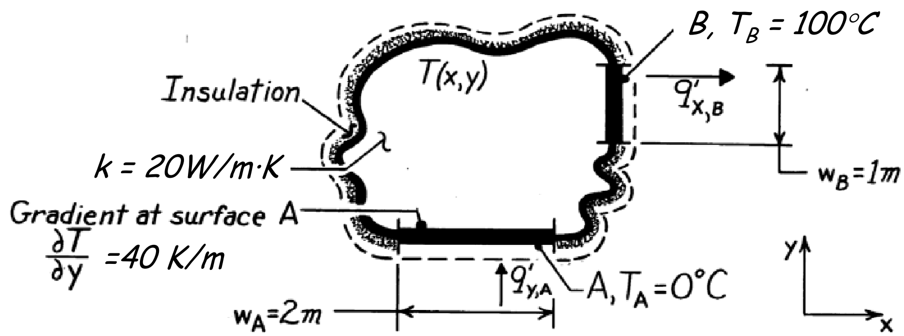
PROBLEM 2.14

KNOWN: Two-dimensional body with specified thermal conductivity and two isothermal surfaces of prescribed temperatures; one surface, A, has a prescribed temperature gradient.

FIND: Temperature gradients, $\partial T/\partial x$ and $\partial T/\partial y$, at the surface B.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) No heat generation, (4) Constant properties.

ANALYSIS: At the surface A, the temperature gradient in the x-direction must be zero. That is, $(\partial T/\partial x)_A = 0$. This follows from the requirement that the heat flux vector must be normal to an isothermal surface. The heat rate at the surface A is given by Fourier's law written as

$$q'_{y,A} = -k \cdot w_A \left.\frac{\partial T}{\partial y}\right]_A = -20\frac{W}{m \cdot K} \times 2m \times 40\frac{K}{m} = -1600 W/m.$$

On the surface B, it follows that

$$\left(\partial T/\partial y\right)_B = 0$$ <

in order to satisfy the requirement that the heat flux vector be normal to the isothermal surface B. Using the conservation of energy requirement, Eq. 1.12c, on the body, find

$$q'_{y,A} - q'_{x,B} = 0 \qquad \text{or} \qquad q'_{x,B} = q'_{y,A}.$$

Note that,

$$q'_{x,B} = -k \cdot w_B \left.\frac{\partial T}{\partial x}\right]_B$$

and hence

$$\left(\partial T/\partial x\right)_B = \frac{-q'_{y,A}}{k \cdot w_B} = \frac{-\left(-1600 \text{ W/m}\right)}{20 \text{ W/m} \cdot \text{K} \times 1\text{m}} = 80 \text{ K/m}.$$ <

COMMENTS: Note that, in using the conservation requirement, $q'_{in} = +q'_{y,A}$ and $q'_{out} = +q'_{x,B}$.