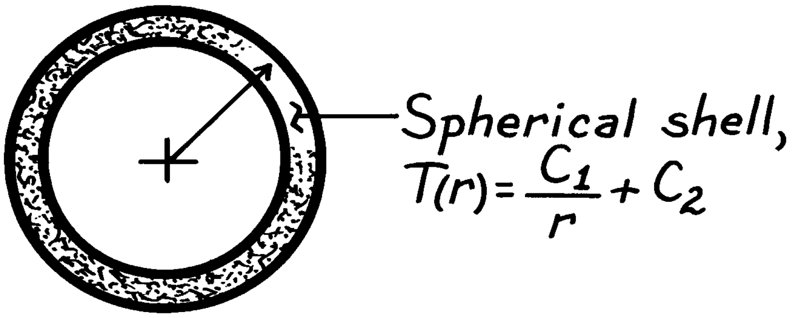
PROBLEM 2.51

KNOWN: Temperature distribution in a spherical shell.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.29, the heat equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{C_1}{r^2}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.28, the radial component of the heat flux is

$$q_r''=-k\frac{\partial T}{\partial r}=k\frac{C_1}{r^2}.$$

Hence, q_r'' decreases with increasing $r^2\left(q_r''\propto 1/r^2\right)$. <

At any radial location, the heat rate is

$$q_r=4\pi r^2 q_r''=4\pi k C_1.$$

Hence, q_r is independent of r. <

COMMENTS: The fact that q_r is independent of r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r^2.