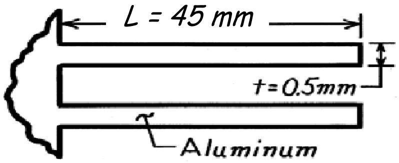
PROBLEM 3.155

KNOWN: Dimensions and number of rectangular aluminum fins. Convection coefficient with and without fins.

FIND: Percentage increase in heat transfer resulting from use of fins.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Negligible fin contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1,* Aluminum, pure: k ≈ 240 W/m·K.

ANALYSIS: Evaluate the fin parameters

$$L_c = L + t/2 = 0.04525\text{m}$$

$$A_p = L_c t = 0.04525\text{m} \times 0.5 \times 10^{-3}\text{m} = 22.63 \times 10^{-6}\ \text{m}^2$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = \left(0.04525\text{m}\right)^{3/2} \left[\frac{30\ \text{W/m}^2\cdot\text{K}}{240\ \text{W/m}\cdot\text{K} \times 22.63 \times 10^{-6}\text{m}^2}\right]^{1/2}$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = 0.715$$

It follows from Fig. 3.19 that $\eta_f \approx 0.75$. Hence,

$$q_f = \eta_f q_{max} = 0.75\ h_w 2wL\ \theta_b$$

$$q_f = 0.75 \times 30\ \text{W/m}^2\cdot\text{K} \times 2 \times 0.05\text{m} \times \left(w\ \theta_b\right) = 2.25\ \text{W/m}\cdot\text{K}\left(w\ \theta_b\right)$$

With the fins, the heat transfer from the walls is

$$q_w = N\ q_f + \left(1 - Nt\right) w\ h_w\ \theta_b$$

$$q_w = 250 \times 2.25 \frac{\text{W}}{\text{m}\cdot\text{K}}\left(w\ \theta_b\right) + \left(1\text{m} - 250 \times 5 \times 10^{-4}\text{m}\right) \times 30\ \text{W/m}^2\cdot\text{K}\ \left(w\ \theta_b\right)$$

$$q_w = \left(563 + 26.3\right)\frac{\text{W}}{\text{m}\cdot\text{K}}\left(w\ \theta_b\right) = 589\ w\ \theta_b.$$

Without the fins, $q_{wo} = h_{wo}\ 1\text{m} \times w\ \theta_b = 45\ w\ \theta_b$. Hence the percentage increase in heat transfer is

$$\frac{q_w - q_{wo}}{q_{wo}} = \frac{\left(589 - 45\right) w\ \theta_b}{45\ w\ \theta_b} = 12.09 = 1200\% \quad <$$

COMMENTS: If the infinite fin approximation is made, it follows that $q_f = (hPkA_c)^{1/2}\ \theta_b = [h_w 2wkwt]^{1/2}\ \theta_b = (30 \times 2 \times 240 \times 5\times10^{-4})^{1/2}\ w\ \theta_b = 2.68\ w\ \theta_b$. Hence, q_f is overestimated.