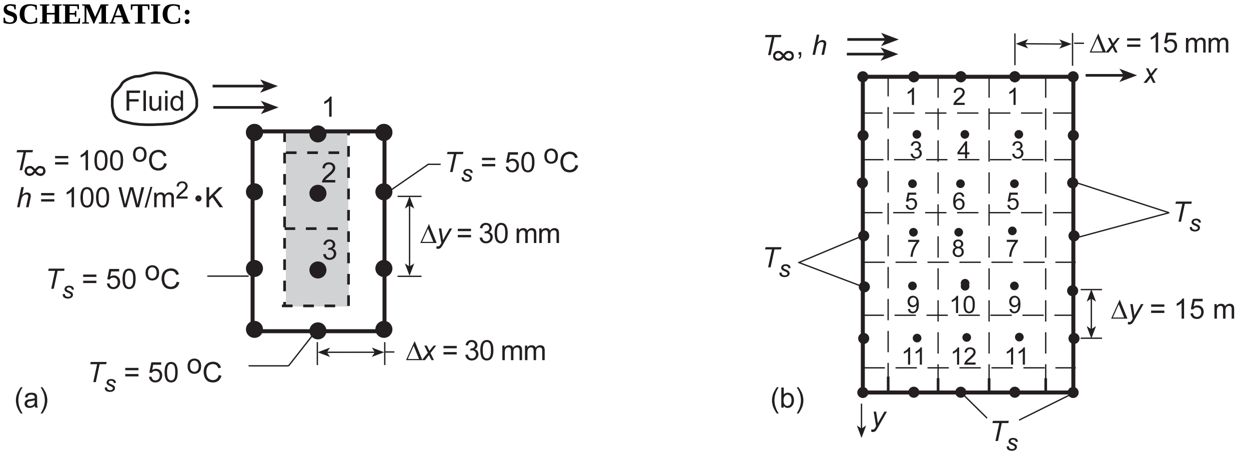
PROBLEM 4.84

KNOWN: Long rectangular bar having one boundary exposed to a convection process (T_∞, h) while the other boundaries are maintained at a constant temperature (T_s).

FIND: (a) Using a grid spacing of 30 mm and the Gauss-Seidel method, determine the nodal temperatures and the heat rate per unit length into the bar from the fluid, (b) Effect of grid spacing and convection coefficient on the temperature field.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) With the grid spacing Δx = Δy = 30 mm, three nodes are created. Using the finite-difference equations as shown in Table 4.2, but written in the form required of the Gauss-Seidel method (see Appendix D), and with $Bi = h\Delta x/k = 100\ W/m^2\cdot K \times 0.030\ m/1\ W/m\cdot K = 3$, we obtain:

Node 1: $$T_1 = \frac{1}{(Bi+2)}\left(T_2 + T_s + BiT_\infty\right) = \frac{1}{5}\left(T_2 + 50 + 3\times 100\right) = \frac{1}{5}\left(T_2 + 350\right) \quad (1)$$

Node 2: $$T_2 = \frac{1}{4}\left(T_1 + 2T_s + T_3\right) = \frac{1}{4}\left(T_1 + T_3 + 2\times 50\right) = \frac{1}{4}\left(T_1 + T_3 + 100\right) \quad (2)$$

Node 3: $$T_3 = \frac{1}{4}\left(T_2 + 3T_s\right) = \frac{1}{4}\left(T_2 + 3\times 50\right) = \frac{1}{4}\left(T_2 + 150\right) \quad (3)$$

Denoting each nodal temperature with a superscript to indicate iteration step, e.g. T_1^k, calculate values as shown below.

k	T_1	T_2	T_3 (°C)	
0	85	60	55	← initial guess
1	82.00	59.25	52.31	
2	81.85	58.54	52.14	
3	81.71	58.46	52.12	
4	81.69	58.45	52.11	

By the 4th iteration, changes are of order 0.02°C, suggesting that further calculations may not be necessary.

Continued...

PROBLEM 4.84 (Cont.)

In finite-difference form, the heat rate from the fluid to the bar is

$$q'_{conv} = h(\Delta x/2)(T_\infty - T_s) + h\Delta x(T_\infty - T_1) + h(\Delta x/2)(T_\infty - T_s)$$

$$q'_{conv} = h\Delta x(T_\infty - T_s) + h\Delta x(T_\infty - T_1) = h\Delta x\left[(T_\infty - T_s) + (T_\infty - T_1)\right]$$

$$q'_{conv} = 100\,W/m^2 \cdot K \times 0.030\,m\left[(100-50)+(100-81.7)\right]^\circ C = 205\,W/m\,. \qquad <$$

(b) Using the *Finite-Difference Equations* option from the *Tools* portion of the IHT menu, the following two-dimensional temperature field was computed for the grid shown in schematic (b), where x and y are in mm and the temperatures are in °C.

y\x	0	15	30	45	60
0	50	80.33	85.16	80.33	50
15	50	63.58	67.73	63.58	50
30	50	56.27	58.58	56.27	50
45	50	52.91	54.07	52.91	50
60	50	51.32	51.86	51.32	50
75	50	50.51	50.72	50.51	50
90	50	50	50	50	50

The improved prediction of the temperature field has a significant influence on the heat rate, where, accounting for the symmetrical conditions,

$$q' = 2h(\Delta x/2)(T_\infty - T_s) + 2h(\Delta x)(T_\infty - T_1) + h(\Delta x)(T_\infty - T_2)$$

$$q' = h(\Delta x)\left[(T_\infty - T_s) + 2(T_\infty - T_1) + (T_\infty - T_2)\right]$$

$$q' = 100\,W/m^2 \cdot K(0.015\,m)\left[50 + 2(19.67) + 14.84\right]^\circ C = 156.3\,W/m \qquad <$$

Additional improvements in accuracy could be obtained by reducing the grid spacing to 5 mm, although the requisite number of finite-difference equations would increase from 12 to 108, significantly increasing problem *set-up* time.

An increase in h would increase temperatures everywhere within the bar, particularly at the heated surface, as well as the rate of heat transfer by convection to the surface.

COMMENTS: (1) Using the matrix-inversion method, the exact solution to the system of equations (1, 2, 3) of part (a) is T_1 = 81.70°C, T_2 = 58.44°C, and T_3 = 52.12°C. The fact that only 4 iterations were required to obtain agreement within 0.01°C is due to the close initial guesses.

(2) Note that the rate of heat transfer by convection to the top surface of the rod must balance the rate of heat transfer by conduction to the sides and bottom of the rod.