PROBLEM 1.33

KNOWN: Exact and approximate expressions for the linearized radiation coefficient, h_r and h_{ra}, respectively.

FIND: (a) Comparison of the coefficients with $\varepsilon = 0.05$ and 0.9 and surface temperatures which may exceed that of the surroundings ($T_{sur} = 25°C$) by 10 to 100°C; also comparison with a free convection coefficient correlation, (b) Plot of the relative error $(h_r - r_{ra})/h_r$ as a function of the furnace temperature associated with a workpiece at $T_s = 25°C$ having $\varepsilon = 0.05$, 0.2 or 0.9.

ASSUMPTIONS: (1) Furnace walls are large compared to the workpiece and (2) Steady-state conditions.

ANALYSIS: (a) The linearized radiation coefficient, Eq. 1.9, follows from the radiation exchange rate equation,

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$

If $T_s \approx T_{sur}$, the coefficient may be approximated by the simpler expression

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3 \qquad \bar{T} = \left(T_s + T_{sur}\right)/2$$

For the condition of $\varepsilon = 0.05$, $T_s = T_{sur} + 10 = 35°C = 308$ K and $T_{sur} = 25°C = 298$ K, find that

$$h_r = 0.05 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\left(308 + 298\right)\left(308^2 + 298^2\right)\mathrm{K^3} = 0.32\,\mathrm{W/m^2 \cdot K} \quad <$$

$$h_{r,a} = 4 \times 0.05 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\left(\left(308 + 298\right)/2\right)^3\mathrm{K^3} = 0.32\,\mathrm{W/m^2 \cdot K} \quad <$$

The free convection coefficient with $T_s = 35°C$ and $T_\infty = T_{sur} = 25°C$, find that

$$h = 0.98\Delta T^{1/3} = 0.98\left(T_s - T_\infty\right)^{1/3} = 0.98\left(308 - 298\right)^{1/3} = 2.1\,\mathrm{W/m^2 \cdot K} \quad <$$

For the range $T_s - T_{sur} = 10$ to 100°C with $\varepsilon = 0.05$ and 0.9, the results for the coefficients are tabulated below. For this range of surface and surroundings temperatures, the radiation and free convection coefficients are of comparable magnitude for moderate values of the emissivity, say $\varepsilon > 0.2$. The approximate expression for the linearized radiation coefficient is valid within 2% for these conditions.

(b) The above expressions for the radiation coefficients, h_r and $h_{r,a}$, are used for the workpiece at $T_s = 25°C$ placed inside a furnace with walls which may vary from 100 to 1000°C. The relative error, $(h_r - h_{ra})/h_r$, will be independent of the surface emissivity and is plotted as a function of T_{sur}. For $T_{sur} > 200°C$, the approximate expression provides estimates which are in error more than 5%. The approximate expression should be used with caution, and only for surface and surrounding temperature differences of 50 to 100°C.

		Coefficients ($W/m^2 \cdot K$)		
T_s (°C)	ε	h_r	$h_{r,a}$	h
35	0.05	0.32	0.32	2.1
	0.9	5.7	5.7	
135	0.05	0.51	0.50	4.7
	0.9	9.2	9.0