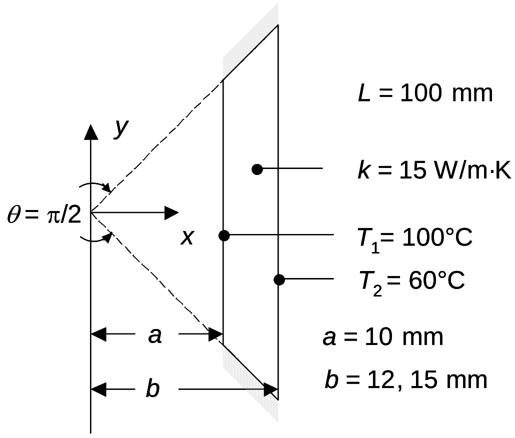
PROBLEM 4.12

KNOWN: Dimensions of a two-dimensional object, applied boundary conditions and thermal conductivity.

FIND: (a) Shape factor for the object if the dimensions are a = 10 mm, b = 12 mm. (b) Shape factor for a = 10 mm, b = 15 mm. (c) Shape factor for cases (a) and (b) using the alternative conduction analysis (d) For T_1 = 100°C and T_2 = 60°C, determine the heat transfer rate per unit depth for k = 15 W/m·K for cases (a) and (b).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

PROPERTIES: Given: k = 15 W/m·K.

ANALYSIS: (a) The geometry and applied boundary conditions correspond to Case 11 of Table 4.1(a). Noting that the diagonals of the square channel of Case 11 are adiabats, the shape factor for $b/a = W/w = 12/10 = 1.2$ is one-fourth the shape factor given in Table 4.1(a),

$$S = 0.25 \times \frac{2\pi L}{0.785 \ln(W/w)} = \frac{2.00L}{\ln(W/w)} = \frac{2.00 \times 0.10\text{m}}{\ln(1.2)} = 1.097 \qquad <$$

(b) For $b/a = W/w = 15/10 = 1.5$,

$$S = 0.25 \times \frac{2\pi L}{0.930 \ln(W/w) - 0.050} = \frac{1.69L}{\ln(W/w) - 0.0534} = \frac{1.69 \times 0.10\text{m}}{\ln(1.5) - 0.0534} = 0.480 \qquad <$$

(c) From the one-dimensional alternative conduction analysis with the top surface described by $y = x$ and $A = 2yL$,

$$q_x = -kA\frac{dT}{dx} = -2kLx\frac{dT}{dx}$$

Separating variables and integrating yields

$$\int_{x=a}^{b} \frac{dx}{x} = -\frac{2kL}{q_x}\int_{T_1}^{T_2} dT \quad \text{or} \quad \ln(b/a) = \frac{2kL}{q_x}(T_1 - T_2)$$

Hence, $S_{1\text{-D}} = 2L/[\ln(b/a)]$.

Continued…

PROBLEM 4.12 (Cont.)

For *b*/*a* = 1.2, $S_{1\text{-D}}$ = 2 × 0.1m/[ln(1.2)] = 1.097. For *b*/*a* = 1.5, $S_{1\text{-D}}$ = 2 × 0.1m/[ln(1.5)] = 0.493. <

As *b*/*a* becomes small, the influence of the lateral edges is diminished and one-dimensional conditions are approached. Hence, the shape factor estimated using the one-dimensional alternative conduction solution is nearly the same as for the two-dimensional shape factor for *b*/*a* = 1.2. As *b*/*a* is increased, the lateral edge effects become more important, and the shape factors obtained by the two methods begin to diverge in value. As two-dimensional conduction in the object becomes more pronounced, the heat transfer rate is decreased relative to that associated with the assumed one-dimensional conditions.

(d) For *b*/*a* = 1.2, the heat transfer rate is

$$q = Sk(T_1 - T_2) = 1.097 \times 15\ \mathrm{W/m \cdot K} \times (100 - 60)^{\circ}\mathrm{C} = 658\ \mathrm{W} \qquad <$$

for *b/a* = 1.5, the heat transfer rate is

$$q = Sk(T_1 - T_2) = 0.480 \times 15\ \mathrm{W/m \cdot K} \times (100 - 60)^{\circ}\mathrm{C} = 288\ \mathrm{W} \qquad <$$

COMMENTS: The heat transfer rate is independent of the individual values of *a* or *b*. As either *b* or *a* is increased while maintaining a fixed *b*/*a* ratio, the cross-sectional area for heat transfer increases, but the increase is offset by increased thickness through which the conduction occurs. The offsetting effects balance one another, and the net result is no change in the heat transfer rate.