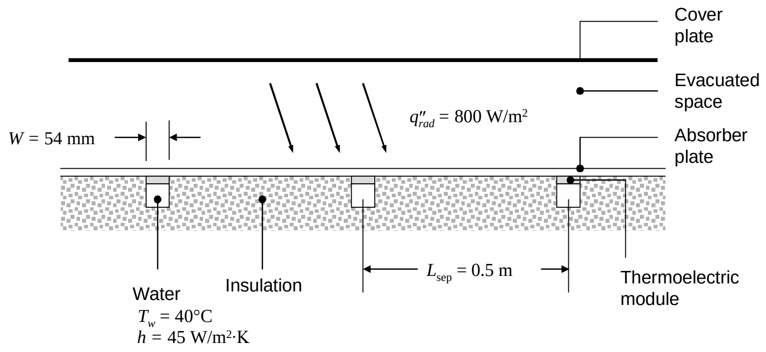
PROBLEM 3.172

KNOWN: Net radiation heat flux on absorber plate. Dimensions of thermoelectric modules, total number of modules, spacing of module rows. Thermoelectric module performance parameters, load electrical resistance. Water temperature and heat transfer coefficient.

FIND: Electrical power produced by one row of thermoelectric modules. Heat transfer rate to water.

SCHEMATIC:

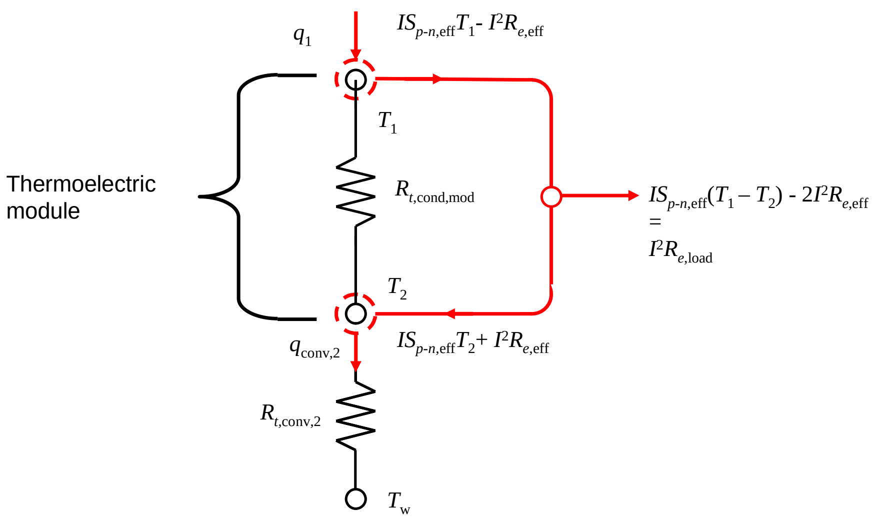


ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible losses through insulation, (4) Negligible losses by convection at absorber plate surface, (5) High thermal conductivity tube wall creates uniform temperature around the tube perimeter, (6) Tubes of square cross section. (7) Water temperature remains at 40°C.

ANALYSIS: The heat absorbed in the absorber plate is known, and under steady-state conditions all of this heat must conduct along the absorber plate and enter the thermoelectric modules, so that the heat associated with one module is given by

$$q_1 = q''_{\text{rad}} L_{\text{sep}} W = 800 \text{ W/m}^2 \times 0.5 \text{ m} \times 0.054 \text{ m} = 21.6 \text{ W}$$

The portion of the equivalent thermal circuit that describes the thermoelectric module is the same as shown in Figure 3.24*b*, see below. The low temperature side of the TEMs exchanges heat with the water through convection. Note that $q_{\text{conv},1}$ has been replaced with the more general term q_1.



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PROBLEM 3.172 (Cont.)

The analysis proceeds as in Example 3.13. The conduction resistance of one module is the same as in the example, namely

$$R_{t,\text{cond,mod}} = \frac{L}{NA_{c,s}k_s} = \frac{2.5 \times 10^{-3}\ \text{m}}{100 \times 1.2 \times 10^{-5}\ \text{m}^2 \times 1.2\ \text{W/m}\cdot\text{K}} = 1.736\ \text{K/W}$$

From Equations 3.125 and 3.126,

$$q_1 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_1 - I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736\ \text{K/W}} + I \times 0.1435\ \text{V/K} \times T_1 - I^2 \times 4\ \Omega \qquad (1)$$

$$q_2 = \frac{1}{R_{t,\text{cond,mod}}}(T_1 - T_2) + IS_{p\text{-}n,\text{eff}}T_2 + I^2R_{e,\text{eff}} = \frac{(T_1 - T_2)}{1.736\ \text{K/W}} + I \times 0.1435\ \text{V/K} \times T_2 + I^2 \times 4\ \Omega \qquad (2)$$

An additional relationship can be written by considering heat transfer by convection to the water. It is assumed that heat exiting the thermoelectric modules conducts around the perimeter of the square tube wall and enters the water uniformly over the entire tube wall area,

$$q_2 = 4hW^2(T_2 - T_w) = 4 \times 45\ \text{W/m}^2\cdot\text{K} \times (0.054\ \text{m})^2 \times (T_2 - (40 + 273)\ \text{K}) \qquad (3)$$

The electric power produced by all $M = 20$ modules, P_{tot}, is equal to the electric power dissipated in the load resistance. Making use of Equation 3.127, and equating the total electrical power generated in the M modules to the electric power dissipated in the load gives

$$P_{\text{tot}} = MP_N = I^2R_{e,\text{load}}$$

$$M\left[IS_{p\text{-}n,\text{eff}}(T_1 - T_2) - 2I^2R_{e,\text{eff}}\right] = I^2R_{e,\text{load}}$$

$$20\left[I \times 0.1435\ \text{V/K} \times (T_1 - T_2) - 2I^2 \times 4\ \Omega\right] = I^2 \times 60\ \Omega \qquad (4)$$

With q_1 known, Equations 1 through 4 can be solved for the four unknowns, T_1, T_2, I, and q_2. Then P_{tot} can be found from $P_{\text{tot}} = I^2R_{e,\text{load}}$. Solving the equations numerically using IHT yields

$T_1 = 371$ K, $T_2 = 354$ K, $I = 0.22$ A, $q_2 = 21.5$ W, $P_N = 0.15$ W

These are the values for a single module. For an entire row:

$$q_{1,\text{tot}} = 20q_1 = 432\ \text{W}$$
$$q_{2,\text{tot}} = 20q_2 = 429\ \text{W}$$
$$P_{\text{tot}} = 3\ \text{W}$$

Thus, each row of modules generates 3.0 W of electricity and supplies 429 W to heat water. <

COMMENTS: (1) This technology provides combined hot water and electricity generation and could potentially displace photovoltaics. If hot water is stored in a thermal energy storage unit, it can be used to generate electricity 24 hours per day, exploiting nighttime radiation loss to the cold sky. (2) The heat entering the water will cause the water temperature to increase along a row of modules. This would have to be accounted for in a more accurate analysis. (3) The electrical conversion efficiency is $P_{\text{tot}}/Mq_1 = 0.0069$. This efficiency can be improved significantly with careful thermal design. For example, doubling the tube spacing to $L_{\text{sep}} = 1$ m more than triples the electric power generated to $P_{\text{tot}} = 10.4$ W. Can you explain why?