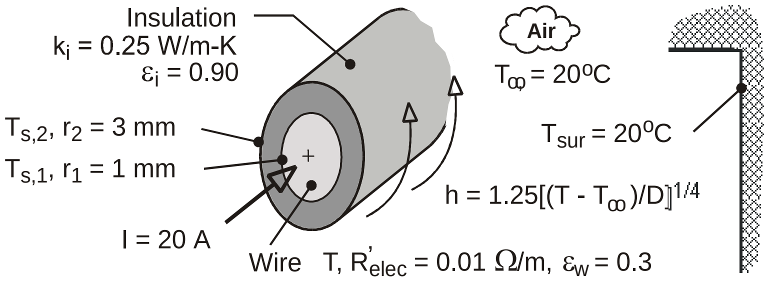
PROBLEM 3.53

KNOWN: Electric current and resistance of wire. Wire diameter and emissivity. Thickness, emissivity and thermal conductivity of coating. Temperature of ambient air and surroundings. Expression for heat transfer coefficient at surface of the wire or coating.

FIND: (a) Heat generation per unit length and volume of wire, (b) Temperature of uninsulated wire, (c) Inner and outer surface temperatures of insulation, including the effect of insulation thickness.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible contact resistance between insulation and wire, (5) Negligible radial temperature gradients in wire, (6) Large surroundings.

ANALYSIS: (a) The rates of energy generation per unit length and volume are, respectively,

$$\dot{E}'_g = I^2 R'_{elec} = (20\,A)^2 (0.01\Omega/m) = 4\ W/m \qquad <$$

$$\dot{q} = \dot{E}'_g / A_c = 4\dot{E}'_g / \pi D^2 = 16\,W/m / \pi(0.002m)^2 = 1.27\times10^6\ W/m^3 \qquad <$$

(b) Without the insulation, an energy balance at the surface of the wire yields

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h (T - T_\infty) + \pi D \varepsilon_W \sigma \left(T^4 - T_{sur}^4\right)$$

where $h = 1.25[(T - T_\infty)/D]^{1/4}$. Substituting,

$$4\,W/m = 1.25\pi(0.002m)^{3/4}(T-293)^{5/4} + \pi(0.002m)0.3\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T^4 - 293^4\right)K^4$$

and a trial-and-error solution yields

$$T = 331\,K = 58°C \qquad <$$

(c) Performing an energy balance at the outer surface,

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h \left(T_{s,2} - T_\infty\right) + \pi D \varepsilon_i \sigma \left(T_{s,2}^4 - T_{sur}^4\right)$$

$$4\,W/m = 1.25\pi(0.006m)^{3/4}\left(T_{s,2}-293\right)^{5/4} + \pi(0.006m)0.9\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_{s,2}^4 - 293^4\right)K^4$$

and an iterative solution yields the following value of the surface temperature

$$T_{s,2} = 307.8\,K = 34.8°C \qquad <$$

The inner surface temperature may then be obtained from the following expression for heat transfer by conduction in the insulation.

Continued …

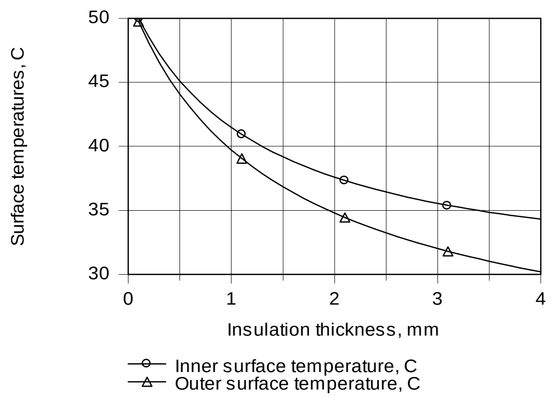
PROBLEM 3.53 (Cont.)

$$q' = \frac{T_{s,i} - T_{s,2}}{R'_{cond}} = \frac{T_{s,i} - T_{s,2}}{\ell n\left(r_2 / r_1\right)/2\pi k_i}$$

$$4\,W = \frac{2\pi\left(0.25\,W/m\cdot K\right)\left(T_{s,i} - 307.8\,K\right)}{\ell n\,(3)}$$

$$T_{s,i} = 310.6\,K = 37.6°C$$ <

As shown below, the effect of increasing the insulation thickness is to *reduce*, not increase, the surface temperatures.



This behavior is due to a reduction in the total resistance to heat transfer with increasing r_2. Although the convection, h, and radiation, $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$, coefficients decrease with increasing r_2, the corresponding increase in the surface area is more than sufficient to provide for a reduction in the total resistance. Even for an insulation thickness of t = 4 mm, h = h + h_r = (7.1 + 5.4) $W/m^2\cdot K$ = 12.5 $W/m^2\cdot K$, and r_{cr} = k/h = 0.25 W/m·K/12.5 $W/m^2\cdot K$ = 0.020m = 20 mm > r_2 = 5 mm. The outer radius of the insulation is therefore well below the critical radius.