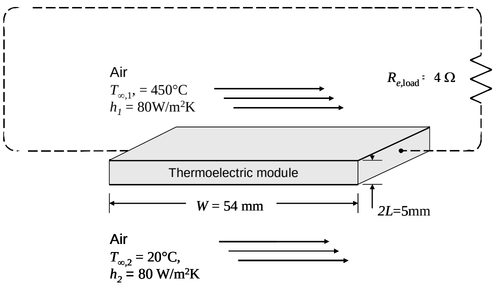
PROBLEM 3.169

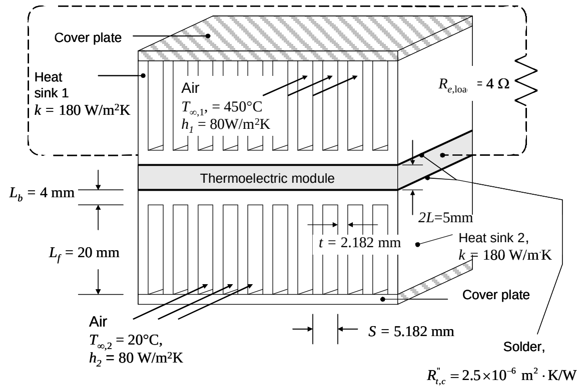
KNOWN: Dimensions of thermoelectric module and heat sinks. Convection conditions, heat sink thermal conductivity, thermoelectric module performance parameters, load electrical resistance. Contact resistance between thermoelectric module and heat sinks.

FIND: (a) Sketch of the equivalent thermal circuit and electrical power generated without the heat sinks. (b) Sketch of the equivalent thermal circuit and electrical power generated with the heat sinks.

SCHEMATIC:



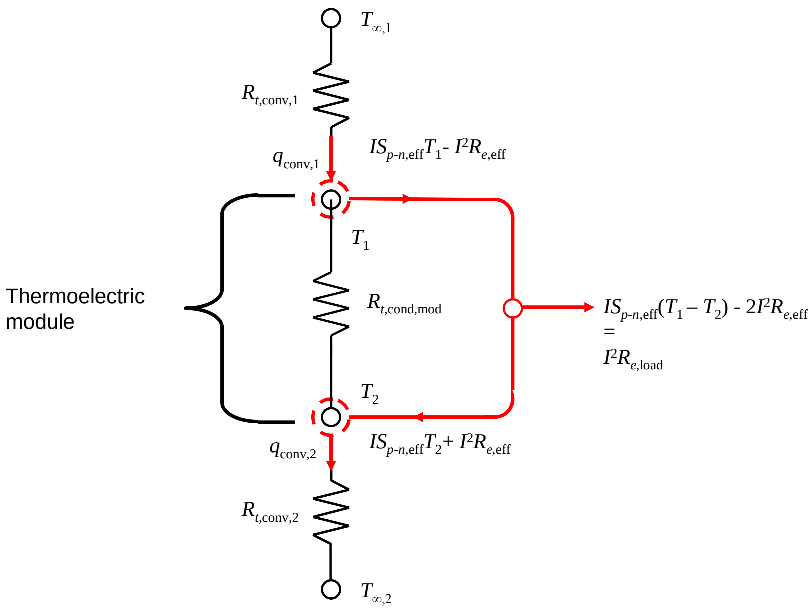
(a)



(b)

ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) Negligible radiation, (4) Adiabatic fin tips for part (b), (5) Convection coefficients same in parts (a) and (b) and the same on the sides of the fin arrays.

ANALYSIS: (a) Without the heat sinks, the equivalent thermal circuit is shown in Figure 3.24*b* as replicated below.



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PROBLEM 3.169 (Cont.)

The analysis can proceed as in Example 3.13. The conduction resistance of one module is the same as in the example, namely

$$R_{t,\text{cond,mod}} = \frac{L}{NA_{c,s}k_s} = \frac{2.5 \times 10^{-3}\ \text{m}}{100 \times 1.2 \times 10^{-5}\ \text{m}^2 \times 1.2\ \text{W/m}\cdot\text{K}} = 1.736\ \text{K/W}$$

From Equations 3.125 and 3.126,

$$q_1 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_1 - I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736\ \text{K/W}} + I \times 0.1435\ \text{V/K} \times T_1 - I^2 \times 4\ \Omega \qquad (1)$$

$$q_2 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_2 + I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736\ \text{K/W}} + I \times 0.1435\ \text{V/K} \times T_2 + I^2 \times 4\ \Omega \qquad (2)$$

Newton's law of cooling may be written at each surface as

$$q_1 = h_1W^2(T_{\infty,1} - T_1) = 80\ \text{W/m}^2\cdot\text{K} \times (0.054\ \text{m})^2 \times \left[(450 + 273)\ \text{K - } T_1\right] \qquad (3)$$

$$q_2 = h_2W^2(T_2 - T_{\infty,2}) = 80\ \text{W/m}^2\cdot\text{K} \times (0.054\ \text{m})^2 \times \left[T_2 - (20 + 273)\ \text{K}\right] \qquad (4)$$

The electric power produced by a single module, P_N, is equal to the electric power dissipated in the load resistance. Equating the expression for P_N from Equation 3.127 to the electric power dissipated in the load gives

$$P_N = IS_{p\text{-}n,\text{eff}}(T_1 - T_2) - 2I^2R_{e,\text{eff}} = I^2R_{e,\text{load}}$$

$$I \times 0.1435\ \text{V/K} \times (T_1 - T_2) - 2I^2 \times 4\ \Omega = I^2 \times 4\ \Omega \qquad (5)$$

Equations 1 through 5 may be solved simultaneously, for example using IHT, to yield I = 0.38 A, and

$$P_N = I^2R_{e,\text{load}} = (0.38\ \text{A})^2 \times 4\ \Omega = 0.59\ \text{W}$$ $<$

(b) The thermal circuit associated with the thermoelectric module is unchanged, but each convection resistance must be replaced with the total thermal resistance, R_{tot}, associated with the contact resistance, fin array base, and overall resistance of the fin array, as shown below. Also, $q_{\text{conv},1}$ and $q_{\text{conv},2}$ have been replaced with the more general terms q_1 and q_2.

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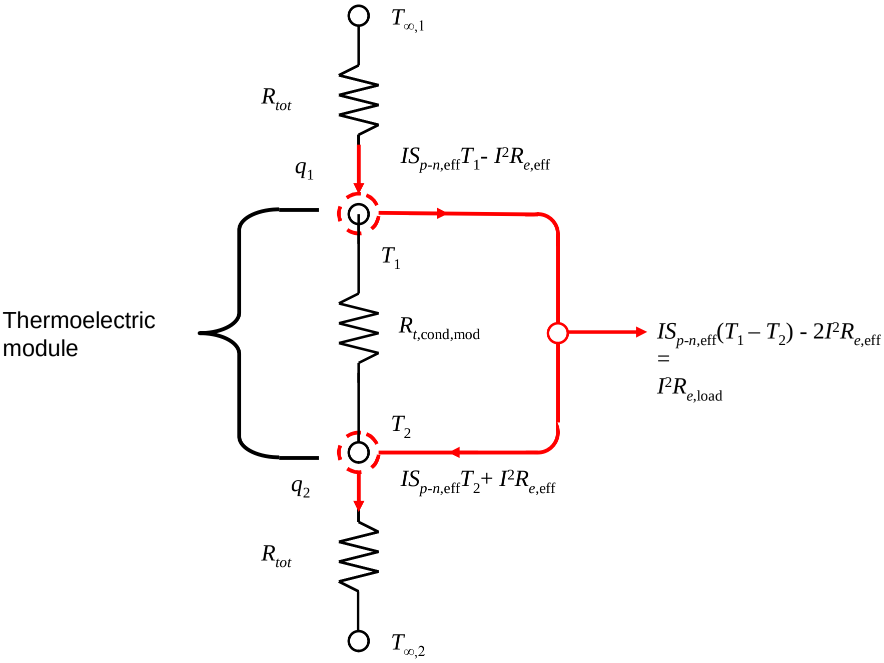
PROBLEM 3.169 (Cont.)



The total thermal resistance is given by

$$R_{\text{tot}} = \frac{R''_{t,c}}{W^2} + R_{\text{base}} + R_{t,o} = \frac{R''_{t,c}}{W^2} + \frac{L_b}{kW^2} + \frac{1}{\eta_o h A_t}$$

where Equation 3.108 has been used to express the overall fin resistance. From Equations 3.104, 3.107, and 3.94,

$$\eta_o A_t = A_b + NA_f\eta_f = (N-1)W(S-t) + 2NWL_f \frac{\tanh mL_f}{mL_f}$$

where

$$m = \sqrt{hP/kA_c} = \sqrt{2h/kt} = \sqrt{2 \times 80 \text{ W/m}^2 \cdot \text{K} / (180 \text{ W/m} \cdot \text{K} \times 0.002182 \text{ m})} = 20.2 \text{ m}^{-1}$$

Thus,

$$\eta_o A_t = 10 \times 0.054 \text{ m} \times 0.003 \text{ m} + 2 \times 11 \times 0.054 \text{ m} \times 0.020 \text{ m} \frac{\tanh(20.2 \text{ m}^{-1} \times 0.02 \text{ m})}{20.2 \text{ m}^{-1} \times 0.02 \text{ m}} = 0.0242 \text{ m}^2$$

The total thermal resistance is then

$$R_{\text{tot}} = \frac{2.5 \times 10^{-6} \text{ K/W}}{(0.054 \text{ m})^2} + \frac{0.004 \text{ m}}{180 \text{ W/m} \cdot \text{K}(0.054 \text{ m})^2} + \frac{1}{0.0242 \text{ m}^2 \times 80 \text{ W/m}^2 \cdot \text{K}} = 0.526 \text{ K/W}$$

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PROBLEM 3.169 (Cont.)

The system of equations from part (a) applies here, except that Equations 3 and 4 are replaced by the revised versions

$$q_1 = \frac{T_{\infty,1} - T_1}{R_{\text{tot}}} = \frac{(450 + 273)\,\text{K} - T_1}{0.526\ \text{K/W}} \tag{3r}$$

$$q_2 = \frac{T_2 - T_{\infty,2}}{R_{\text{tot}}} = \frac{T_2 - (20 + 273)\,\text{K}}{0.526\ \text{K/W}} \tag{4r}$$

Equations 1, 2, 3r, 4r, and 5 may be solved simultaneously to yield I = 2.04 A, and

$$P_N = I^2 R_{e,\text{load}} = (2.04\ \text{A})^2 \times 4\ \Omega = 16.7\ \text{W} \qquad <$$

This is 28 times larger than the result of part (a). $<$

COMMENTS: By adding the two heat sinks to the thermoelectric module, the power produced by the module increases by a factor of 28. Not only does the power depend on the semiconductor properties of the thermoelectric material, but also strongly on the thermal management of the module through, as in this problem, addition of heat sinks.