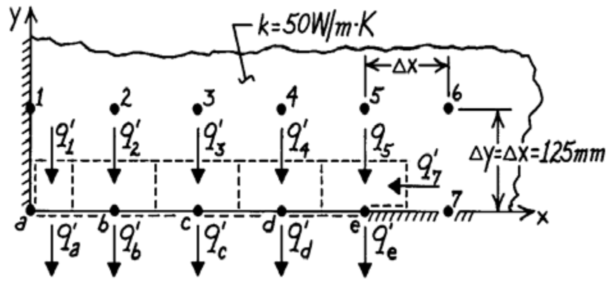
PROBLEM 4.50

KNOWN: Two-dimensional network with prescribed nodal temperatures and thermal conductivity of the material.

FIND: Heat rate per unit length normal to page, q'.

SCHEMATIC:



Node	T_i(°C)
1	120.55
2	120.64
3	121.29
4	123.89
5	134.57
6	150.49
7	147.14

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: Construct control volumes around the nodes on the surface maintained at the uniform temperature T_s and indicate the heat rates. The heat rate per unit length is $q' = q'_a + q'_b + q'_c + q'_d + q'_e$ or in terms of conduction terms between nodes,

$$q' = q'_1 + q'_2 + q'_3 + q'_4 + q'_5 + q'_7.$$

Each of these rates can be written in terms of nodal temperatures and control volume dimensions using Fourier's law,

$$q' = k\cdot\frac{\Delta x}{2}\cdot\frac{T_1 - T_s}{\Delta y} + k\cdot\Delta x\cdot\frac{T_2 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_3 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_4 - T_s}{\Delta y}$$
$$+ k\cdot\Delta x\frac{T_5 - T_s}{\Delta y} + k\cdot\frac{\Delta y}{2}\cdot\frac{T_7 - T_s}{\Delta x}.$$

and since $\Delta x = \Delta y$,

$$q' = k[(1/2)(T_1 - T_s) + (T_2 - T_s) + (T_3 - T_s)$$
$$+ (T_4 - T_s) + (T_5 - T_s) + (1/2)(T_7 - T_s)].$$

Substituting numerical values, find

$$q' = 50\ \text{W/m}\cdot\text{K}[(1/2)(120.55 - 100) + (120.64 - 100) + (121.29 - 100)$$
$$+ (123.89 - 100) + (134.57 - 100) + (1/2)(147.14 - 100)]$$

$$q' = 6711\ \text{W/m}.$$ <

COMMENTS: For nodes a through d, there is no heat transfer into the control volumes in the x-direction. Look carefully at the energy balance for node e, $q'_e = q'_5 + q'_7$, and how q'_5 and q'_7 are evaluated.