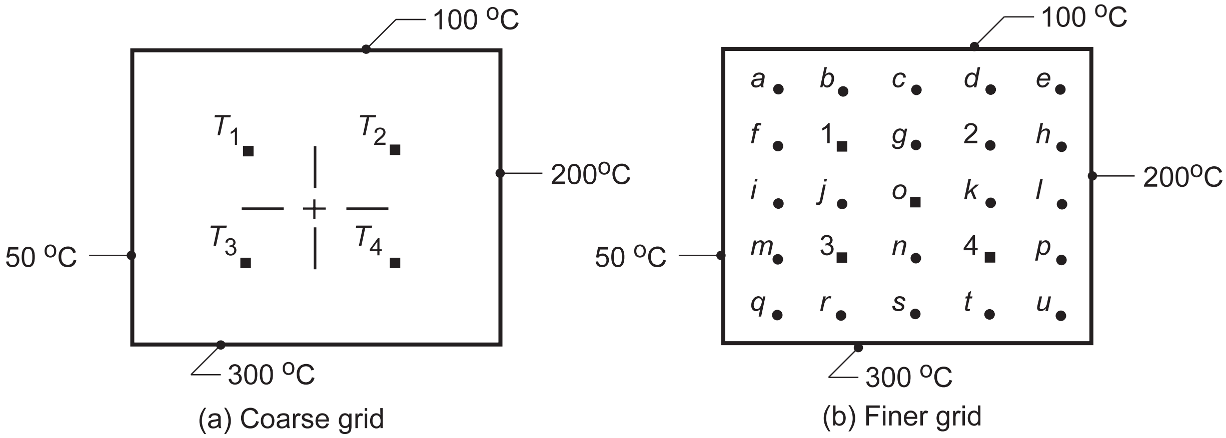
PROBLEM 4.64

KNOWN: Square shape subjected to uniform surface temperature conditions.

FIND: (a) Temperature at the four specified nodes; estimate the midpoint temperature T_o, (b) Reducing the mesh size by a factor of 2, determine the corresponding nodal temperatures and compare results, and (c) For the finer grid, plot the 75, 150, and 250°C isotherms.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equation for each node follows from Eq. 4.29 for an interior point written in the form, $T_i = 1/4\sum T_{neighbors}$. Using the Gauss-Seidel iteration method, Section 4.5.2, the finite-difference equations for the four nodes are:

$$T_1^k = 0.25\left(100 + T_2^{k-1} + T_3^{k-1} + 50\right) = 0.25T_2^{k-1} + 0.25T_3^{k-1} + 37.5$$

$$T_2^k = 0.25\left(100 + 200 + T_4^{k-1} + T_1^{k-1}\right) = 0.25T_1^{k-1} + 0.25T_4^{k-1} + 75.0$$

$$T_3^k = 0.25\left(T_1^{k-1} + T_4^{k-1} + 300 + 50\right) = 0.25T_1^{k-1} + 0.25T_4^{k-1} + 87.5$$

$$T_4^k = 0.25\left(T_2^{k-1} + 200 + 300 + T_3^{k-1}\right) = 0.25T_2^{k-1} + 0.25T_3^{k-1} + 125.0$$

The iteration procedure using a hand calculator is implemented in the table below. Initial estimates are entered on the k = 0 row.

k	T_1 (°C)	T_2 (°C)	T_3 (°C)	T_4 (°C)
0	100	150	150	250
1	112.50	165.63	178.13	210.94
2	123.44	158.60	171.10	207.43
3	119.93	156.40	169.34	206.55
4	119.05	156.40	168.90	206.33
5	118.83	156.29	168.79	206.27
6	118.77	156.26	168.76	206.26
7	118.76	156.25	168.76	206.25

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PROBLEM 4.64 (Cont.)

By the seventh iteration, the convergence is approximately 0.01°C. The midpoint temperature can be estimated as

$$T_o = (T_1 + T_2 + T_3 + T_4)/2 = (118.76 + 156.25 + 168.76 + 206.25)^\circ C/4 = 162.5^\circ C$$

(b) Because all the nodes are interior ones, the nodal equations can be written by inspection directly into the IHT workspace and the set of equations solved for the nodal temperatures (°C).

Mesh	T_o	T_1	T_2	T_3	T_4
Coarse	162.5	118.8	156.3	168.8	206.3
Fine	162.5	117.4	156.1	168.9	207.6

The maximum difference for the interior points is 1.4°C (node 1), but the estimate at the center, T_o, is the same, independently of the mesh size. In terms of the boundary surface temperatures,

$$T_o = (50 + 100 + 200 + 300)^\circ C/4 = 162.5^\circ C$$

Why must this be so?

(c) To generate the isotherms, it would be necessary to employ a contour-drawing routine using the tabulated temperature distribution (°C) obtained from the finite-difference solution. Using these values as a guide, try sketching a few isotherms.

-	100	100	100	100	100	-
50	86.0	105.6	119	131.7	151.6	200
50	88.2	117.4	138.7	156.1	174.6	200
50	99.6	137.1	162.5	179.2	190.8	200
50	123.0	168.9	194.9	207.6	209.4	200
50	173.4	220.7	240.6	246.8	239.0	200
-	300	300	300	300	300	-

COMMENTS: Recognize that this finite-difference solution is only an approximation to the temperature distribution, since the heat conduction equation has been solved for only four (or 25) discrete points rather than for all points if an analytical solution had been obtained.