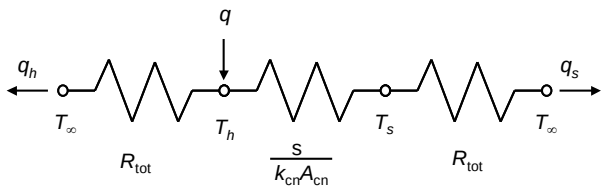
PROBLEM 3.11

KNOWN: Temperature of the heating island and sensing island, as well as the surrounding silicon nitride wafer temperature of Example 3.4.

FIND: The thermal conductivity of the carbon nanotube, k_{cn}, for the conditions of the problem statement and T_h = 332.6 K, without evaluating the thermal resistances of the supports.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer, (4) Isothermal heating and sensing islands, (5) Negligible radiation and convection effects.

ANALYSIS: We begin by defining an *excess temperature*, $\theta \equiv T - T_\infty$ and modifying the thermal circuit as shown in the schematic. In the modified circuit, the total thermal resistance, R_{tot}, represents the combined effects of the two beams that support either the heated island or the sensing island.

From the modified thermal circuit, it is evident that an expression for R_{tot} can be derived as

$$q = q_h + q_s = \frac{T_h - T_\infty}{R_{tot}} + \frac{T_s - T_\infty}{R_{tot}} = \frac{\theta_h + \theta_s}{R_{tot}} \quad \text{or} \quad R_{tot} = \frac{\theta_h + \theta_s}{q}$$

For conduction through the supporting beams of the heated island, and through the carbon nanotube, we may write

$$q = q_h + q_s = \frac{T_h - T_\infty}{R_{tot}} + \frac{T_h - T_s}{s/(k_{cn}A_{cn})} = \frac{\theta_h}{R_{tot}} + \frac{\theta_h - \theta_s}{s/(k_{cn}A_{cn})}$$

Substituting the expression for R_{tot} into the preceding equation, and rearranging the resulting expression yields

$$k_{cn} = \left[1 - \frac{\theta_h}{\theta_h + \theta_s}\right]\left[\frac{1}{\theta_h - \theta_s}\right]\frac{sq}{A_{cn}} = \left[1 - \frac{32.6\ \text{K}}{32.6\ \text{K} + 8.4\ \text{K}}\right]\left[\frac{1}{32.6\ \text{K} - 8.4\ \text{K}}\right]\frac{5\times10^{-6}\ \text{m}\times 11.3\times10^{-6}\ \text{W}}{1.54\times10^{-16}\ \text{m}^2}$$

$$= 3113\ \text{W/m}\cdot\text{K} \quad <$$

COMMENTS: (1) The analysis is simplified if the heating island temperature is known. (2) This solution is independent of the thermal resistance posed by the support beams, making the measured thermal conductivity of the carbon nanotube less susceptible to experimental error.