PROBLEM 2.22

KNOWN: Ideal gas behavior for air, hydrogen and carbon dioxide.

FIND: The thermal conductivity of each gas at 300 K. Compare calculated values to values from Table A.4.

ASSUMPTIONS: (1) Ideal gas behavior.

PROPERTIES: Table A.4 (T = 300 K): Air; c_p = 1007 J/kg·K, k = 0.0263 W/m·K, Hydrogen; c_p = 14,310 J/kg·K, k = 0.183 W/m·K, Carbon dioxide; c_p = 851 J/kg·K, k = 0.0166 W/m·K. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol, $d = 0.372 \times 10^{-9}$ m, Hydrogen; $\mathcal{M}$ = 2.018 kg/kmol, $d = 0.274 \times 10^{-9}$ m, Carbon Dioxide; $\mathcal{M}$ = 44.01 kg/kmol, $d = 0.464 \times 10^{-9}$ m.

ANALYSIS: For <u>air</u>, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315 \text{ kJ/kmol}\cdot\text{K}}{28.97 \text{ kg/kmol}} = 0.287 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.007\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.720\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad \gamma = \frac{c_p}{c_v} = \frac{1.007}{0.720} = 1.399$$

From Equation 2.12

$$k = \frac{9\gamma - 5}{4}\frac{c_v}{\pi d^2}\sqrt{\frac{\mathcal{M}k_B T}{\mathcal{N}\pi}}$$

$$= \frac{9 \times 1.399 - 5}{4} \times \frac{720 \text{ J/kg}\cdot\text{K}}{\pi\left(0.372 \times 10^{-9}\text{m}\right)^2}\sqrt{\frac{28.97 \text{ kg/kmol} \times 1.381 \times 10^{-23} \text{ J/K} \times 300 \text{ K}}{\pi \times 6.024 \times 10^{23} \text{ mol}^{-1} \times 1000 \text{ mol/kmol}}}$$

$$= 0.025\frac{\text{W}}{\text{m}\cdot\text{K}} \qquad <$$

The thermal conductivity of air at T = 300 K is 0.0263 W/m·K. Hence, the computed value is within 5 % of the reported value.

For <u>hydrogen</u>, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315 \text{ kJ/kmol}\cdot\text{K}}{2.018 \text{ kg/kmol}} = 4.120 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 14.31\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 4.120\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 10.19\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad \gamma = \frac{c_p}{c_v} = \frac{14.31}{10.19} = 1.404$$

Equation 2.12 may be used to calculate

$$k = 0.173 \ \frac{\text{W}}{\text{m}\cdot\text{K}} \qquad <$$

Continued...

PROBLEM 2.22 (Cont.)

The thermal conductivity of hydrogen at T = 300 K is 0.183 W/m·K. Hence, the computed value is within 6 % of the reported value.

For carbon dioxide, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315 \text{ kJ/kmol}\cdot\text{K}}{44.01 \text{ kg/kmol}} = 0.189 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 0.851\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.189\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.662\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad \gamma = \frac{c_p}{c_v} = \frac{0.851}{0.662} = 1.285$$

Equation 2.12 may be used to calculate

$$k = 0.0158 \ \frac{\text{W}}{\text{m}\cdot\text{K}}$$ <

The thermal conductivity of carbon dioxide at T = 300 K is 0.0166 W/m·K. Hence, the computed value is within 5 % of the reported value.

COMMENTS: The preceding analysis may be used to *estimate* the thermal conductivity at various temperatures. However, the analysis is not valid for extreme temperatures or pressures. For example, (1) the thermal conductivity is predicted to be independent of the pressure of the gas. As pure vacuum conditions are approached, the thermal conductivity will suddenly drop to zero, and the preceding analysis is no longer valid. Also, (2) for temperatures considerably higher or lower than normally-encountered room temperatures, the agreement between the predicted and actual thermal conductivities can be poor. For example, for carbon dioxide at T = 600 K, the predicted thermal conductivity is k = 0.0223 W/m·K, while the actual (tabular) value is k = 0.0407 W/m·K. For extreme temperatures, thermal correction factors must be included in the predictions of the thermal conductivity.